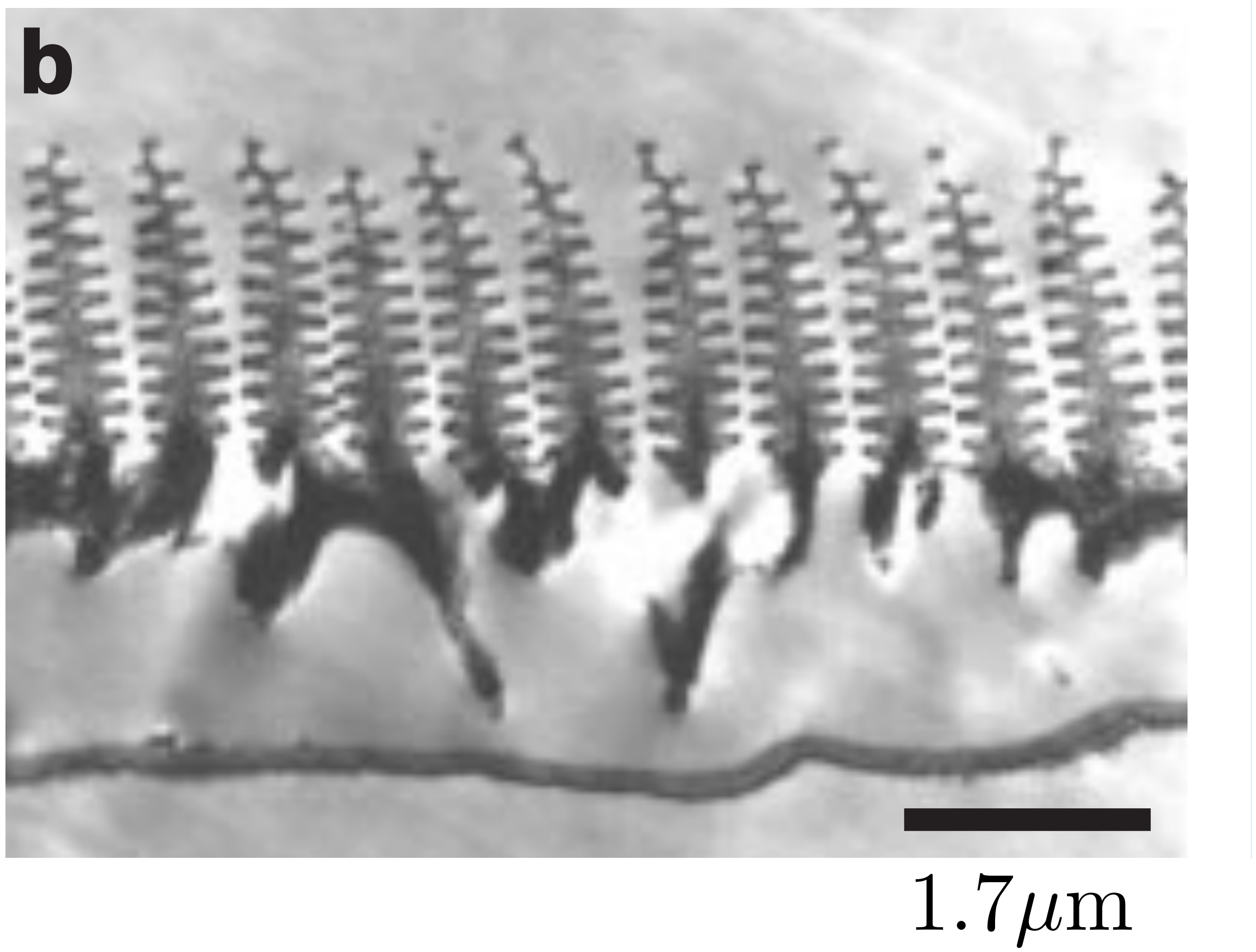
MAE 545: Lecture 2 (2/9)

Structural colors

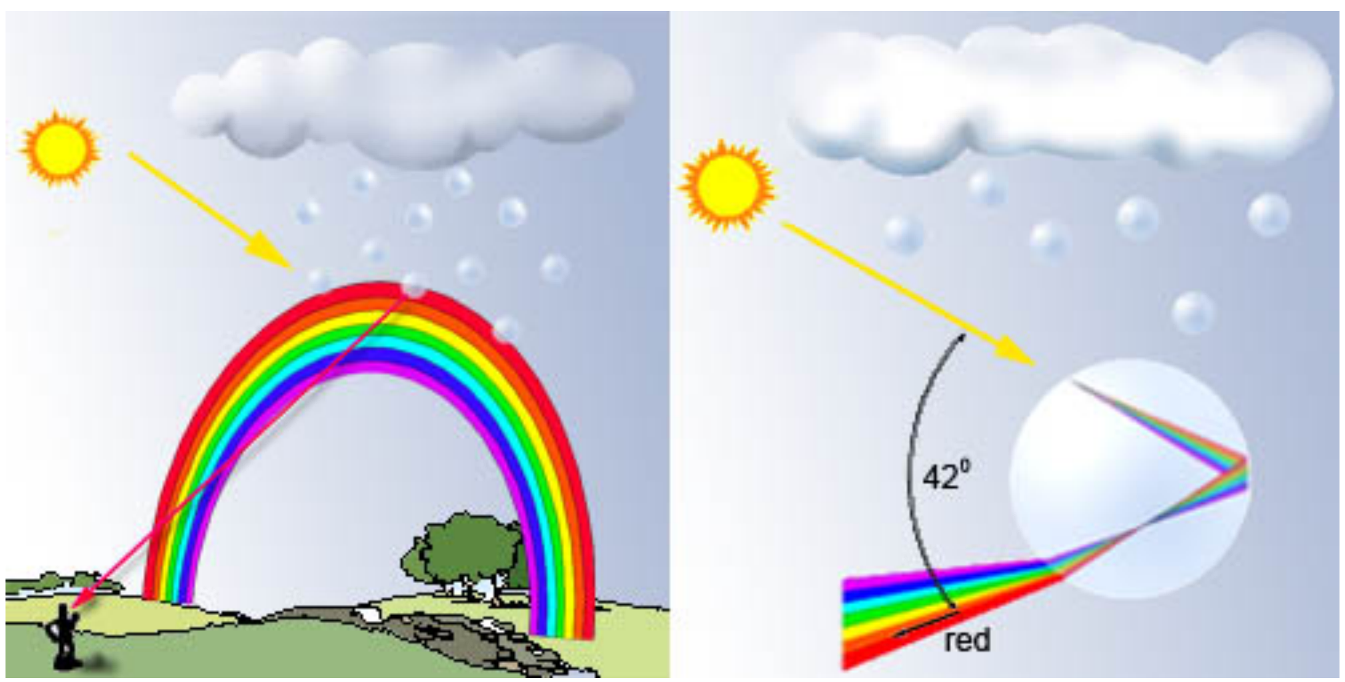


Structural color

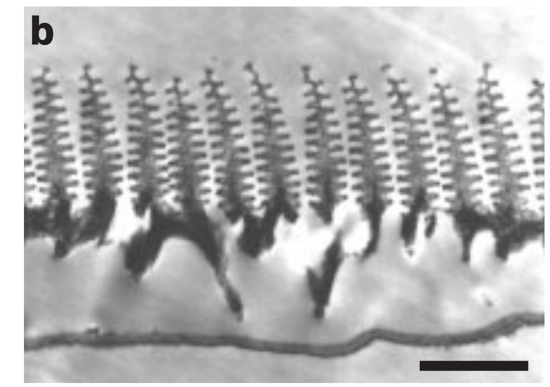
Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.

White light coming from the sun consists of all colors.

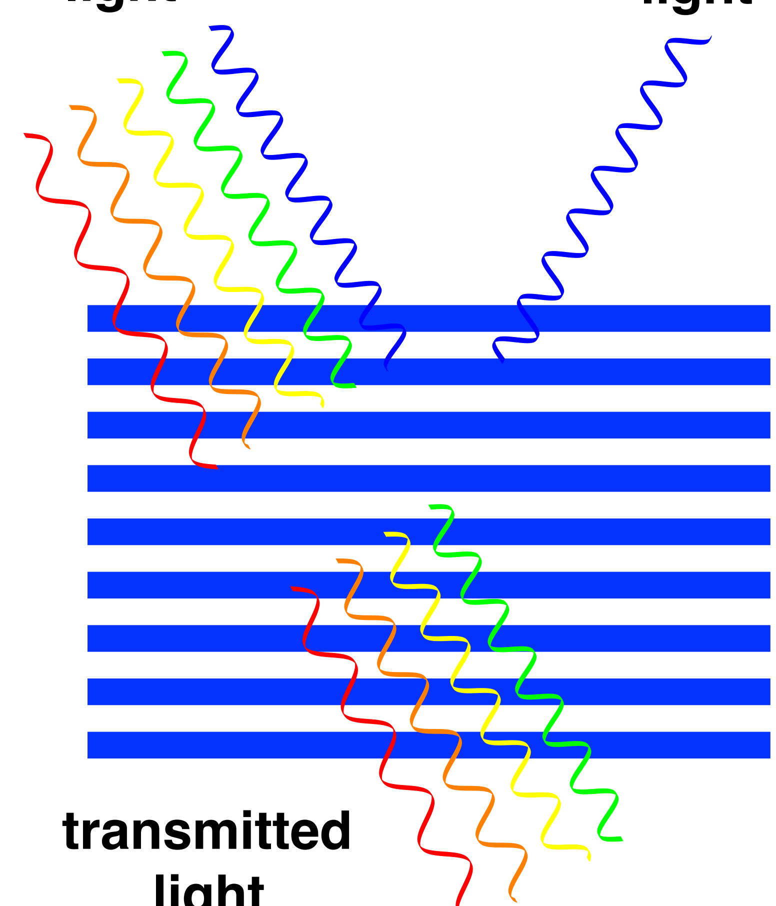
rainbow



structural color



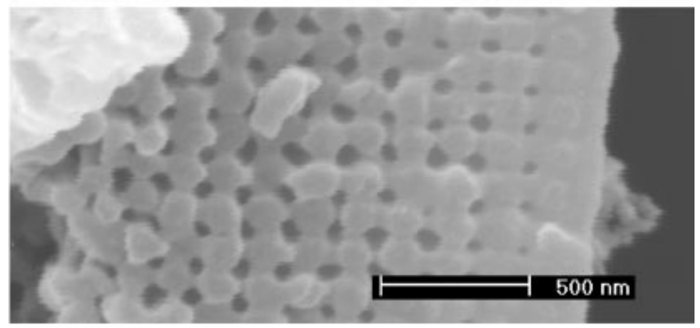
$1.7\mu m$



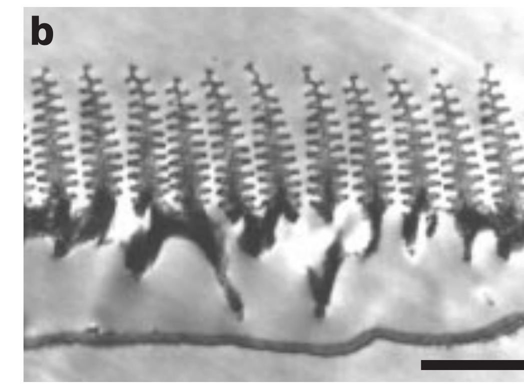
Structural colors

Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.

