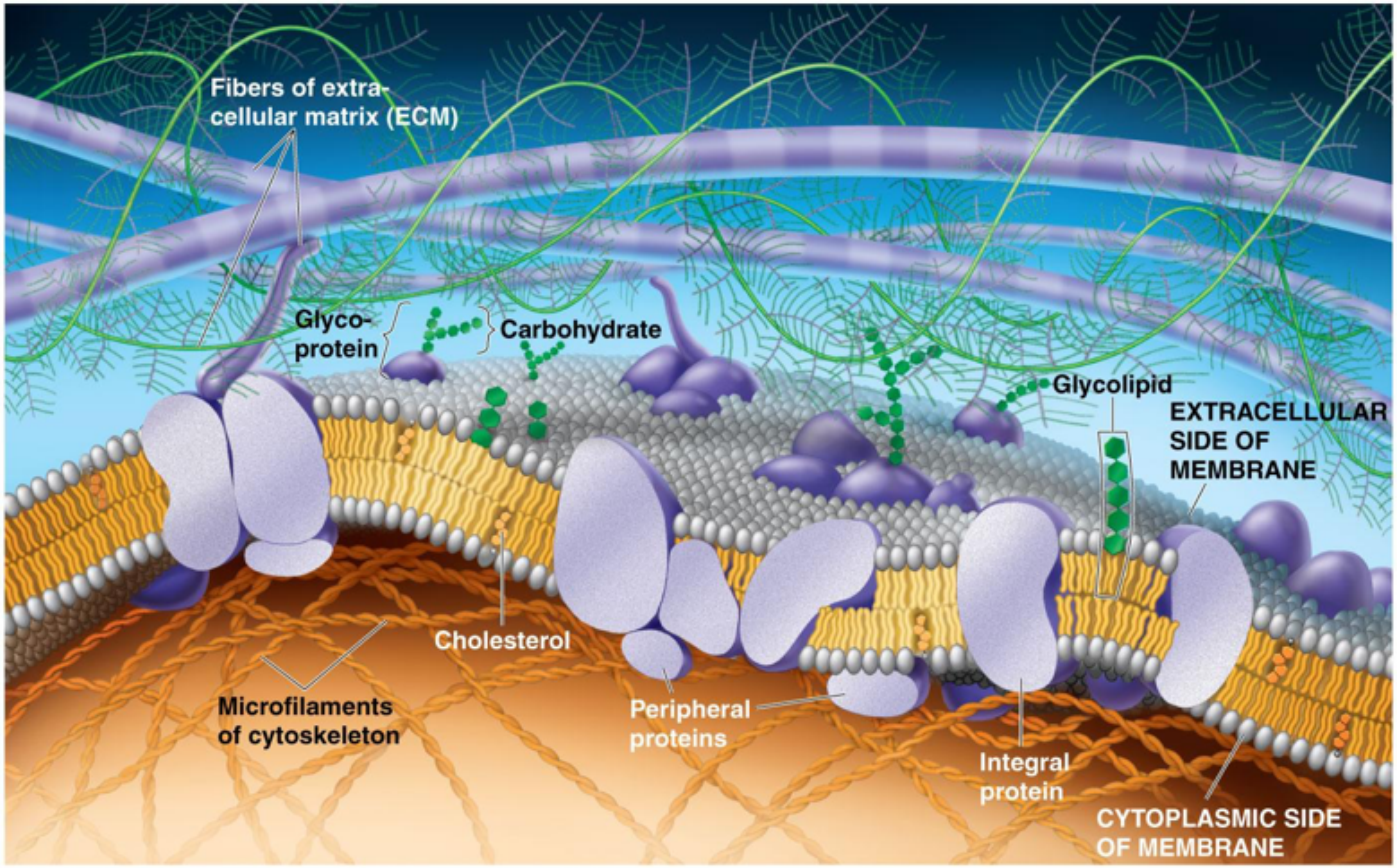