Peacock feather eyes

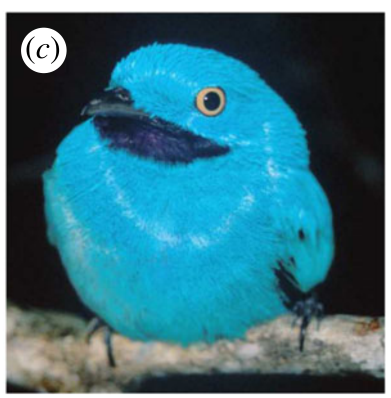


Morpho butterfly

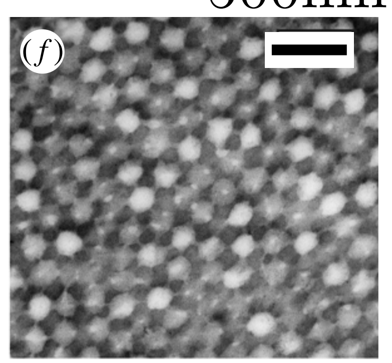


$1.7\mu m$

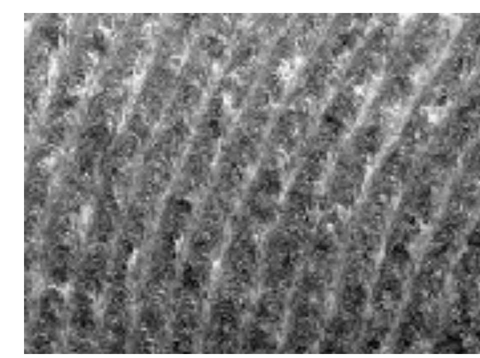
Plum-throated Cotinga



500nm

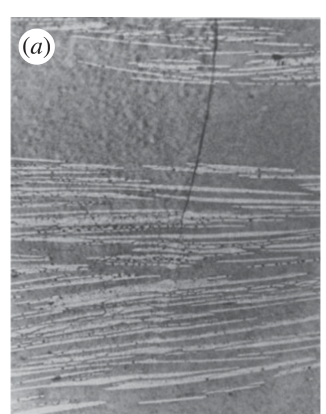


Marble berry



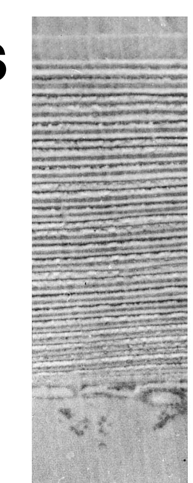
250nm

bleak fish



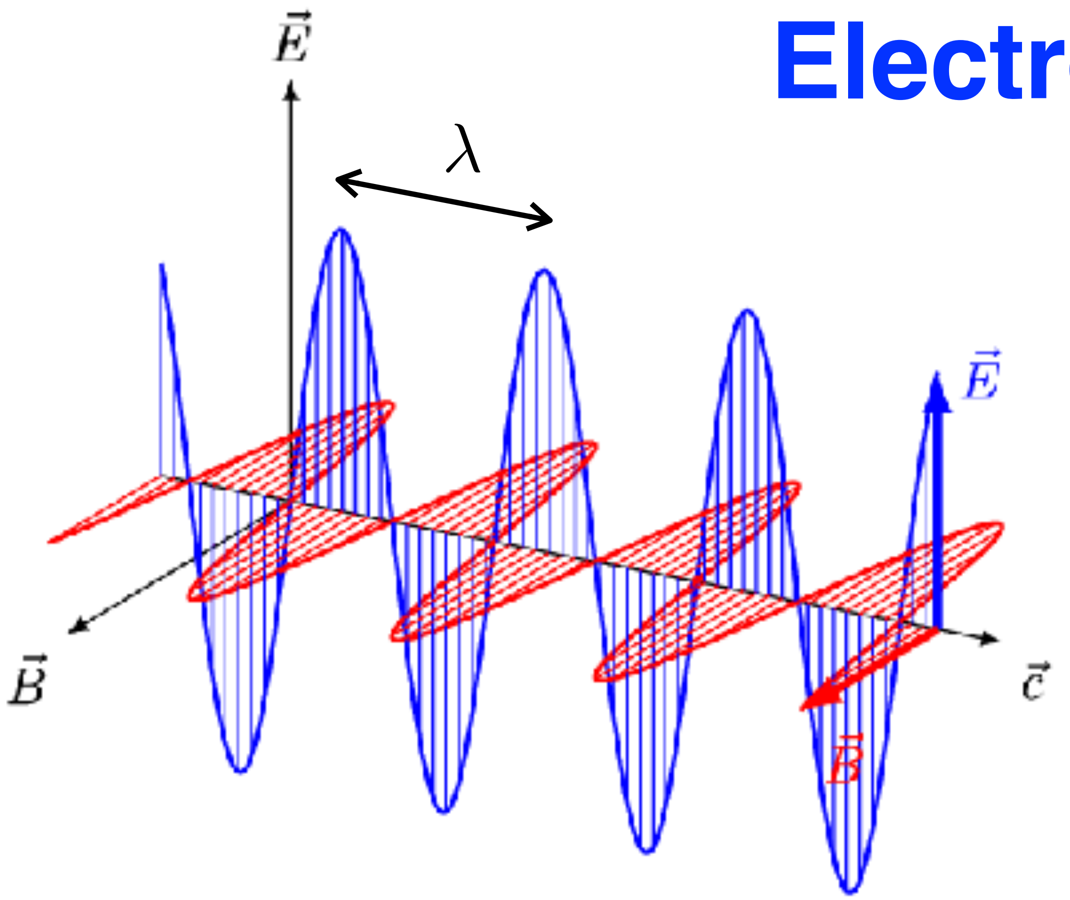
$1\mu m$

Chrysina aurigans beetle



$1\mu m$

Electromagnetic waves



electric field

magnetic field

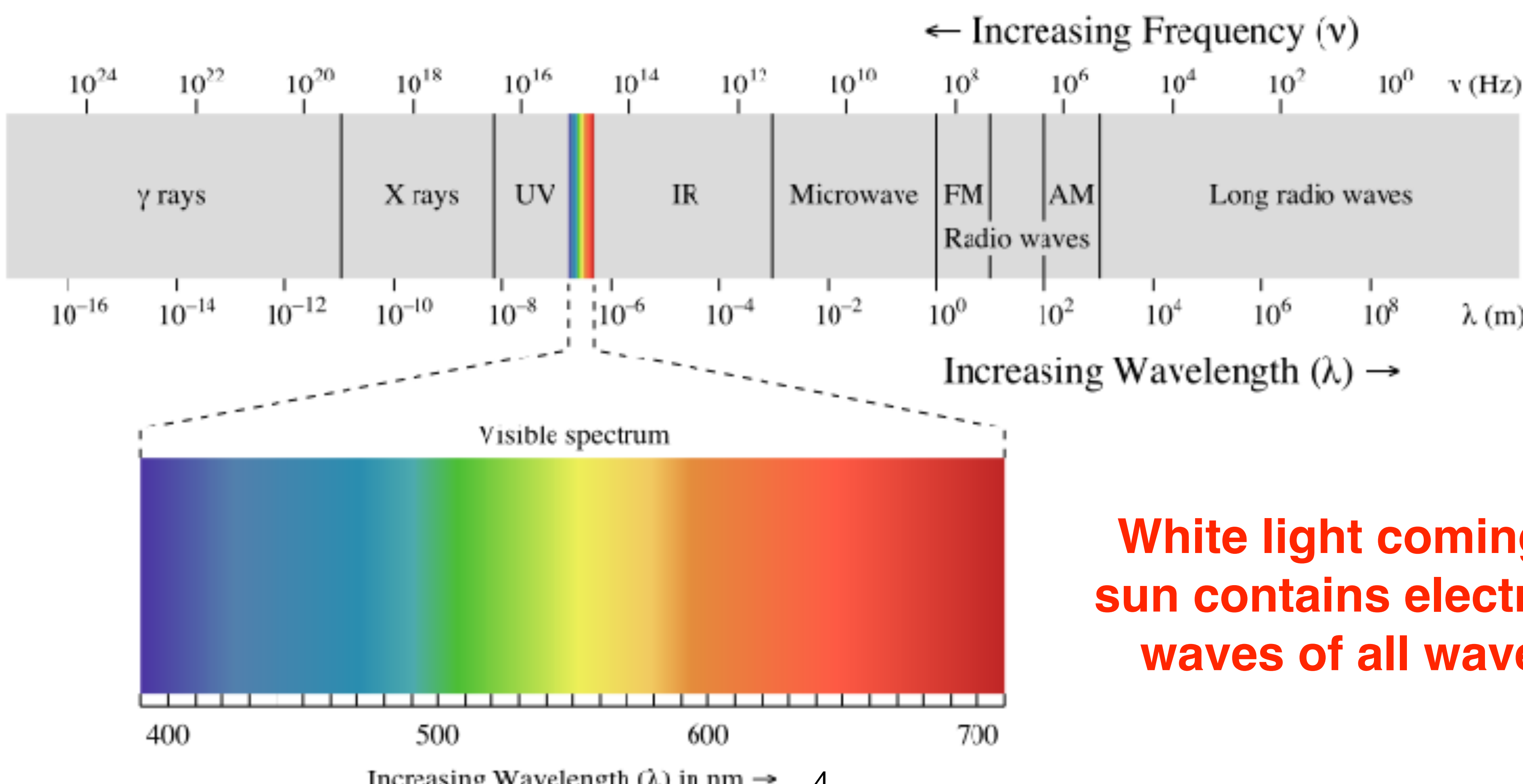
$$c^2 \vec{B}_0 = \vec{c} \times \vec{E}_0$$

speed of light

$$c_0 = \lambda \nu = 3 \times 10^8 \mathrm{m/s}$$

wavelength λ

frequency ν



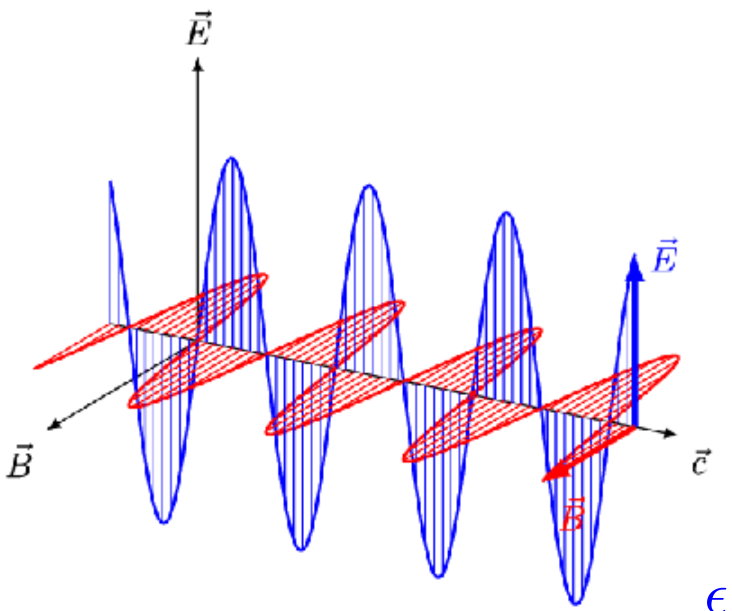
White light coming from the sun contains electromagnetic waves of all wavelengths!

Wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Solutions are traveling waves with velocity c.

electromagnetic waves

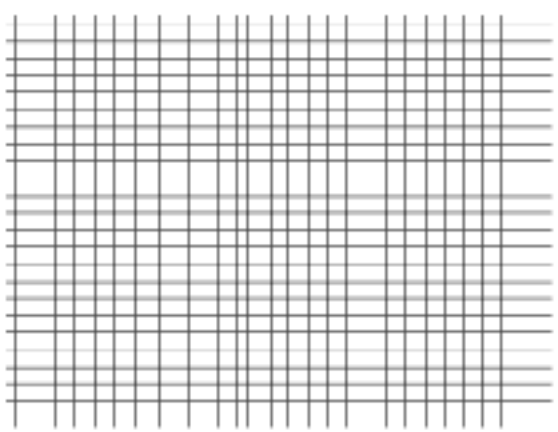


$$c = \frac{c_0}{\sqrt{\epsilon\mu}}$$

ϵ **permittivity**

μ **permeability**

sound waves

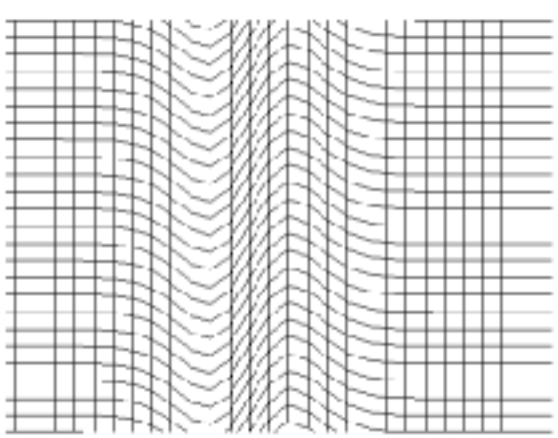


$$c = \sqrt{\frac{K}{\rho}}$$

K **bulk modulus**

ρ **mass density**

shear waves



$$c = \sqrt{\frac{\mu}{\rho}}$$

μ **shear modulus**

ρ **mass density**

waves in ropes under tension

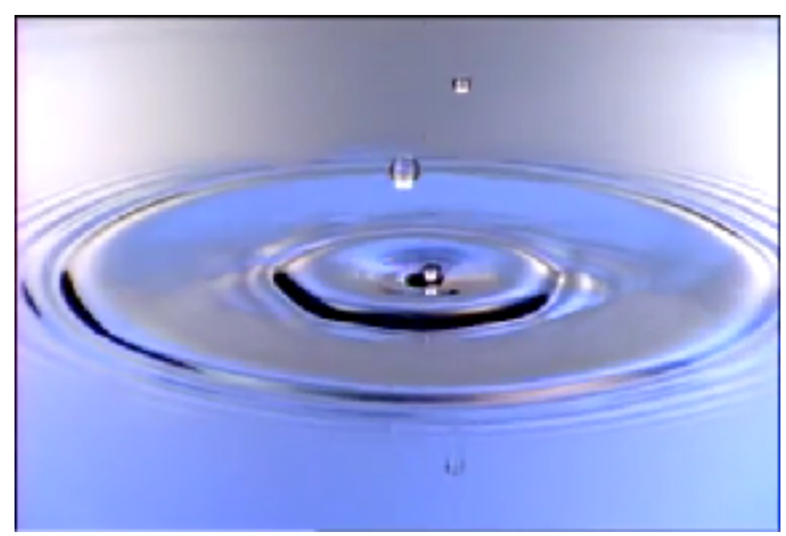
$$c = \sqrt{\frac{F}{\rho A}}$$

F **tensile force**

ρ **mass density**

A **cross-section area**

waves on liquid surfaces



shallow water

$$c = \sqrt{gh}$$

deep water

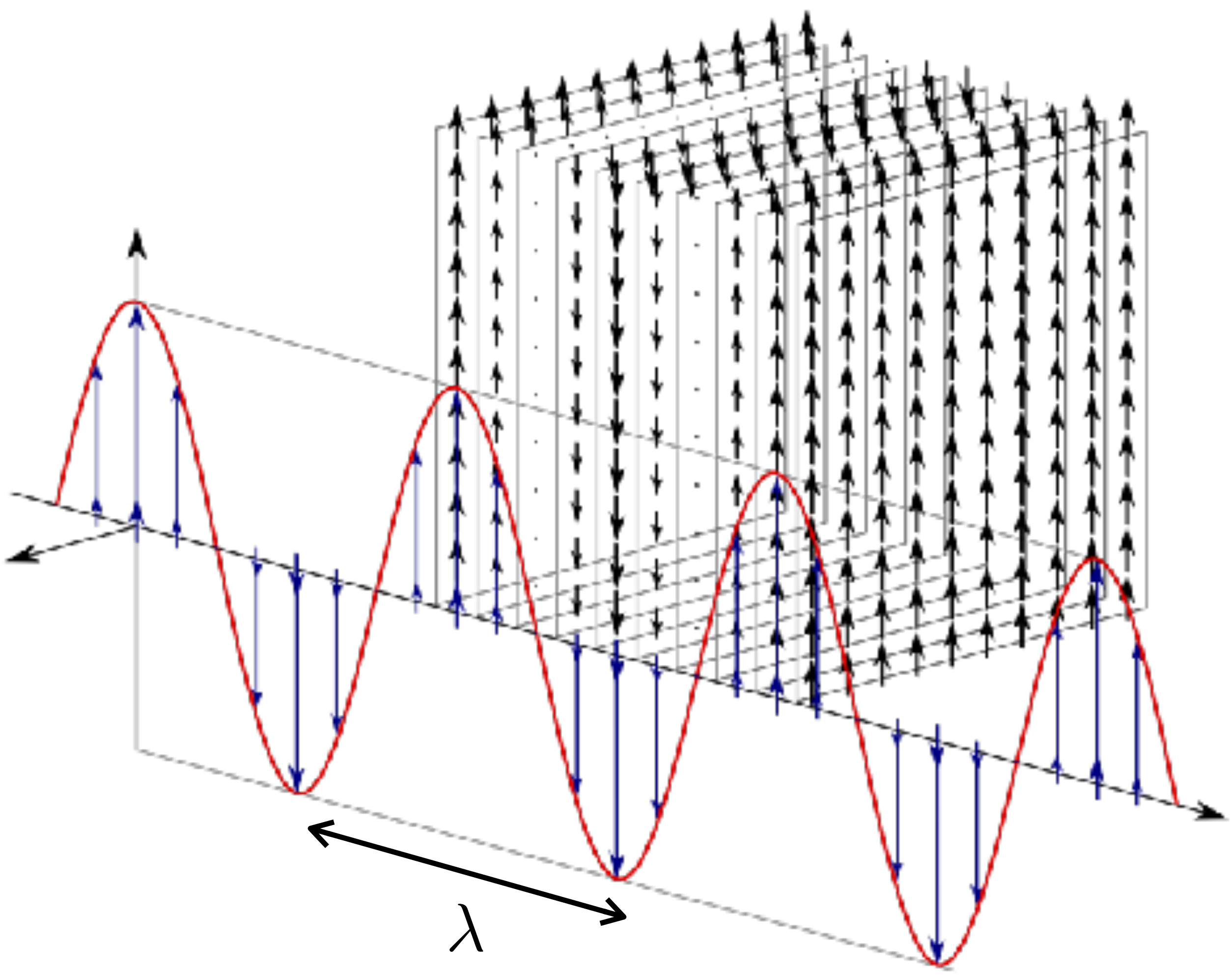
$$c = \sqrt{\frac{g\lambda}{2\pi}}$$

g **gravitational const.**

h **water depth**

λ **wavelength**

Plane waves



Solutions of wave equation can be described as a linear superposition of plane waves:

$$u(x,t) = \sum_{\vec{k}} A_{\vec{k}} e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$k = \frac{2\pi}{\lambda}$ **wavevector**

$\omega = 2\pi\nu$ **angular frequency**

Plane waves travel in direction of $\vec{k}$ with velocity:

$$c = \frac{\omega}{k} = \lambda\nu$$

Note: velocity of plane waves may depend on the wavevector $c(\vec{k})$!