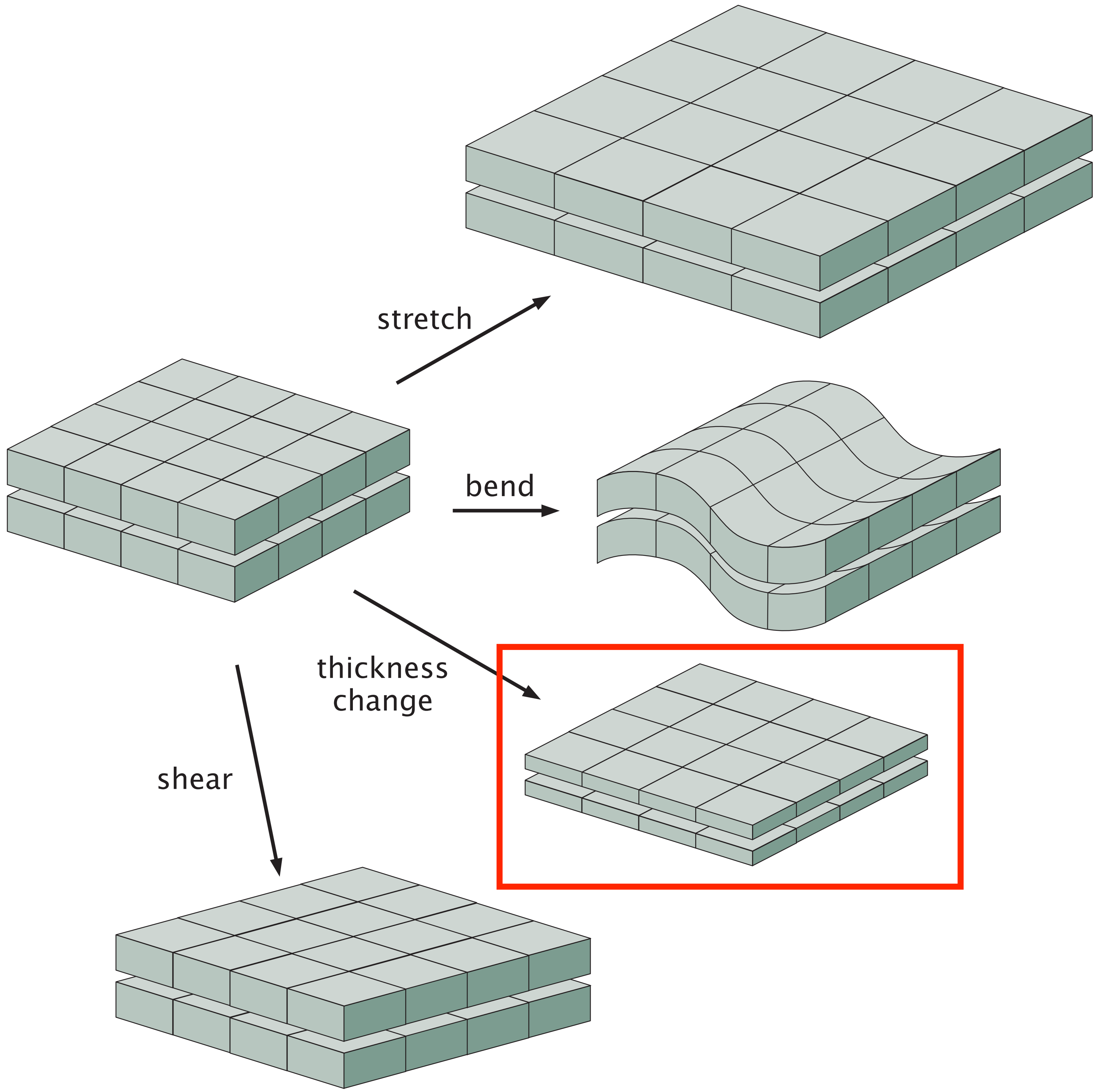
MAE 545: Lecture 17 (11/19)