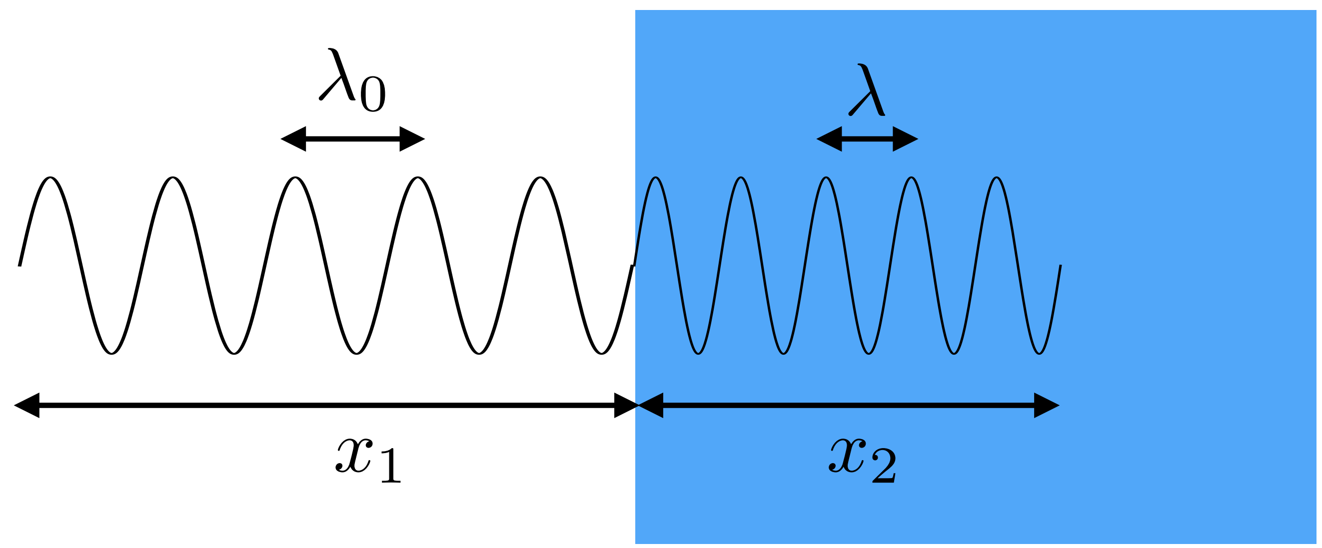
Planes of constant phases:

$$\vec{k}\cdot\vec{r} = \text{const}$$

Propagation of light in medium

vacuum

medium with index of refraction *n>1*



	vacuum	medium
speed of light	$c_0 = 3 \times 10^8 \mathrm{m/s}$	$c = c_0/n$
frequency	ν_0	$\nu = \nu_0$
wavelength	λ_0	$\lambda = \lambda_0/n$
	$c_0 = \nu_0 \lambda_0$	$c = \nu \lambda$

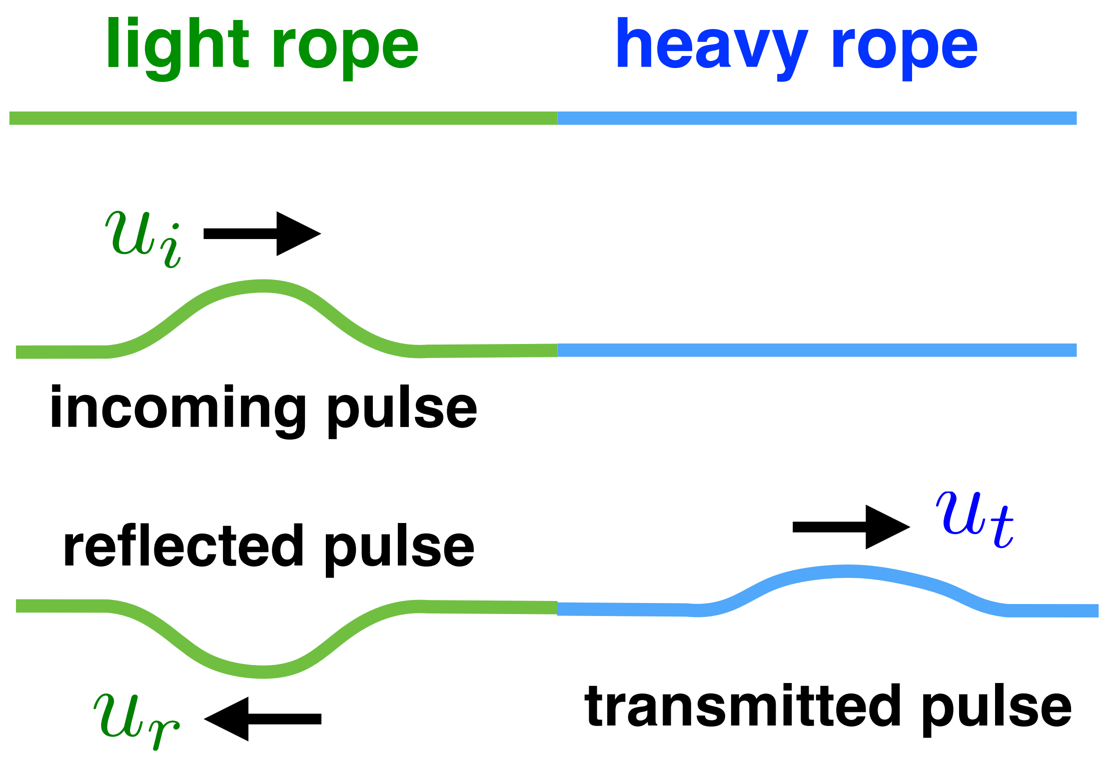
---

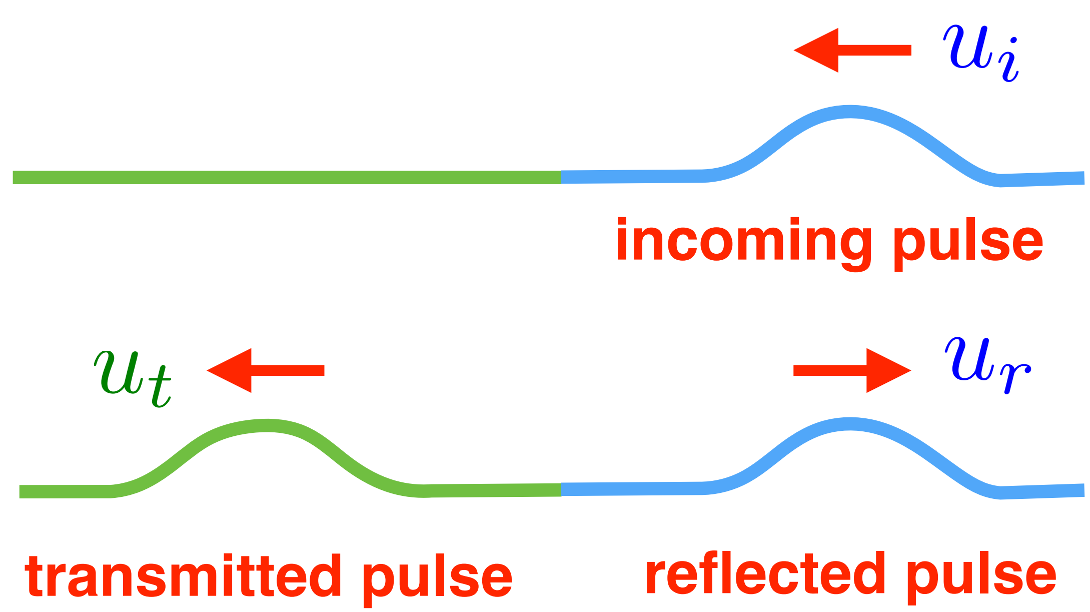
total number of cycles

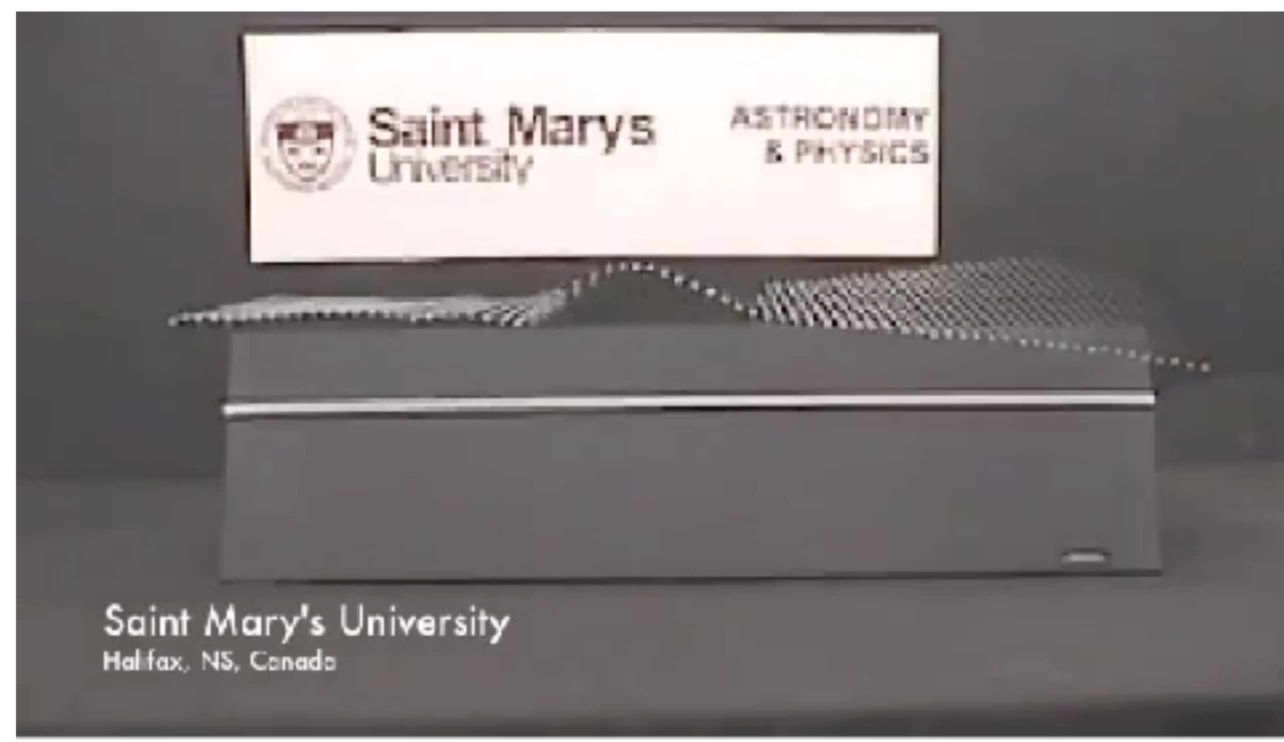
$$\frac{x_1}{\lambda_0} + \frac{x_2}{\lambda} = \frac{x_1 + n x_2}{\lambda_0}$$

Optical path length is geometric distance multiplied by the index of refraction!

Reflection of waves







amplitude of reflected pulse

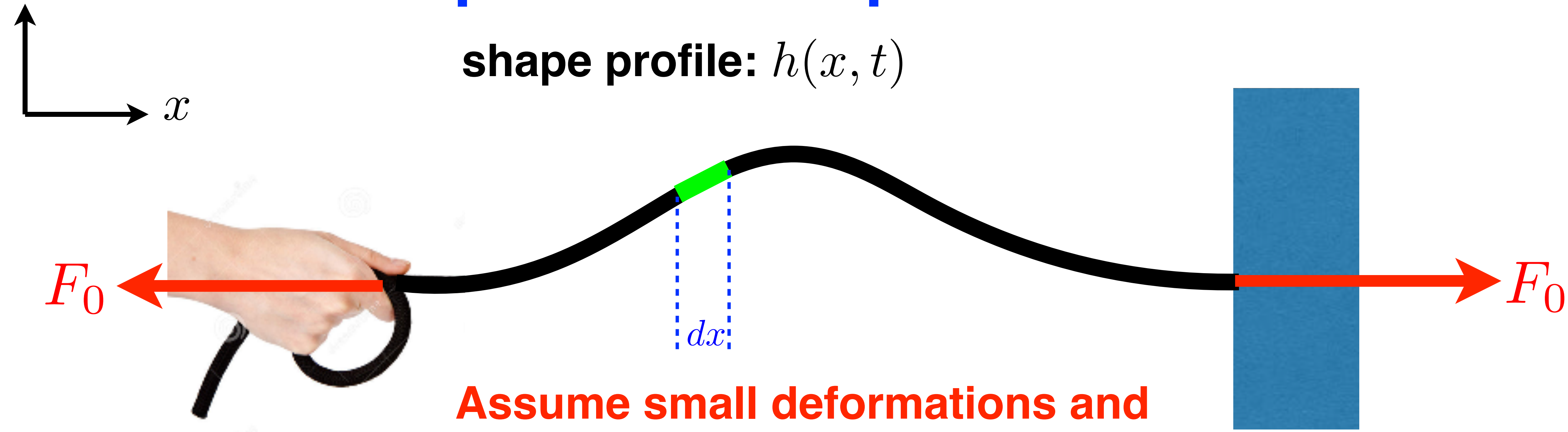
$$\frac{u_r}{u_i} = \frac{c_2 - c_1}{c_1 + c_2}$$

amplitude of transmitted pulse

$$\frac{u_t}{u_i} = \frac{2c_2}{c_1 + c_2}$$

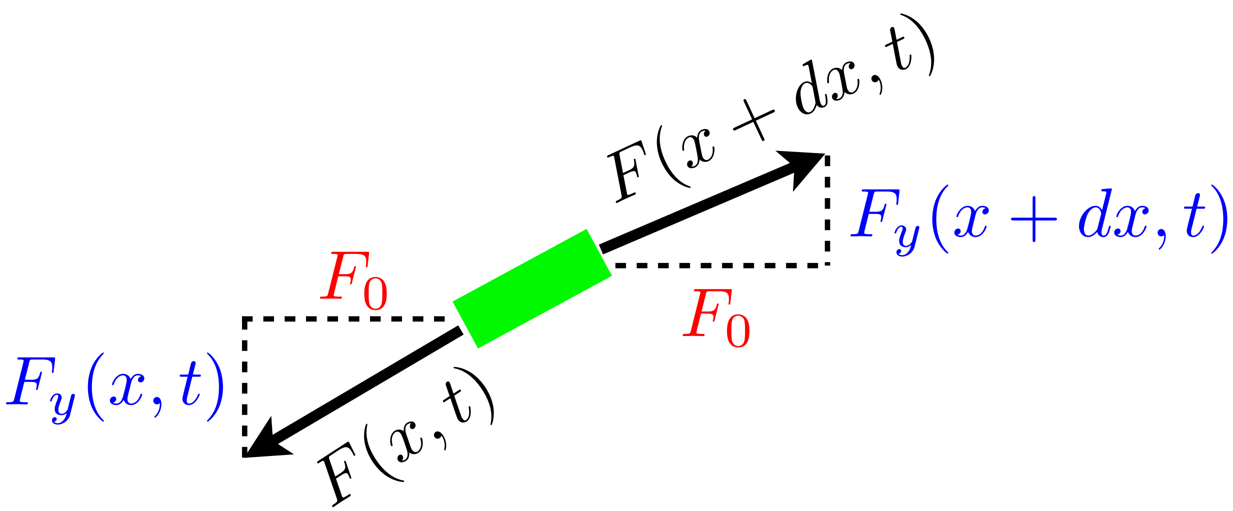
Wave equation for rope under tension

shape profile: $h(x,t)$



Assume small deformations and ignore movement in *x* direction!

Forces acting on a small rope element:



Forces act only in direction of the rope:

$$\frac{F_y(x,t)}{F_0} = \frac{\partial h(x,t)}{\partial x}$$

Second Newton's law for a small rope element:

$$\rho A dx \frac{\partial^2 h}{\partial t^2} = F_y(x+dx,t) - F_y(x,t)$$

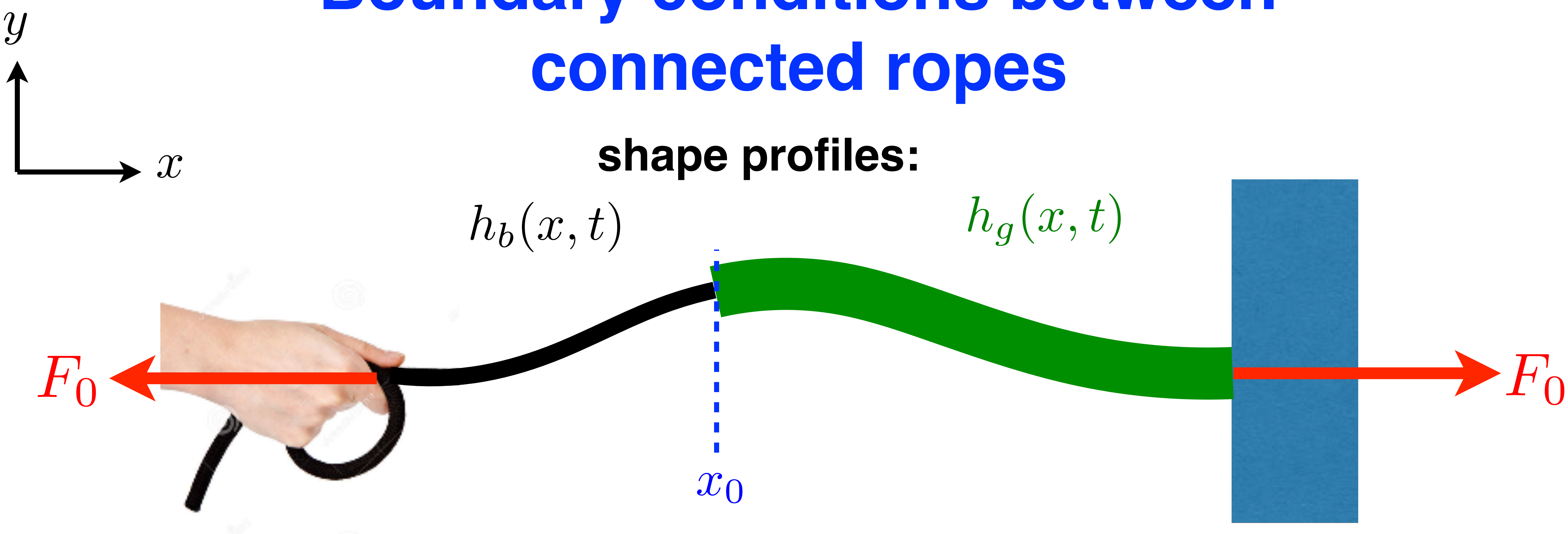
$$\rho A \frac{\partial^2 h}{\partial t^2} = \frac{\partial F_y}{\partial x} = F_0 \frac{\partial^2 h}{\partial x^2}$$

Wave equation:

$$\frac{\partial^2 h}{\partial t^2} = \frac{F_0}{\rho A}\frac{\partial^2 h}{\partial x^2} \equiv c^2 \frac{\partial^2 h}{\partial x^2}$$

Boundary conditions between connected ropes

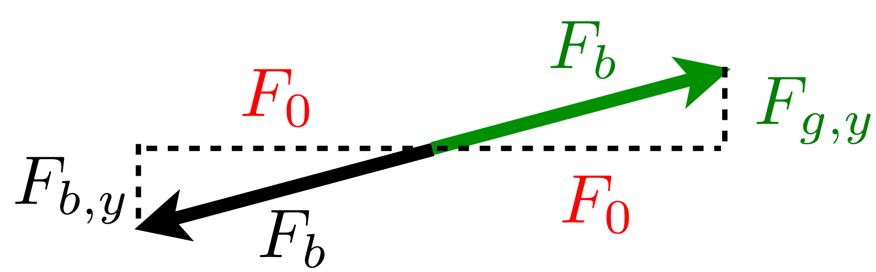
shape profiles:



Continuity: ropes are connected

$$h_b(x_0,t) = h_g(x_0,t)$$

Forces acting on the massless point, where ropes are connected:



Force balance:

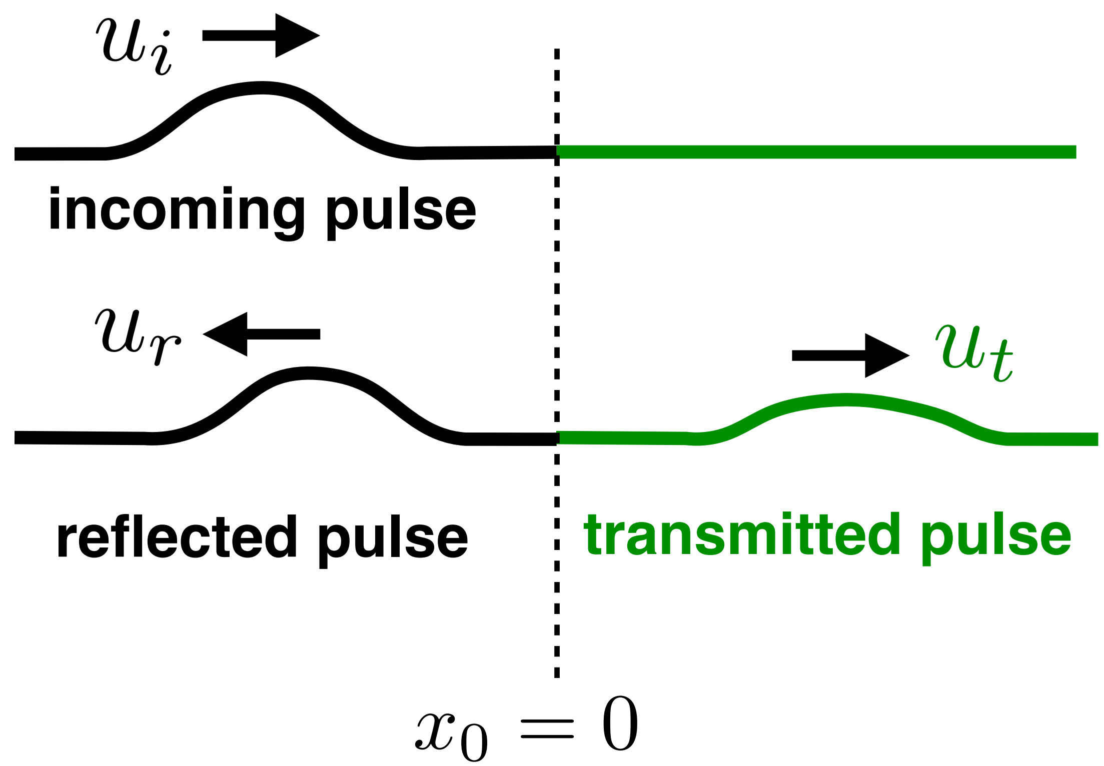
Newton's law for this massless point:

$$F_{g,y} - F_{b,y} = ma = 0 \quad \longrightarrow \quad \frac{\partial h_b}{\partial x}(x_0,t) = \frac{\partial h_g}{\partial x}(x_0,t)$$

Reflection of waves on ropes

wave speed in black rope

$$c_1 = \frac{\omega}{k_1}$$



wave speed in green rope

$$c_2 = \frac{\omega}{k_2}$$

Solutions of wave equations can be expanded in Fourier series:

incoming pulse **reflected pulse**

$$u_b(x,t) = \sum_{\omega} \left(A_\omega e^{i(k_1 x - \omega t)} + B_\omega e^{i(-k_1 x - \omega t)} \right)$$

transmitted pulse

$$u_g(x,t) = \sum_{\omega} \left(C_\omega e^{i(k_2 x - \omega t)} \right)$$

boundary conditions:

$$u_b(0,t) = u_g(0,t)$$

$$\frac{\partial u_b}{\partial x}(0,t) = \frac{\partial u_g}{\partial x}(0,t)$$

$$A_\omega + B_\omega = C_\omega$$

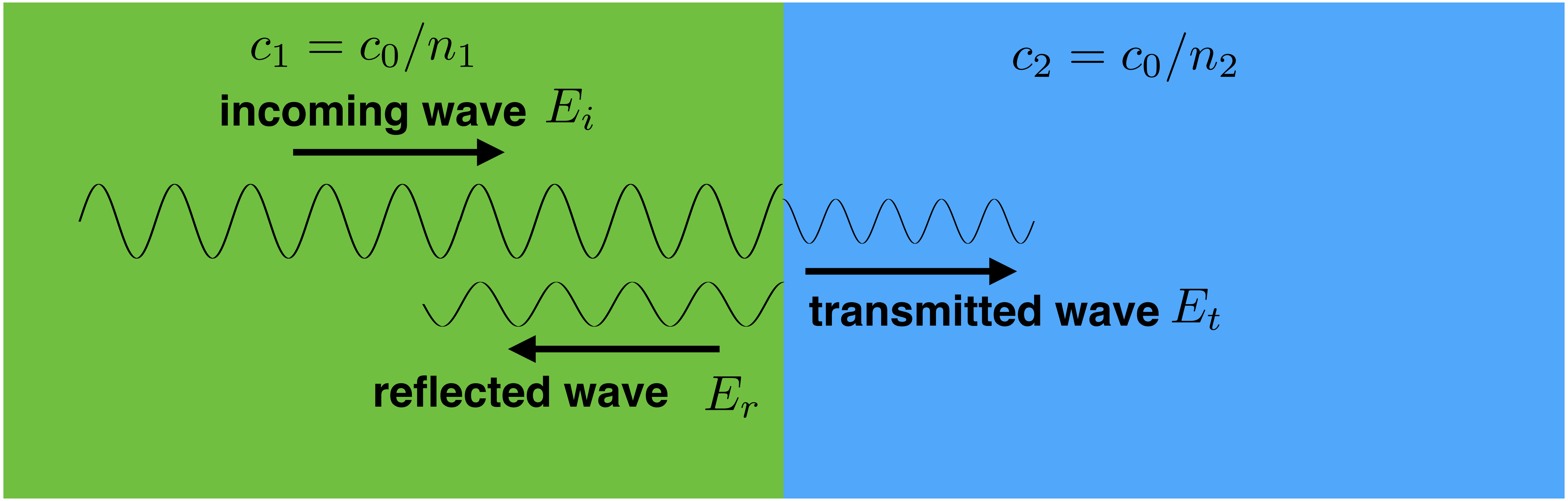
$$ik_1(A_\omega - B_\omega) = ik_2 C_\omega$$

amplitudes of reflected and transmitted waves:

$$B_\omega = A_\omega \frac{(c_2 - c_1)}{(c_1 + c_2)}$$

$$C_\omega = A_\omega \frac{2c_2}{(c_1 + c_2)}$$

Reflection of light at the interface between two media



boundary conditions for incident waves normal to the interface:

$$E_1 = E_2 \qquad \frac{\partial E_1}{\partial x} = \frac{\partial E_2}{\partial x}$$

amplitude of reflected electric field

$$r \equiv \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

amplitude of transmitted electric field

$$t \equiv \frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2}$$

energy density of electromagnetic waves

$$\propto n|E|^2$$

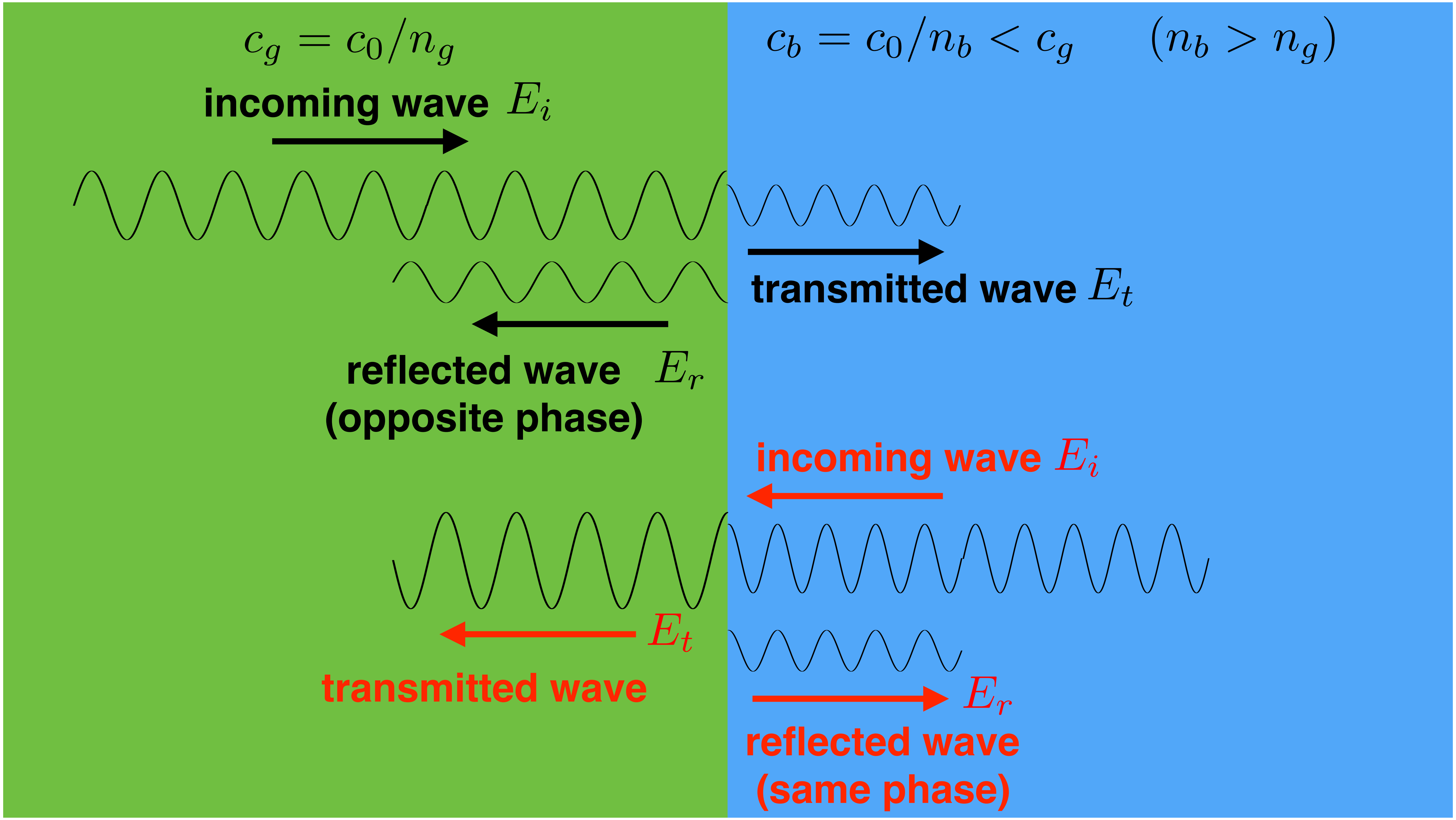
reflectance

$$R \equiv \frac{n_1|E_r|^2}{n_1|E_i|^2} = |r|^2$$

transmittance

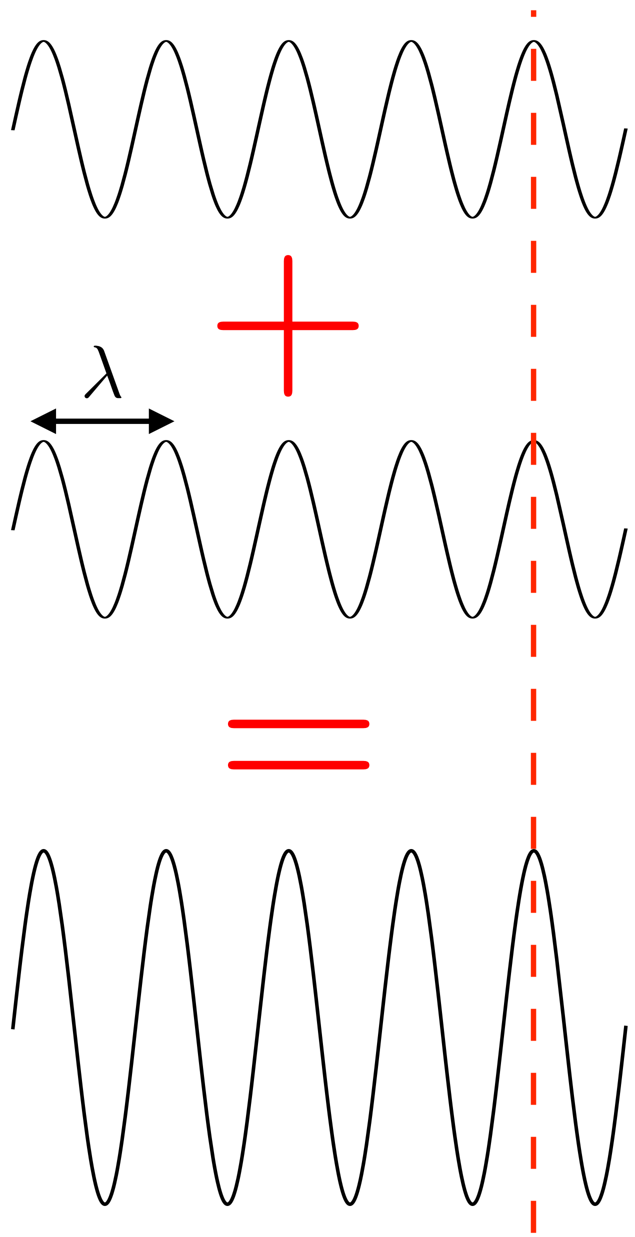
$$T \equiv \frac{n_2|E_t|^2}{n_1|E_i|^2} = |t|^2\frac{n_2}{n_1} = 1 - R$$