Mechanics of cell membranes



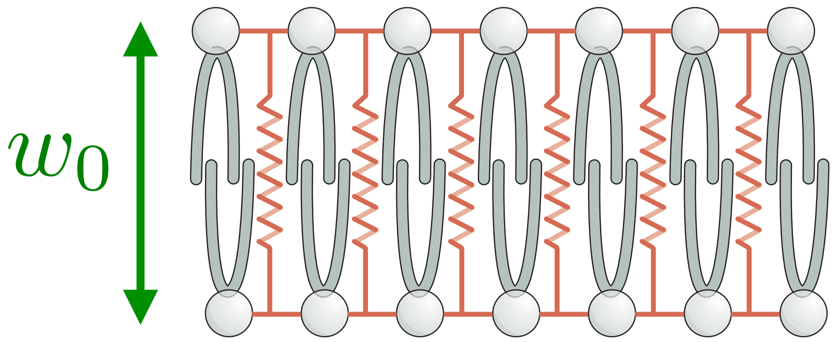
Membrane deformations



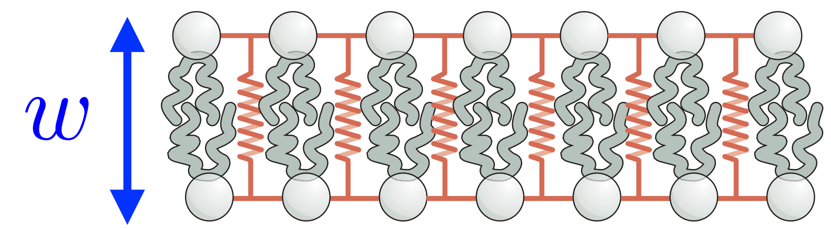
R. Phillips et al., Physical Biology of the Cell

Membrane thickness deformation

undeformed bilayer



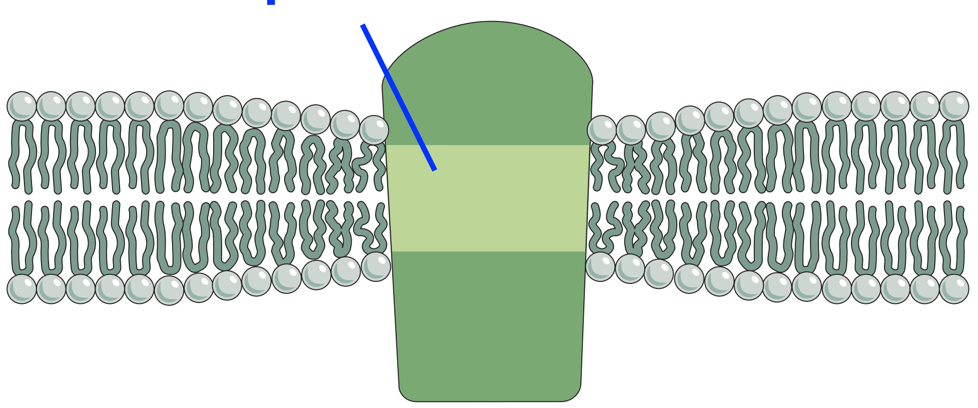
deformed bilayer



$$E_t = \frac{K_t}{2} \int \sqrt{g}\, dx^1 dx^2 \left(\frac{w(x^1, x^2) - w_0}{w_0} \right)^2$$