Reflection of light at the interface between two media



Interference

constructive interference

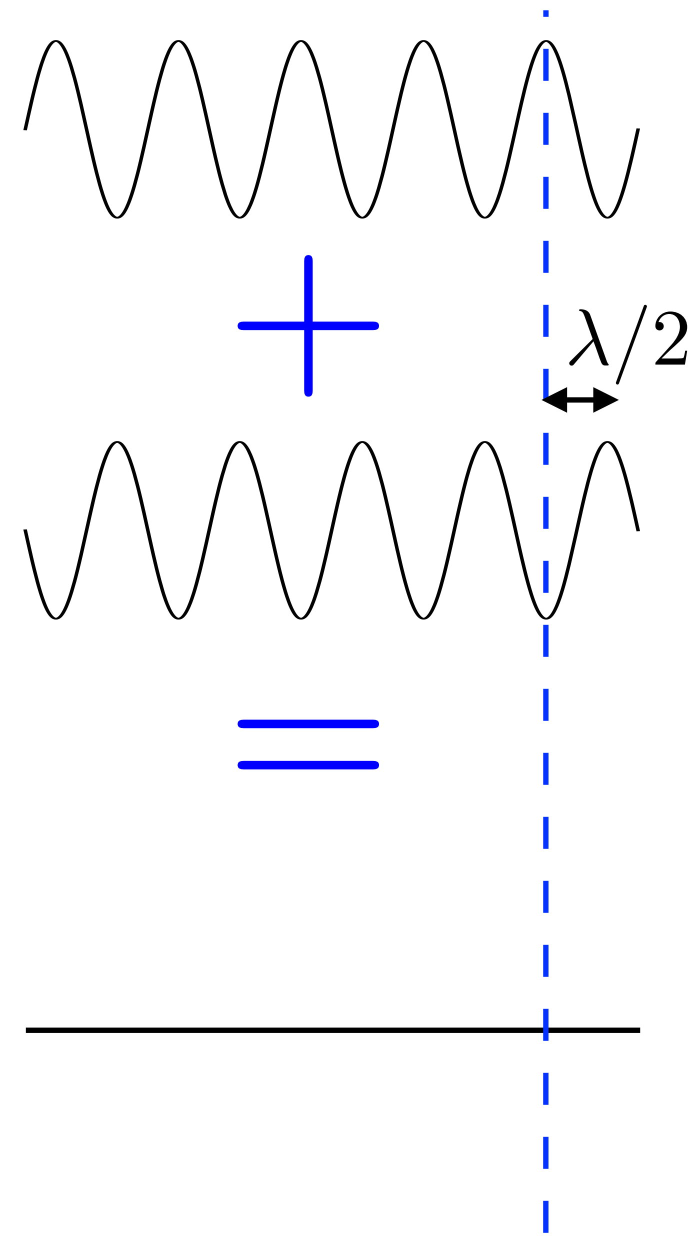


Constructive interference occurs when the two waves are in phase: waves offset by $m\lambda$,

$$m = 0, \pm 1, \pm 2, \ldots$$

$$e^{ikm\lambda} = e^{i2\pi m} = +1$$

destructive interference

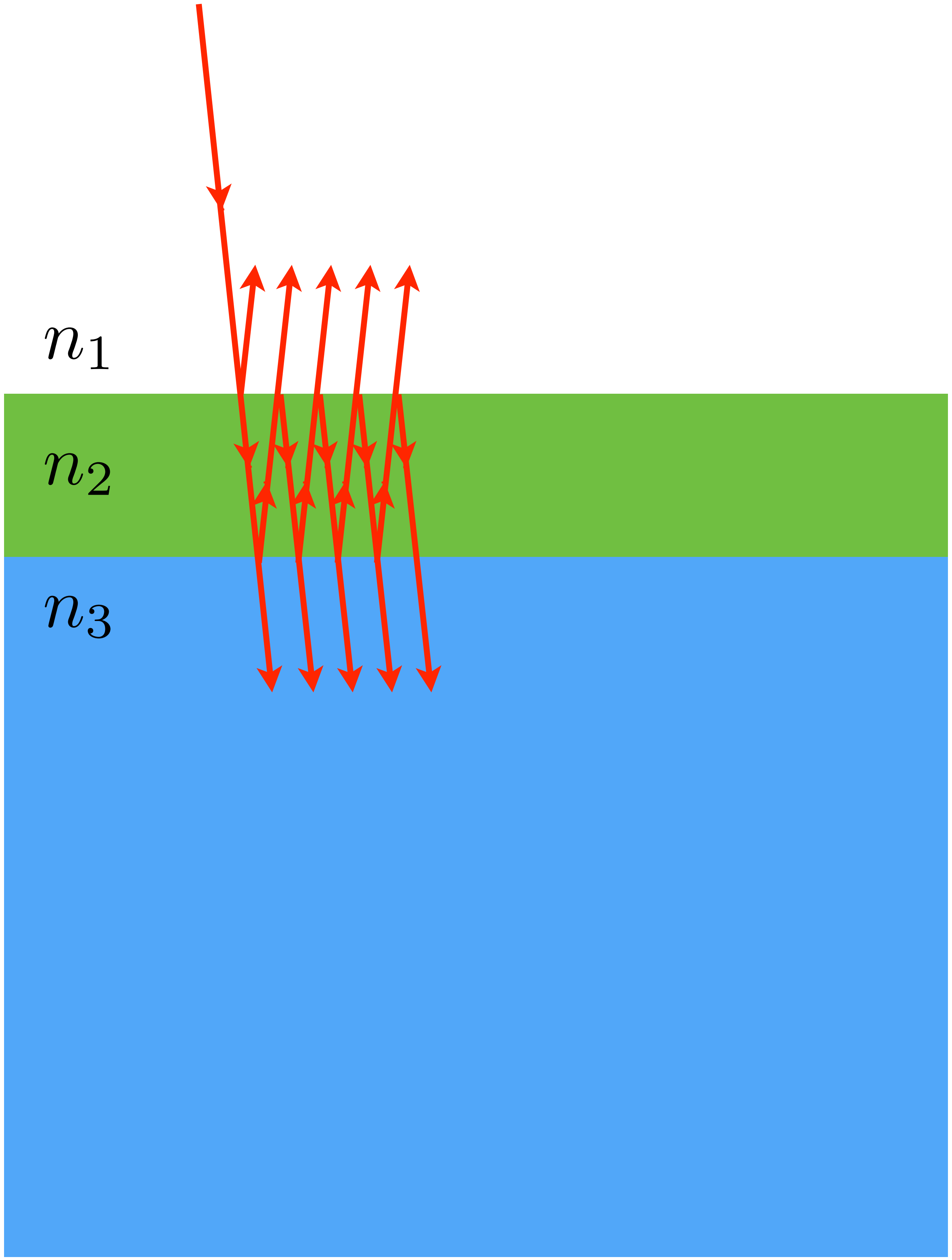


Destructive interference occurs when the two waves are out of phase: waves offset by $(m + 1/2)\lambda$,

$$m = 0, \pm 1, \pm 2, \ldots$$

$$e^{ik(m+1/2)\lambda} = e^{i(2\pi m + \pi)} = -1$$

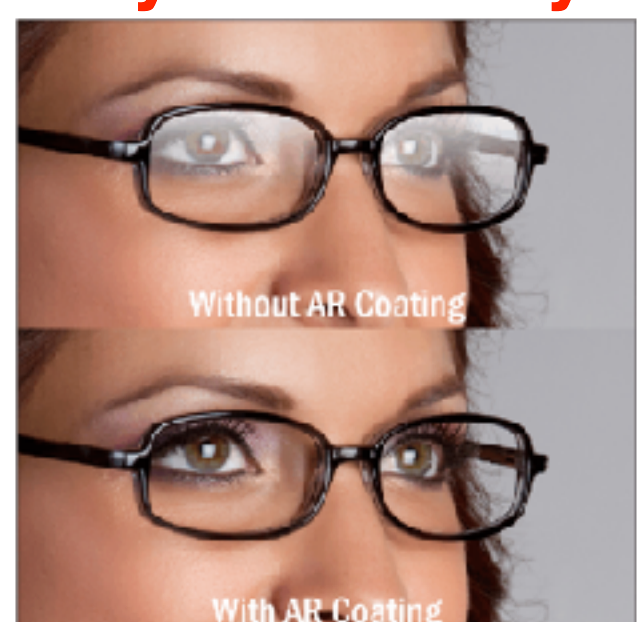
Interference on thin films



Constructive interference of reflected rays results in strongly reflected rays with very little transmission.

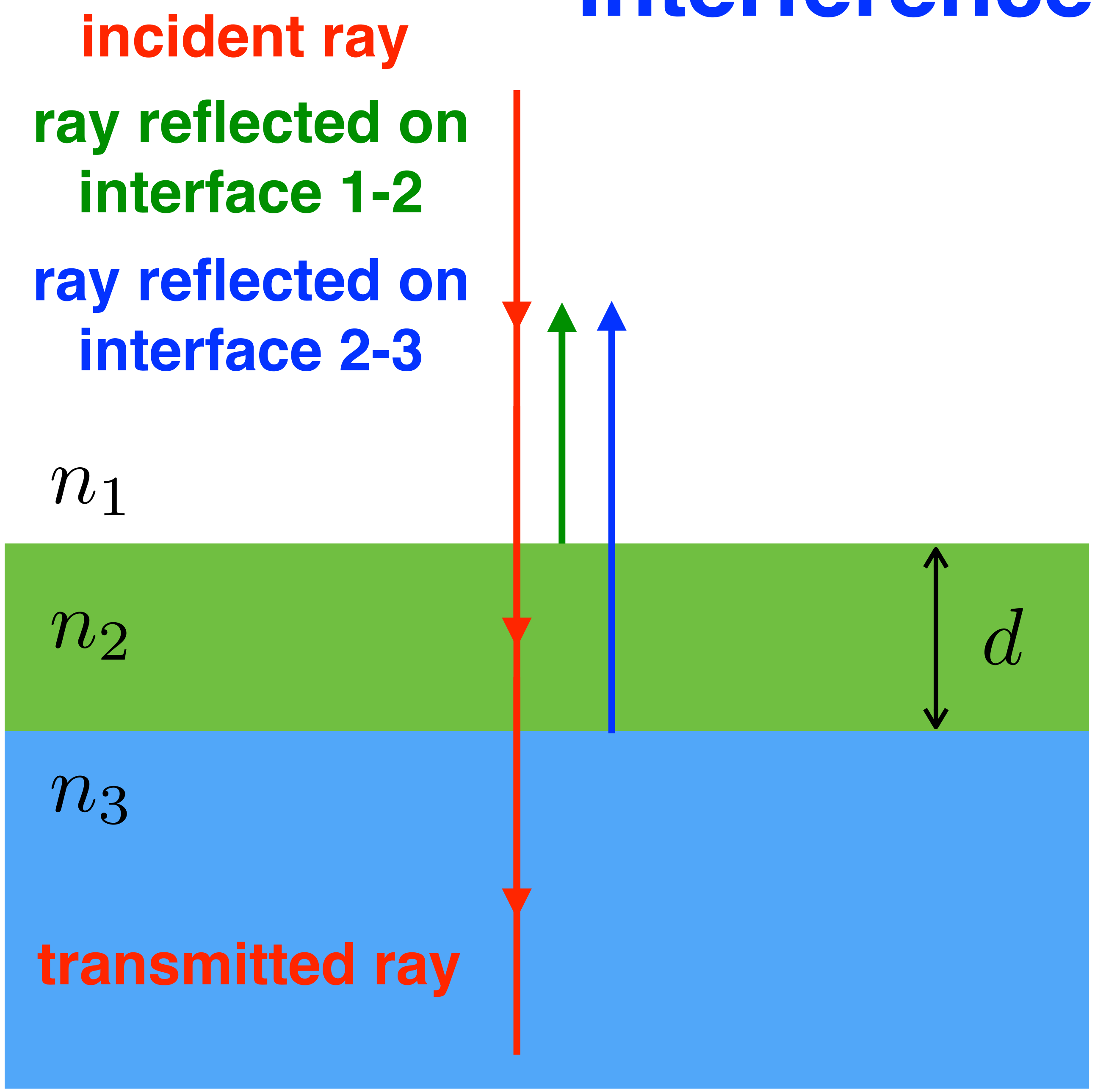
mirrors

Deconstructive interference of reflected rays results in almost perfectly transmitted rays with very little reflection.