$$K_t \approx 60 k_B T/\mathrm{nm}^2$$

Membrane proteins can locally deform the thickness of lipid bilayer



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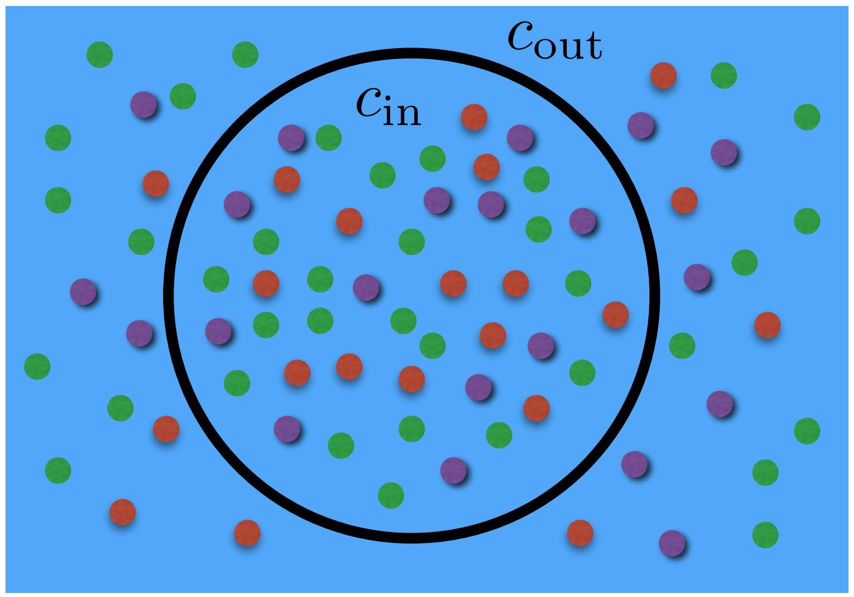
Osmotic pressure

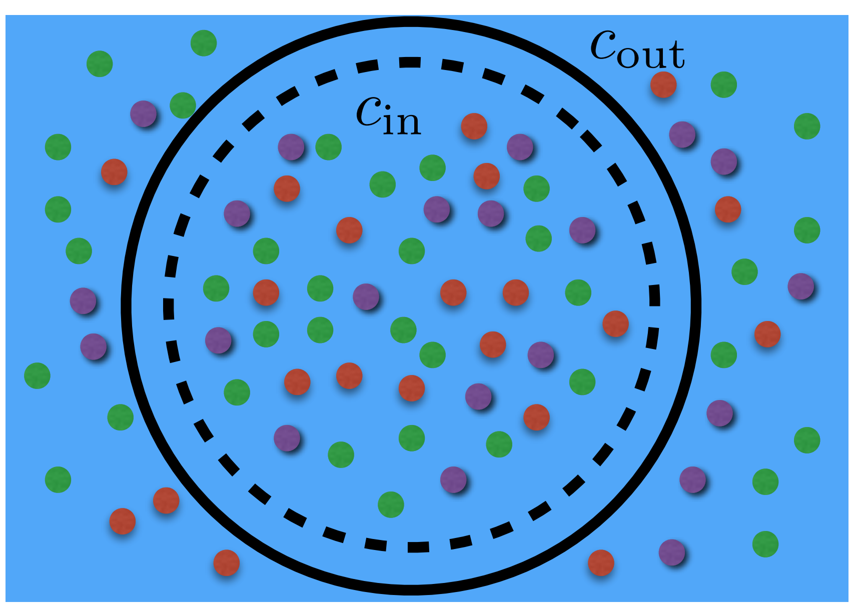
$$\Delta p = p_{\text{in}} - p_{\text{out}} = k_B T (c_{\text{in}} - c_{\text{out}})$$

$c_{\text{in}} > c_{\text{out}}$



Water flows in the cell until the mechanical equilibrium is reached.