antireflective coatings

Interference on thin films



difference between optical path lengths of the two reflected rays

$$OPD = 2n_2d$$

no additional phase difference due to reflections

$n_1 < n_2 < n_3$ $\quad$ $n_1 > n_2 > n_3$

constructive interference of reflected rays

$$OPD = m\lambda$$

destructive interference of reflected rays

$$OPD = (m + 1/2)\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

additional π phase difference due to reflections

$n_1 < n_2 > n_3$ $\quad$ $n_1 > n_2 < n_3$

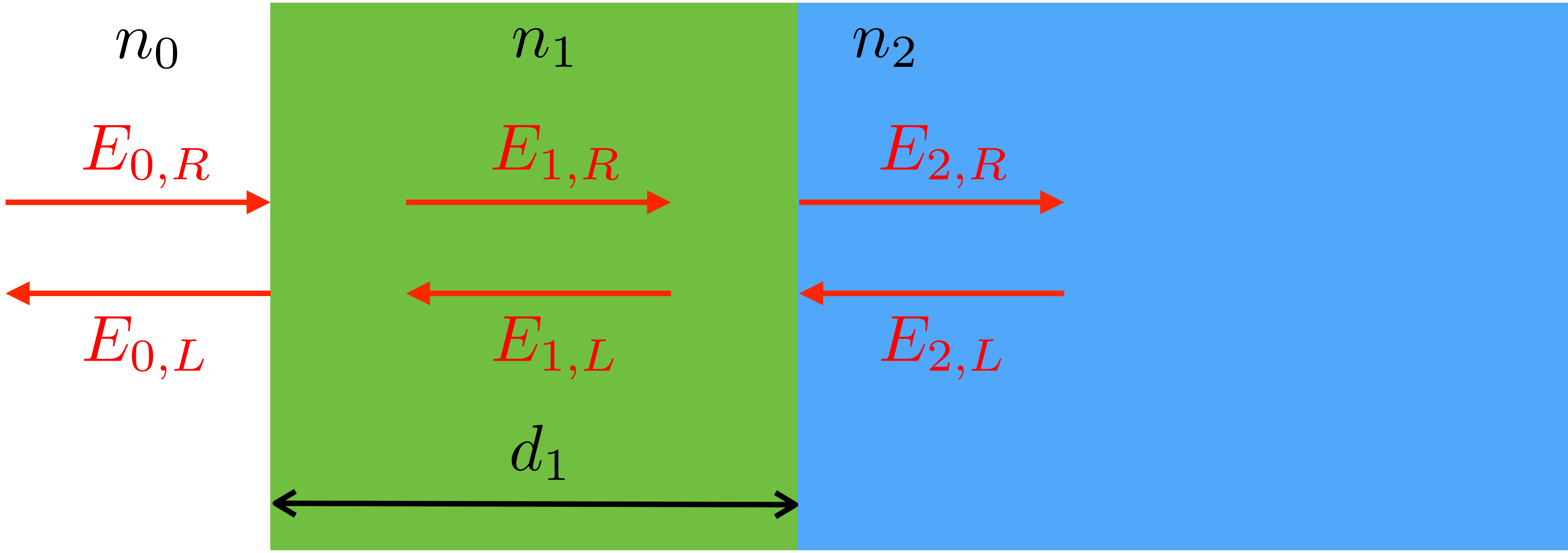
constructive interference of reflected rays

$$OPD = (m + 1/2)\lambda$$

destructive interference of reflected rays

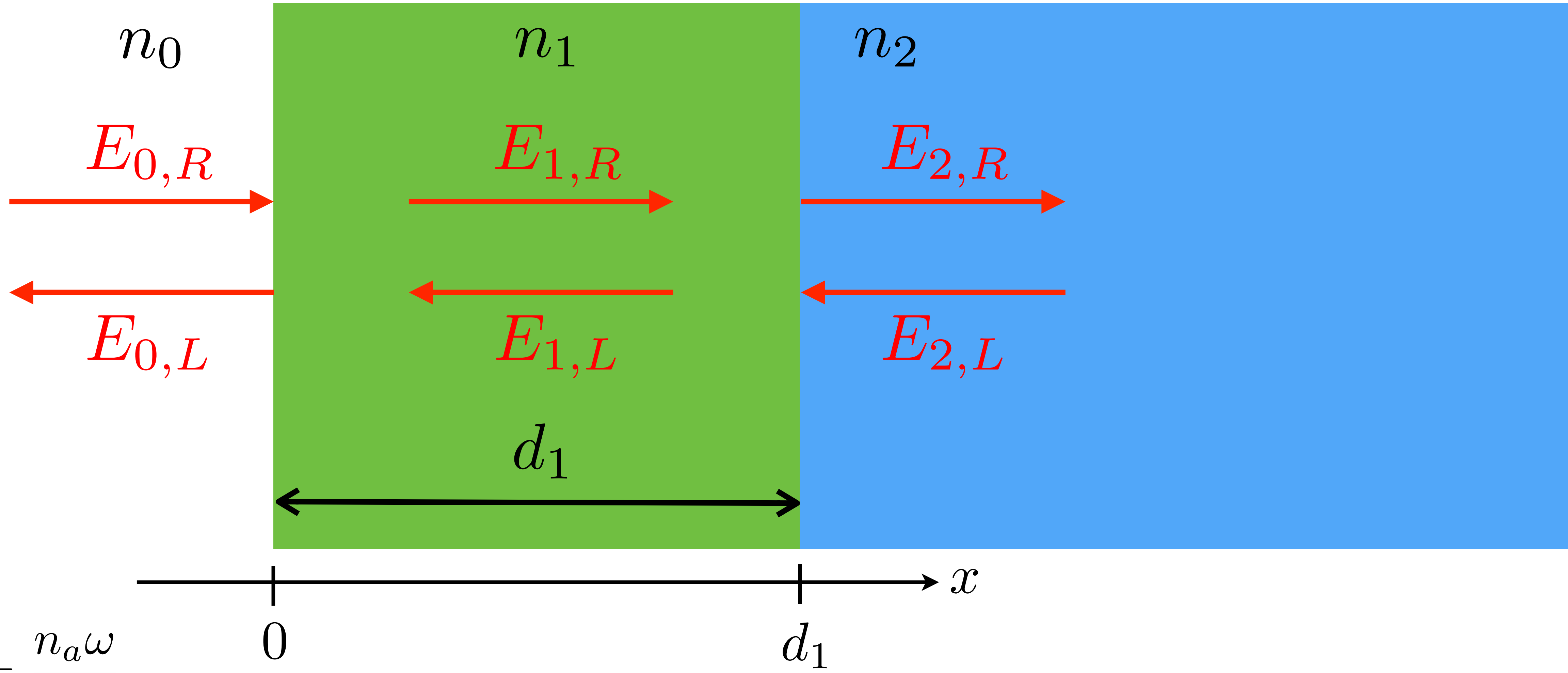
$$OPD = m\lambda$$

Transfer matrices



How can we relate the amplitudes of electromagnetic waves in the region 0 (white) to the amplitudes of electromagnetic waves in the region 2 (blue)?

Transfer matrices



$$k_a = \frac{2\pi n_a}{\lambda} = \frac{n_a \omega}{c_0}$$

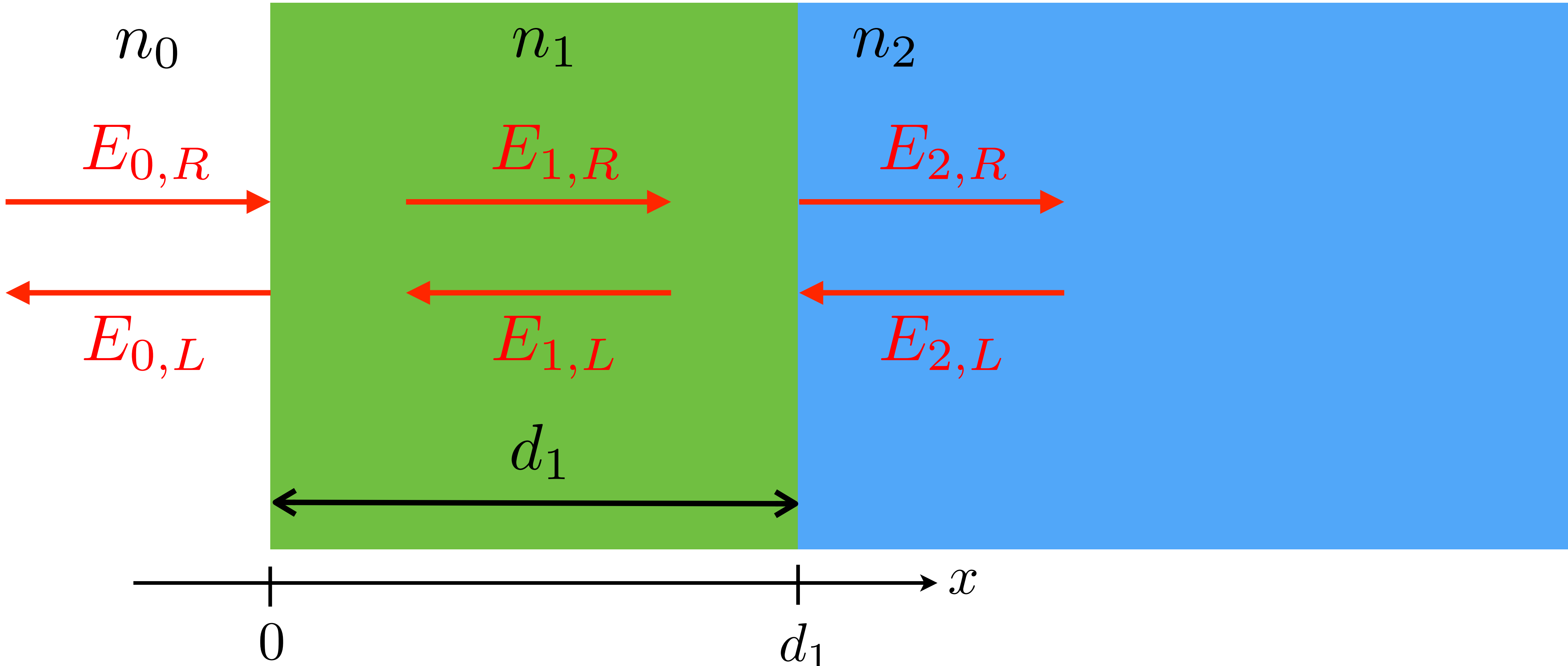
Electromagnetic waves in different regions:

$$E_0(x,t) = E_{0,R} e^{i(k_0 x - \omega t)} + E_{0,L} e^{i(-k_0 x - \omega t)}$$
$$E_1(x,t) = E_{1,R} e^{i(k_1 x - \omega t)} + E_{1,L} e^{i(-k_1 x - \omega t)}$$
$$E_2(x,t) = E_{2,R} e^{i(k_2 x - \omega t)} + E_{2,L} e^{i(-k_2 x - \omega t)}$$

Boundary conditions:

$$E_0(0,t) = E_1(0,t) \qquad E_1(d_1,t) = E_2(d_1,t)$$
$$\frac{\partial E_0}{\partial x}(0,t) = \frac{\partial E_1}{\partial x}(0,t) \qquad \frac{\partial E_1}{\partial x}(d_1,t) = \frac{\partial E_2}{\partial x}(d_1,t)$$

Transfer matrices



Boundary conditions:

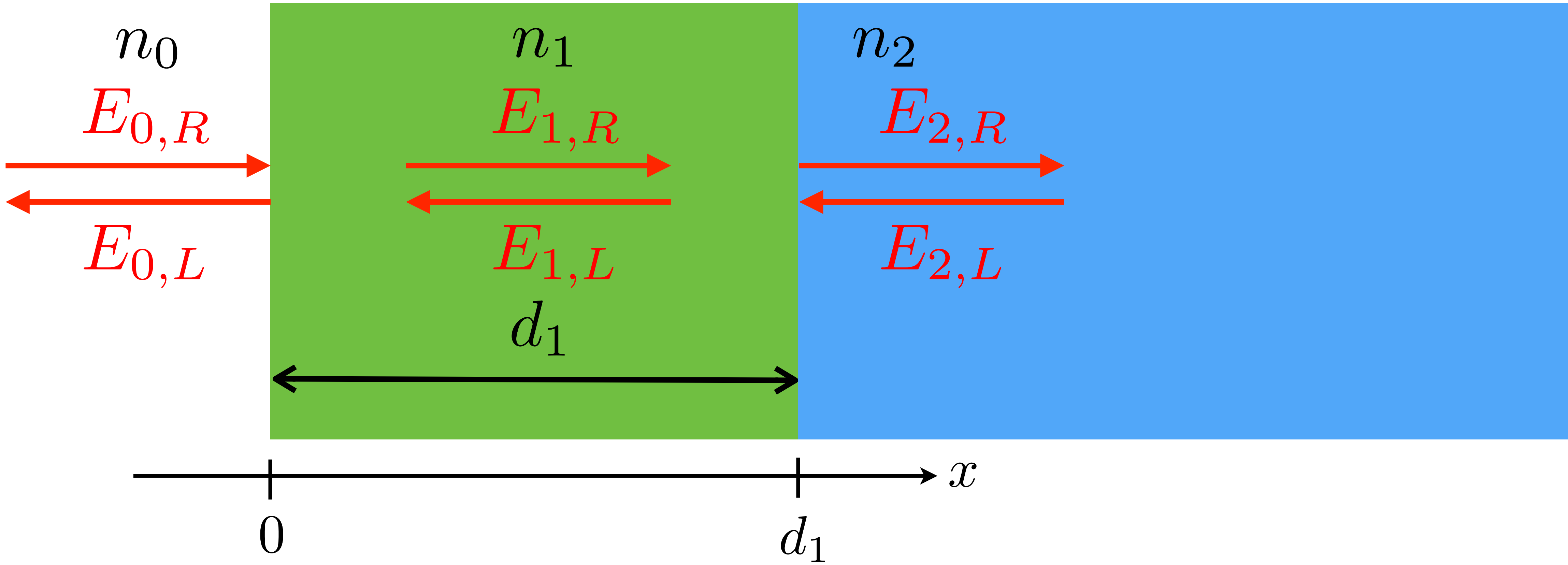
$$E_0(0,t) = E_1(0,t) \qquad E_1(d_1,t) = E_2(d_1,t)$$

$$\frac{\partial E_0}{\partial x}(0,t) = \frac{\partial E_1}{\partial x}(0,t) \qquad \frac{\partial E_1}{\partial x}(d_1,t) = \frac{\partial E_2}{\partial x}(d_1,t)$$

We would like to relate boundary conditions at two different interfaces via a transfer matrix *M_1*:

$$\begin{pmatrix} E_2(d_1,t) \\ \frac{\partial E_2}{\partial x}(d_1,t) \end{pmatrix} = M_1 \begin{pmatrix} E_0(0,t) \\ \frac{\partial E_0}{\partial x}(0,t) \end{pmatrix}$$

Transfer matrices



Electromagnetic waves in regions 1:

$$E_1(x,t) = E_{1,R}e^{i(k_1x-\omega t)} + E_{1,L}e^{i(-k_1x-\omega t)}$$

Relation between boundary conditions:

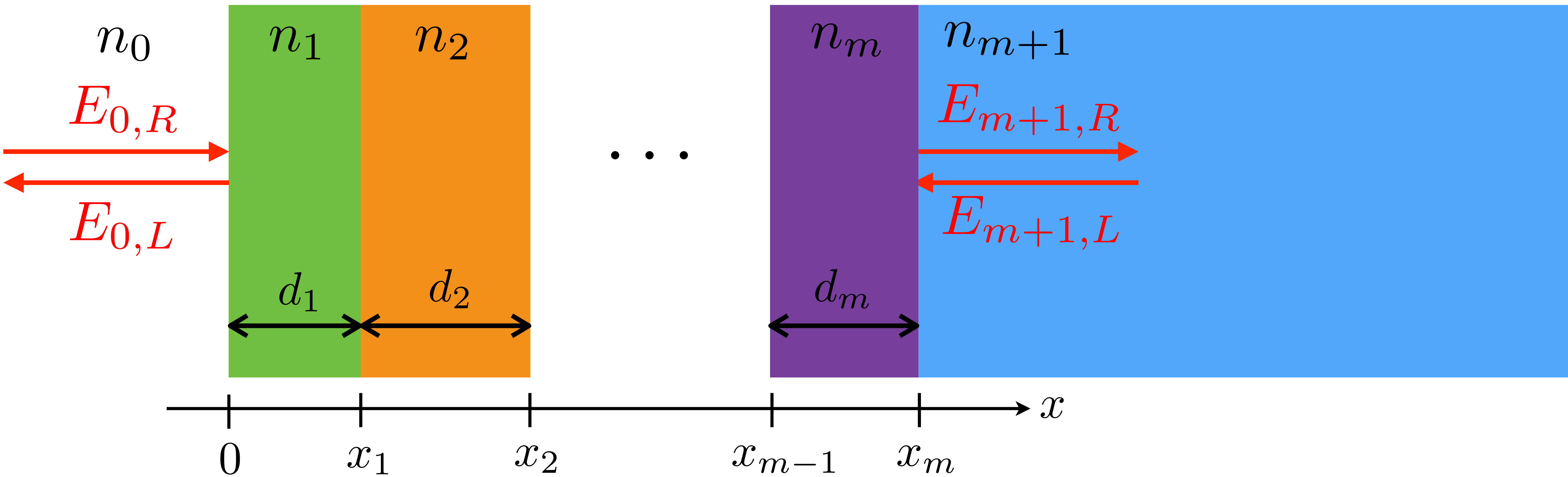
$$\begin{pmatrix} E_1(d_1,t) \\ \frac{\partial E_1}{\partial x}(d_1,t) \end{pmatrix} = M_1 \begin{pmatrix} E_1(0,t) \\ \frac{\partial E_1}{\partial x}(0,t) \end{pmatrix}$$

$$\begin{pmatrix} E_{1,R}e^{ik_1d_1} + E_{1,L}e^{-ik_1d_1} \\ ik_1E_{1,R}e^{ik_1d_1} - ik_1E_{1,L}e^{-ik_1d_1} \end{pmatrix} = M_1 \begin{pmatrix} E_{1,R} + E_{1,L} \\ ik_1E_{1,R} - ik_1E_{1,L} \end{pmatrix}$$

Transfer matrix *M*₁ can be obtained by solving equations above:

$$M_1 = \begin{pmatrix} \cos(k_1d_1), & \frac{\sin(k_1d_1)}{k_1} \\ -k_1\sin(k_1d_1), & \cos(k_1d_1) \end{pmatrix}$$

Transfer matrices



Transfer matrix for *m* layers:

$$\begin{pmatrix} E_{m+1}(x_m, t) \\ \frac{\partial E_{m+1}}{\partial x}(x_m, t) \end{pmatrix} = M \begin{pmatrix} E_0(0, t) \\ \frac{\partial E_0}{\partial x}(0, t) \end{pmatrix}$$

$$M = M_m \cdot \ldots \cdot M_2 \cdot M_1$$

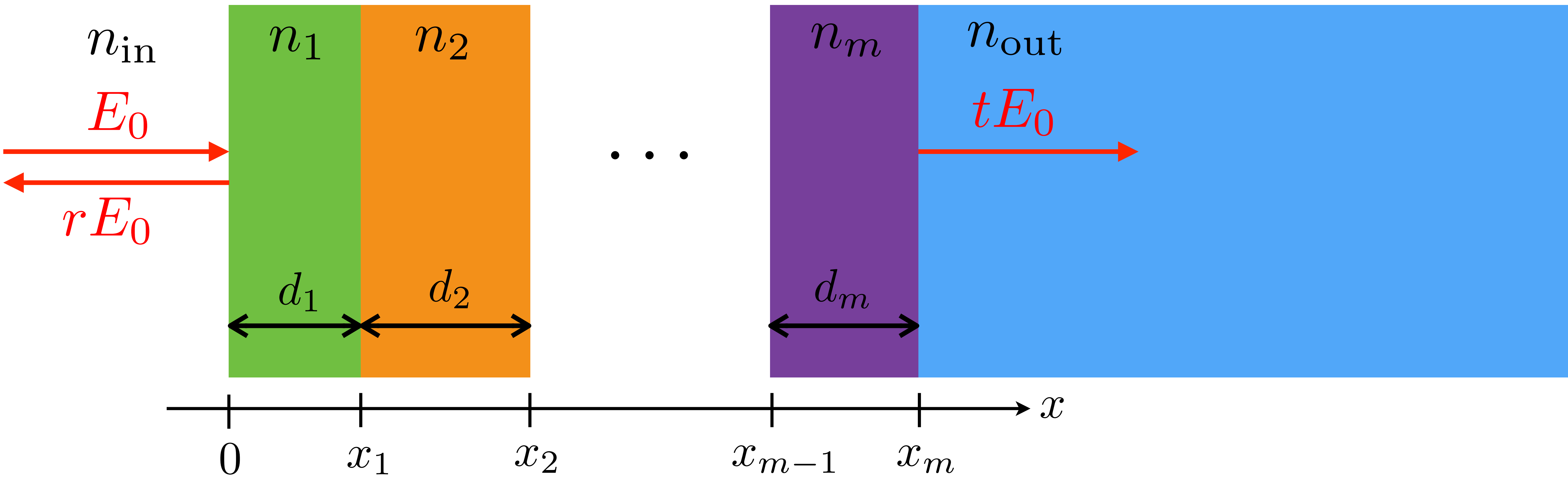
$$M_a = \begin{pmatrix} \cos(k_a d_a), & \frac{\sin(k_a d_a)}{k_a} \\ -k_a \sin(k_a d_a), & \cos(k_a d_a) \end{pmatrix}$$

Note:

$$\det(M) = \det(M_a) = 1$$

$$k_a = \frac{2\pi n_a}{\lambda} = \frac{n_a \omega}{c_0}$$

Transfer matrices



Incoming and outgoing electromagnetic waves:

$$E_{\rm in}(x,t) = E_0 e^{i(k_{\rm in}x-\omega t)} + rE_0 e^{i(-k_{\rm in}x-\omega t)}$$

$$E_{\rm out}(x,t) = tE_0 e^{i(k_{\rm out}x-\omega t)}$$

$$\begin{pmatrix} E_{\rm out}(x_m,t) \\ \frac{\partial E_{\rm out}}{\partial x}(x_m,t) \end{pmatrix} = \begin{pmatrix} M_{11}, & M_{12} \\ M_{21}, & M_{22} \end{pmatrix} \begin{pmatrix} E_{\rm in}(0,t) \\ \frac{\partial E_{\rm in}}{\partial x}(0,t) \end{pmatrix}$$

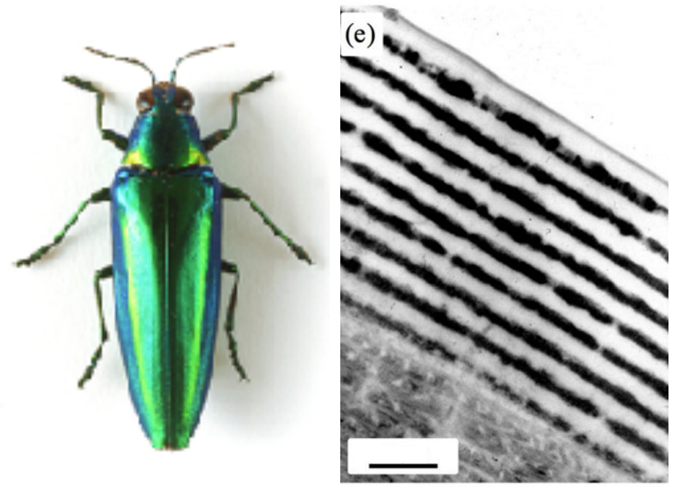
Amplitudes of reflected and transmitted waves:

$$r = \frac{(M_{21}+k_{\rm in}k_{\rm out}M_{12}) + i(k_{\rm in}M_{22}-k_{\rm out}M_{11})}{(-M_{21}+k_{\rm in}k_{\rm out}M_{12}) + i(k_{\rm out}M_{11}+k_{\rm in}M_{22})}$$

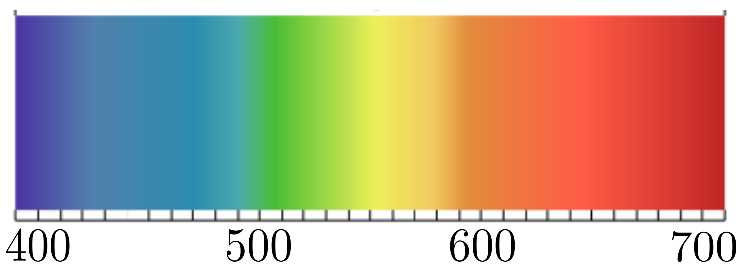
$$t = \frac{2ik_{\rm in}e^{-ix_m k_{\rm out}}}{(-M_{21}+k_{\rm in}k_{\rm out}M_{12}) + i(k_{\rm out}M_{11}+k_{\rm in}M_{22})}$$

Example

Chrysochroa raja bettle



$1\mu\text{m}$



wavelength $[\text{nm}]$

Typical refraction indices:

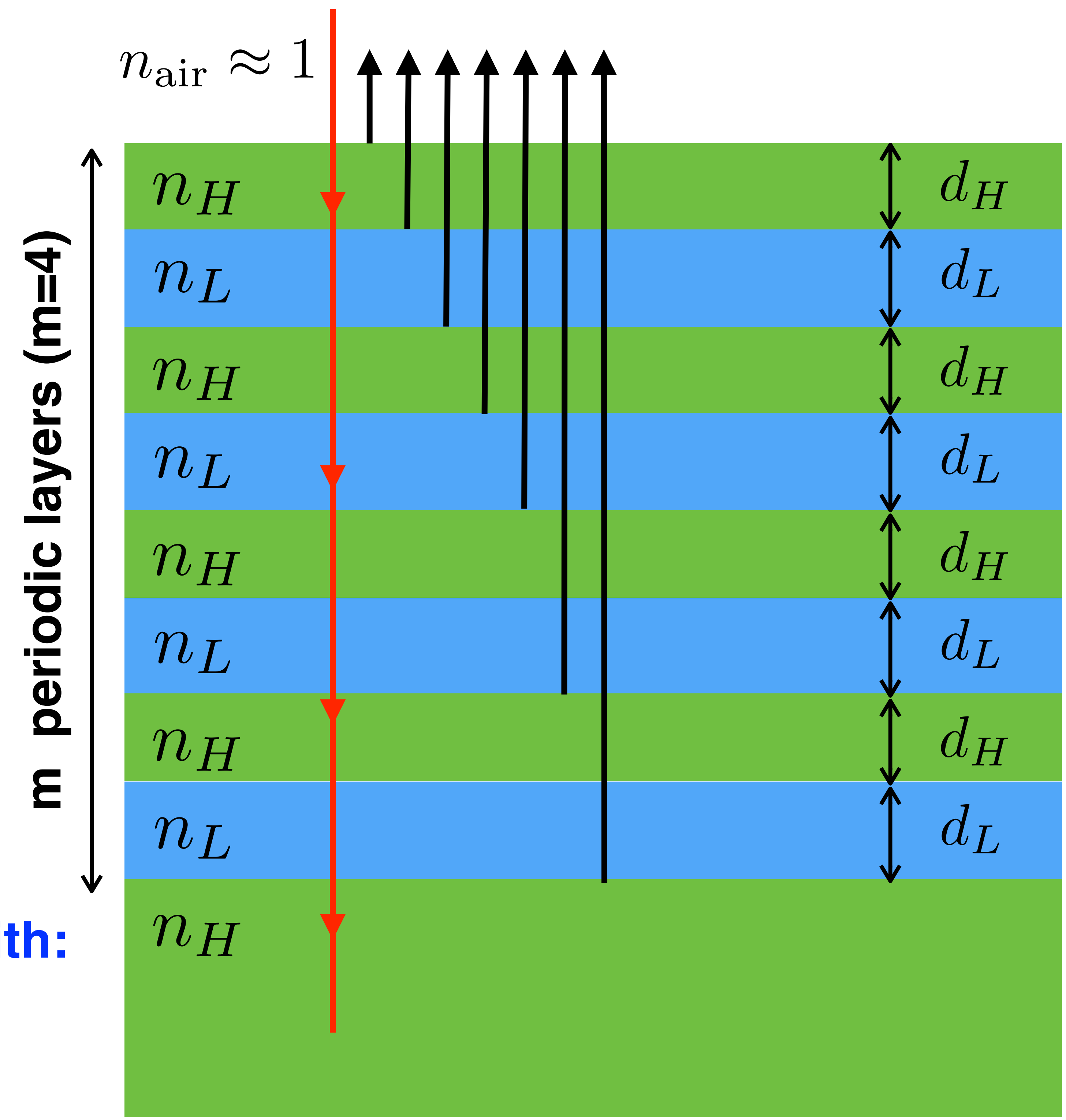
$$n_H = 1.69 \qquad n_L = 1.56$$

Constructive interference of reflected rays can be achieved with:

$$d_H = \frac{\lambda_0}{4n_H} = 74\,\text{nm}$$

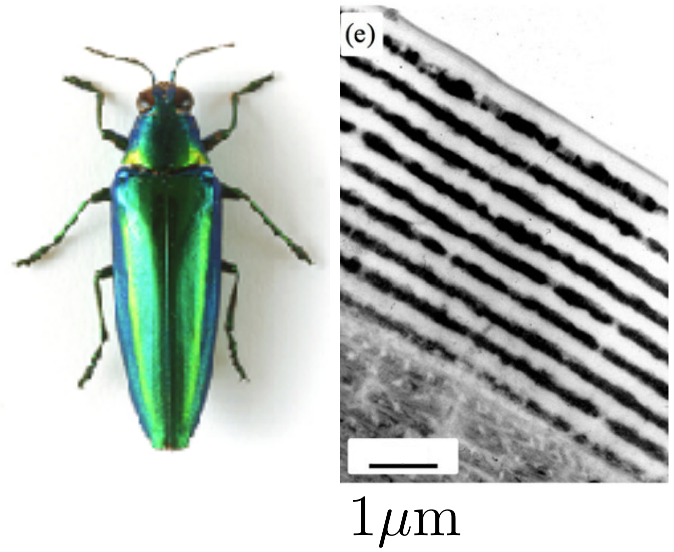
$$d_L = \frac{\lambda_0}{4n_L} = 80\,\text{nm}$$

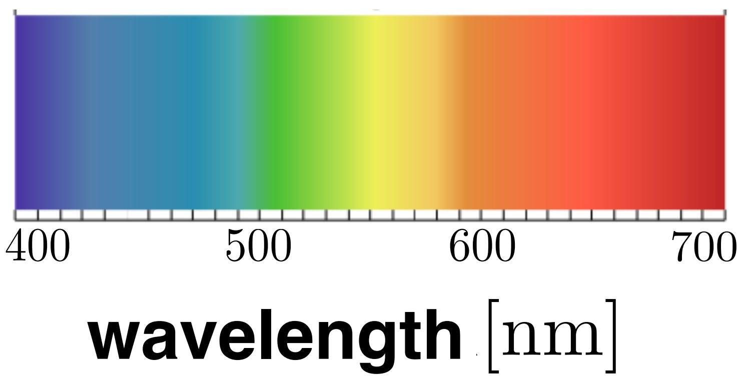
We would like to design periodic structure, which preferentially reflects green color with $\lambda_0 = 500\,\text{nm}$.