$c_{\text{in}} > c_{\text{out}}$



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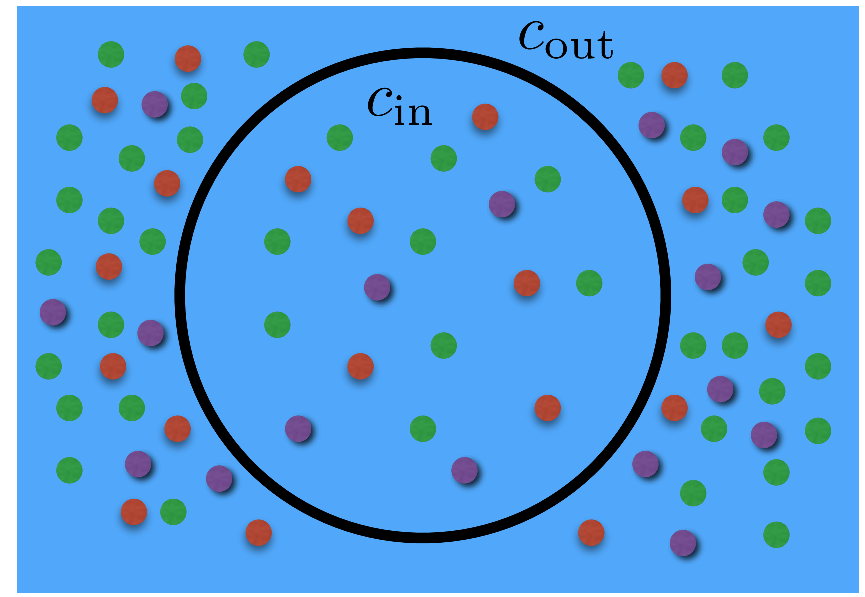
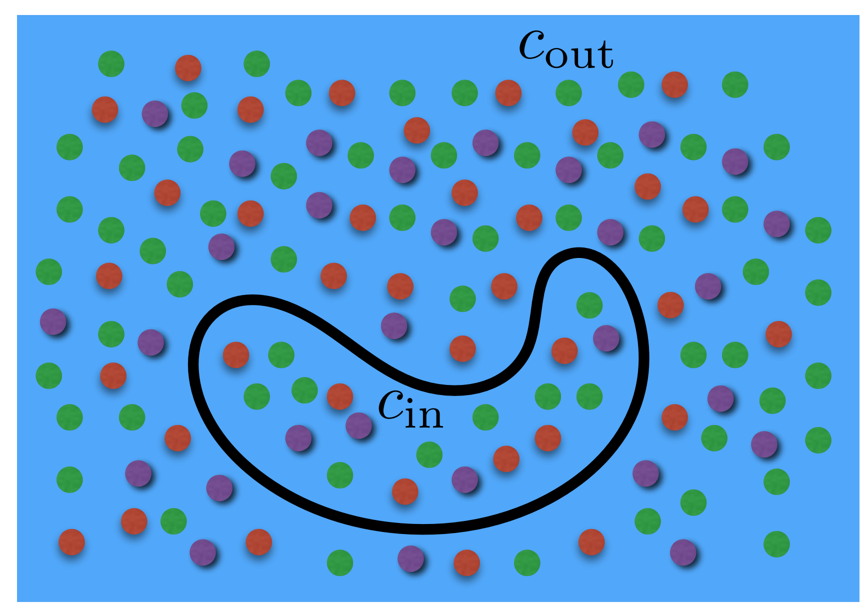


Water flows out of the cell until concentrations become equal.

$c_{\text{in}} = c_{\text{out}}$



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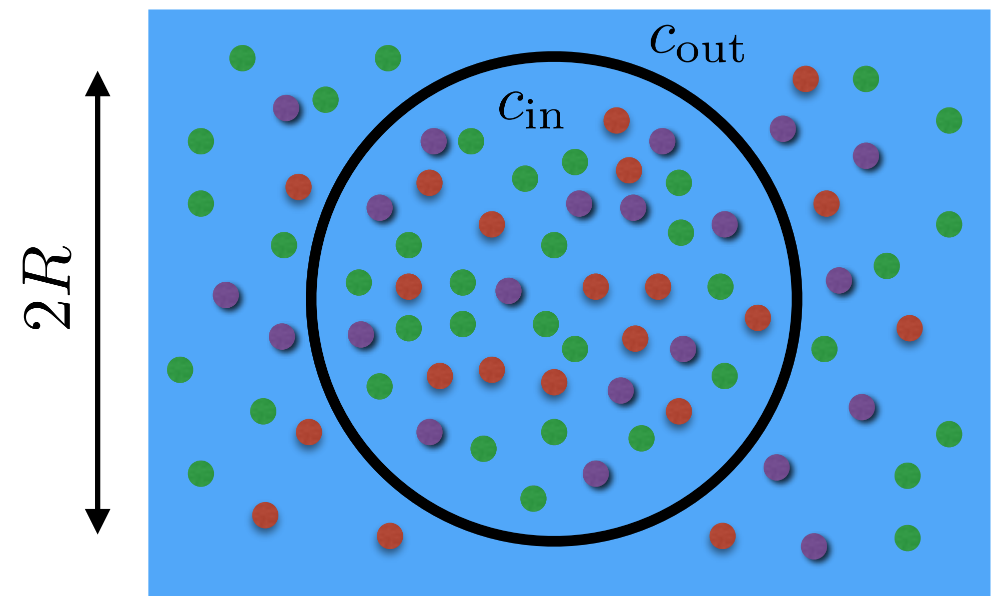
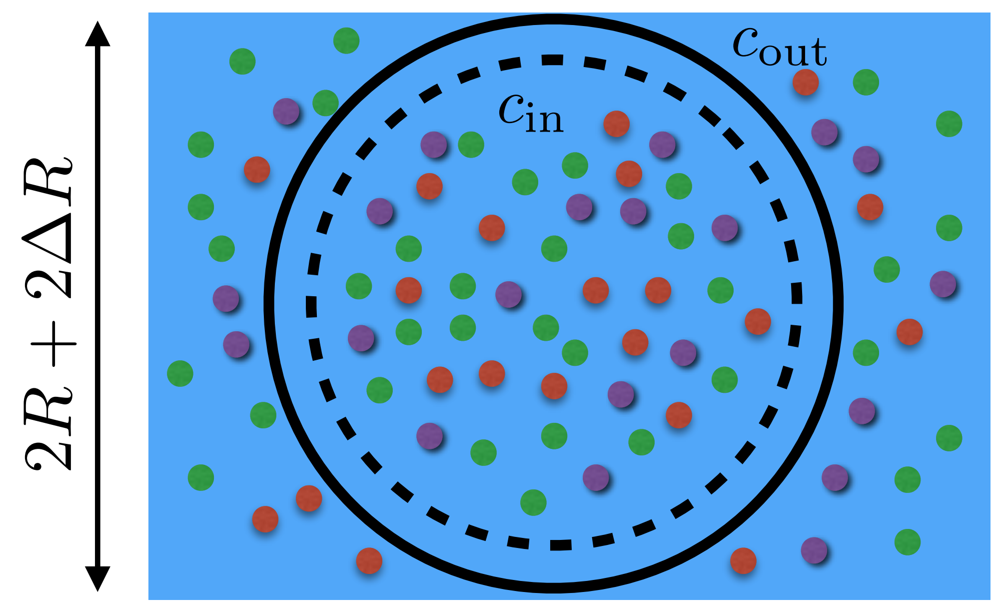


$A = 4\pi R^2$

$V = \frac{4\pi R^3}{3}$

Water flows in the cell until the mechanical equilibrium is reached.

$c_{\text{in}} > c_{\text{out}}$



$\Delta A = 8\pi R \Delta R$

$\Delta V = 4\pi R^2 \Delta R$

The radius of swollen cell can be estimated by minimizing the free energy.

$$E = A\frac{B}{2}\left(\frac{\Delta A}{A}\right)^2 - \Delta p \Delta V$$

$$E = 8\pi B \Delta R^2 - 4\pi R^2 \Delta p \Delta R$$

$$\frac{\Delta R}{R} = \frac{R\Delta p}{4B}$$

Membrane tension

$$\tau = B\frac{\Delta A}{A} = B\frac{2\Delta R}{R} = \frac{R\Delta p}{2}$$

(Young-Laplace equation)

$$\Delta p = \tau\,(1/R_1 + 1/R_2)$$

Osmotic pressure

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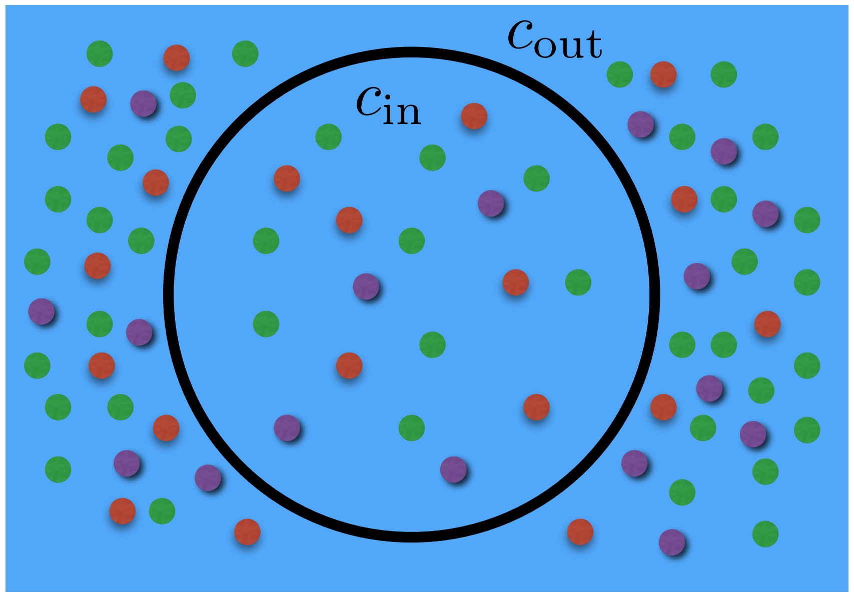
$c_{\text{in}} < c_{\text{out}}$



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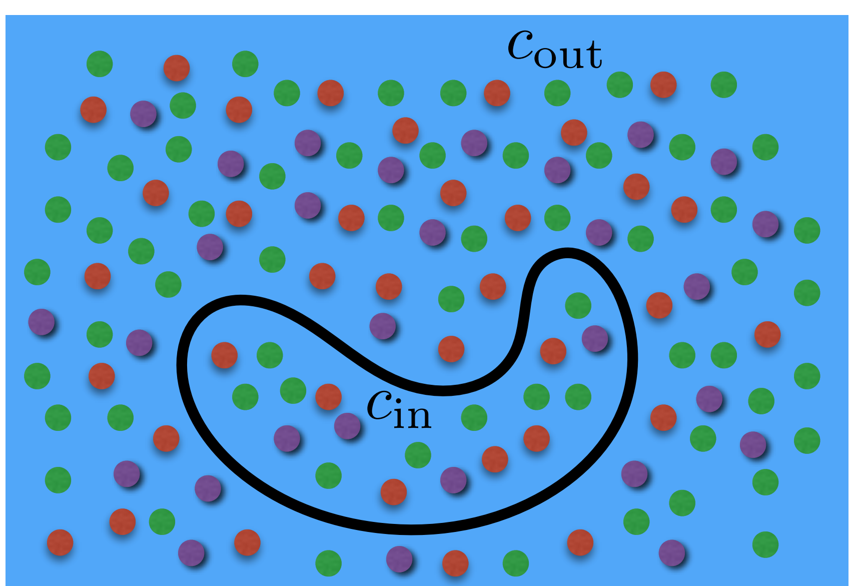


Total concentration of molecules inside a cell (vesicle)

$$c_{\text{in}} = \frac{N}{V}$$

Preferred cell (vesicle) volume

$$V_0 = \frac{N}{c_{\text{out}}}$$

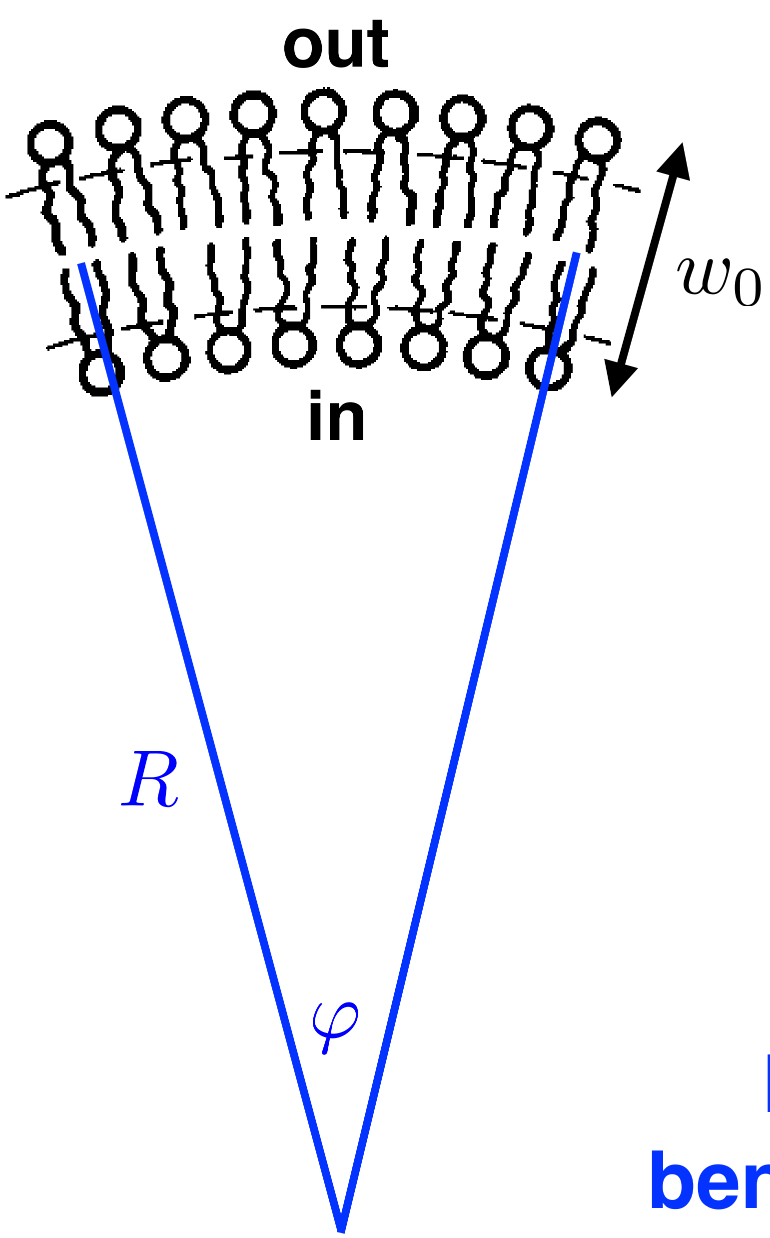
Energy cost for modifying the volume

$$E_v = -\int_{V_0}^{V} \Delta p(V) dV$$

$$E_v = -k_B T \left[N \ln\left(\frac{V}{V_0}\right) - c_{\text{out}} (V - V_0) \right]$$

$$E_v = \frac{1}{2} k_B T c_{\text{out}} V_0 \left(\frac{V - V_0}{V_0}\right)^2$$

Area difference between lipid layers



Length difference for 2D example on the left

$$\Delta\ell = \ell_{\text{out}} - \ell_{\text{in}} = (R + w_0/2)\varphi - (R - w_0/2)\varphi$$

$$\Delta\ell = w_0\varphi = \frac{w_0\ell}{R}$$

Area difference between lipid layers in 3D

$$\Delta A = A_{\text{out}} - A_{\text{in}} = w_0 \int dA \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Lipids can move within a given layer, but flipping between layers is unlikely. This sets a preferred area difference ΔA_0 .

Non-local bending energy

$$E = \frac{k_r}{2Aw_0^2}\left(\Delta A - \Delta A_0\right)^2$$

$$k_r \approx 3\kappa \approx 60 k_B T$$