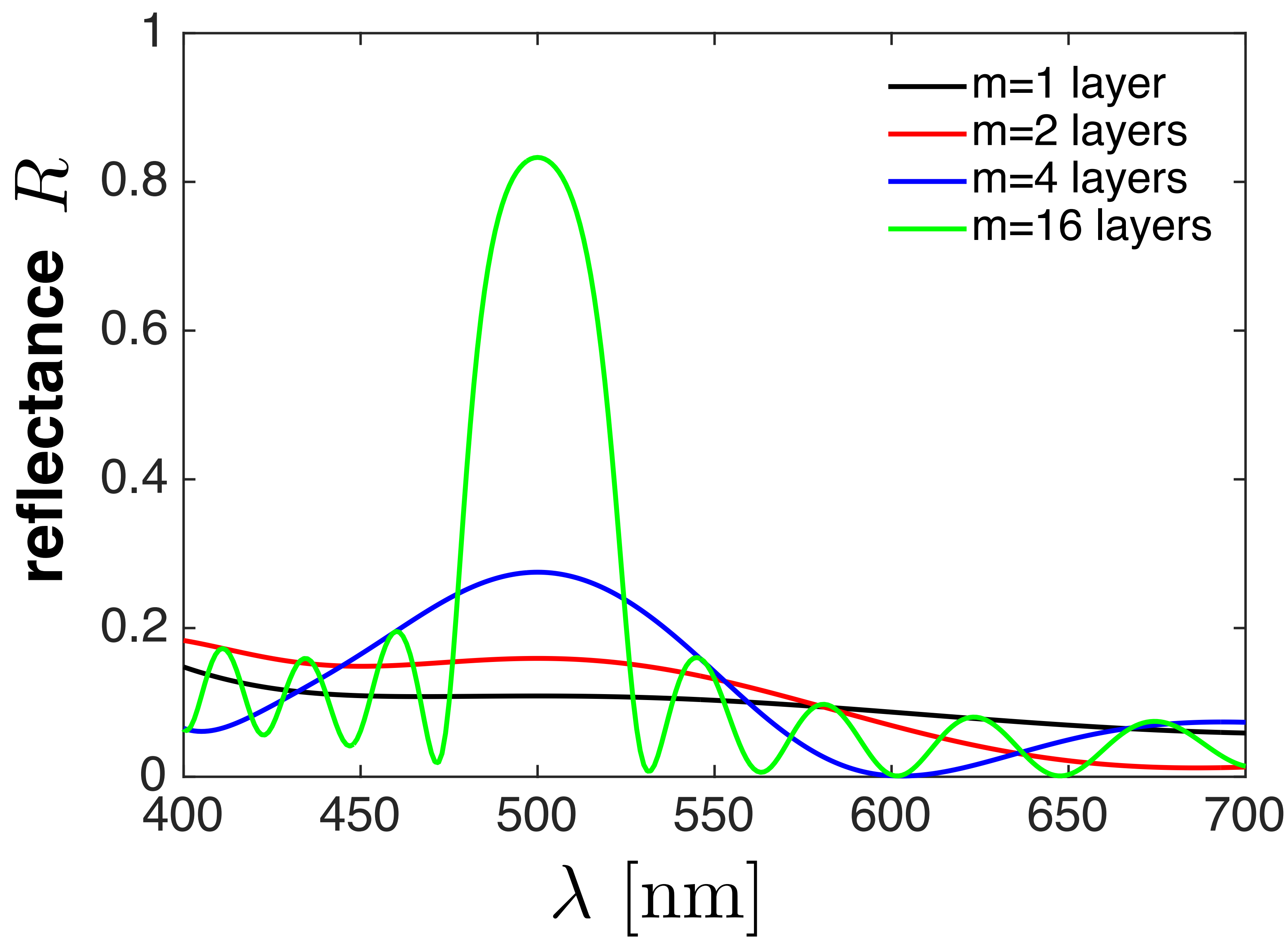
Example

Chrysochroa raja bettle



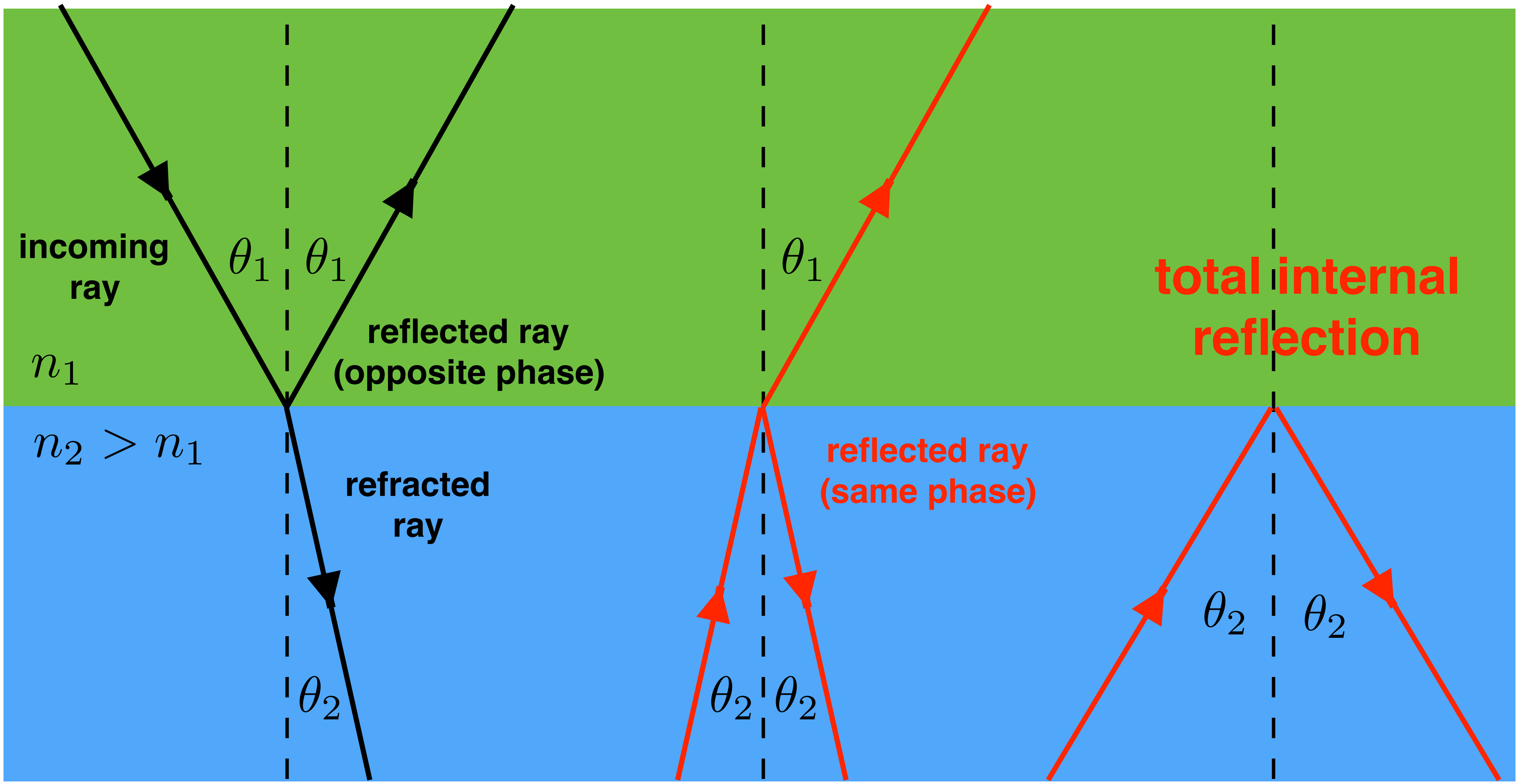


Multiple periodic layers increase the reflectance of target wavelength $\lambda_0 = 500\,\mathrm{nm}$!



In periodic structures high reflectance is achieved for a range of wavelengths around the target wavelength. This range is called band gap.

Refraction of light



Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

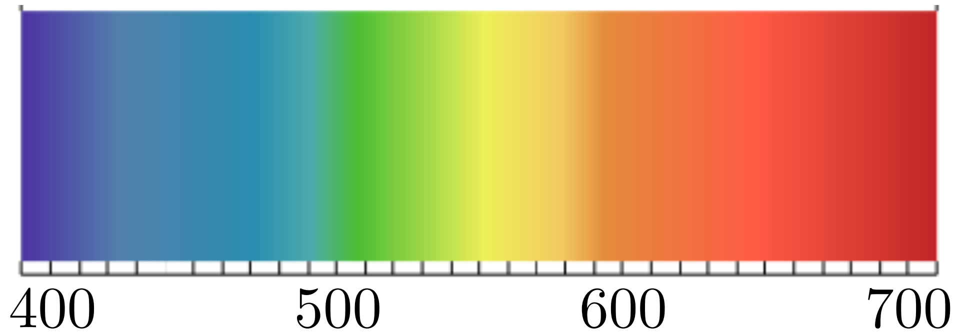
Total internal reflection

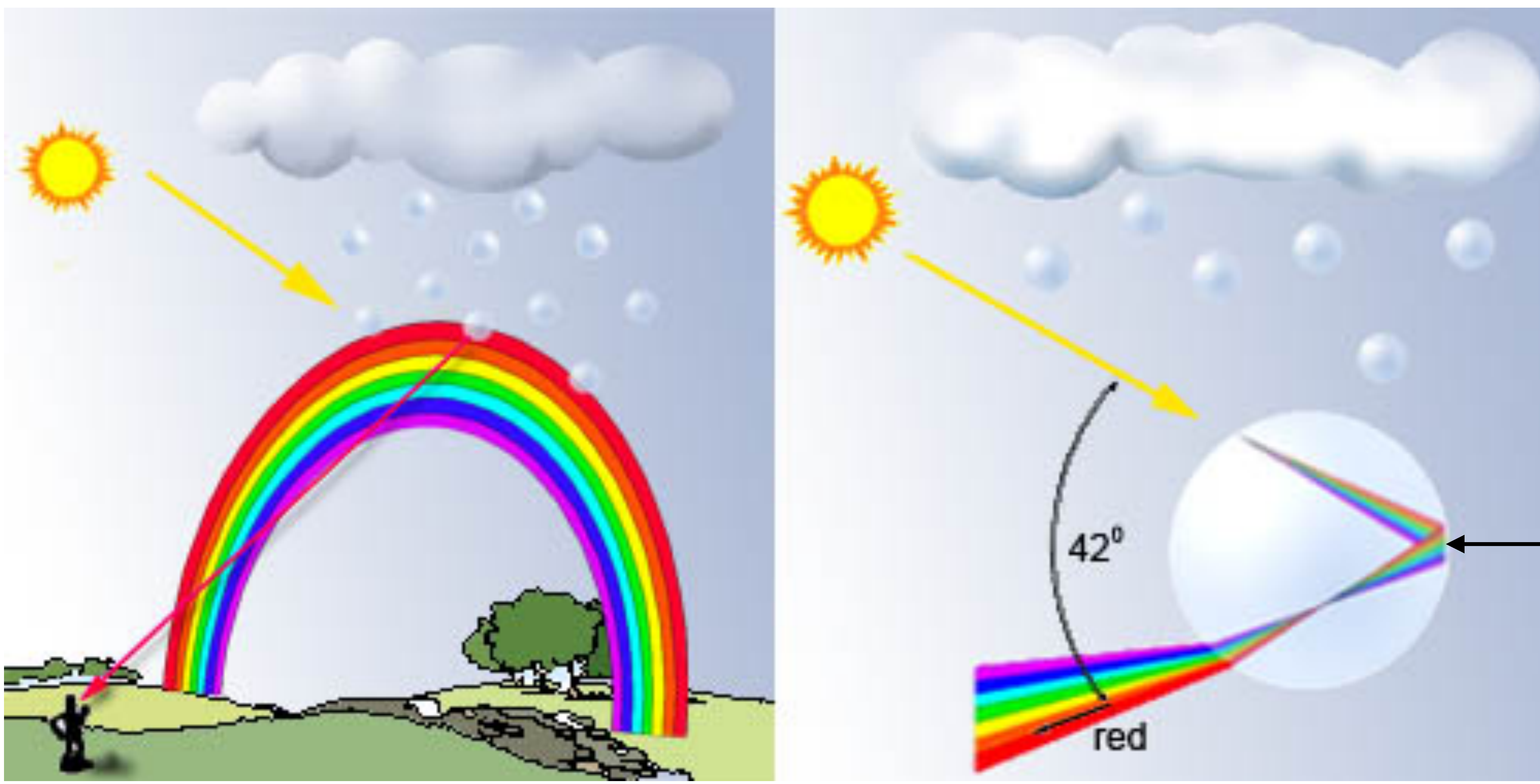
$$\theta_2 > \arcsin(n_1/n_2)$$

Rainbow

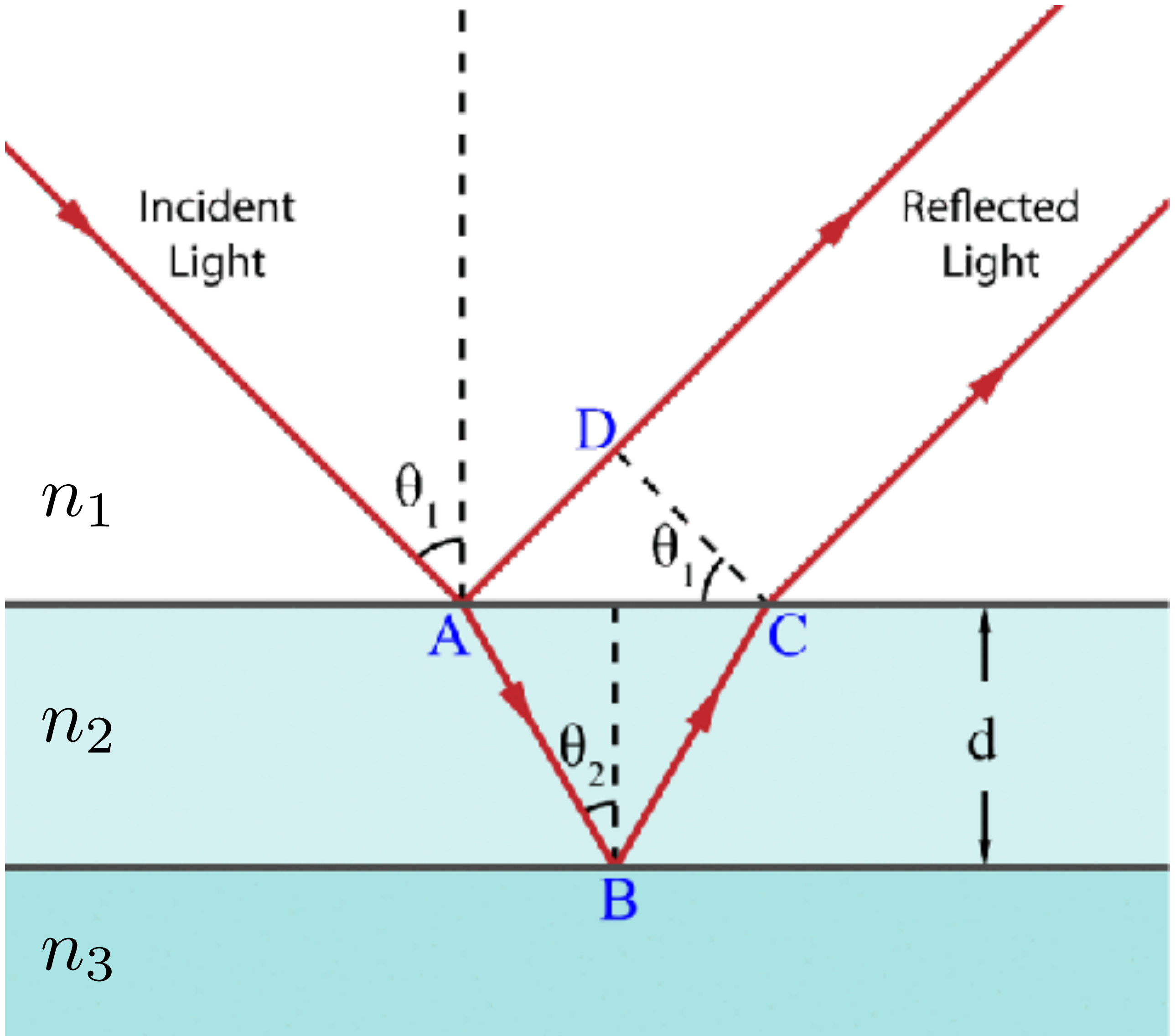
Rainbow forms because refraction index *n* in water droplets depends on the color (wavelength) of light.

$$n_{\text{purple}} > n_{\text{blue}} > n_{\text{green}} > n_{\text{yellow}} > n_{\text{orange}} > n_{\text{red}}$$





Interference on thin films



difference between optical path lengths of the two reflected rays

$$OPD = n_2\left(\overline{AB} + \overline{BC}\right) - n_1 \overline{AD}$$

$$OPD = 2n_2 d\cos(\theta_2)$$

no additional phase difference due to reflections

$n_1 < n_2 < n_3$ $\quad$ $n_1 > n_2 > n_3$

constructive interference

$$OPD = m\lambda$$

destructive interference

$$OPD = (m + 1/2)\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

additional π phase difference due to reflections

$n_1 < n_2 > n_3$ $\quad$ $n_1 > n_2 < n_3$

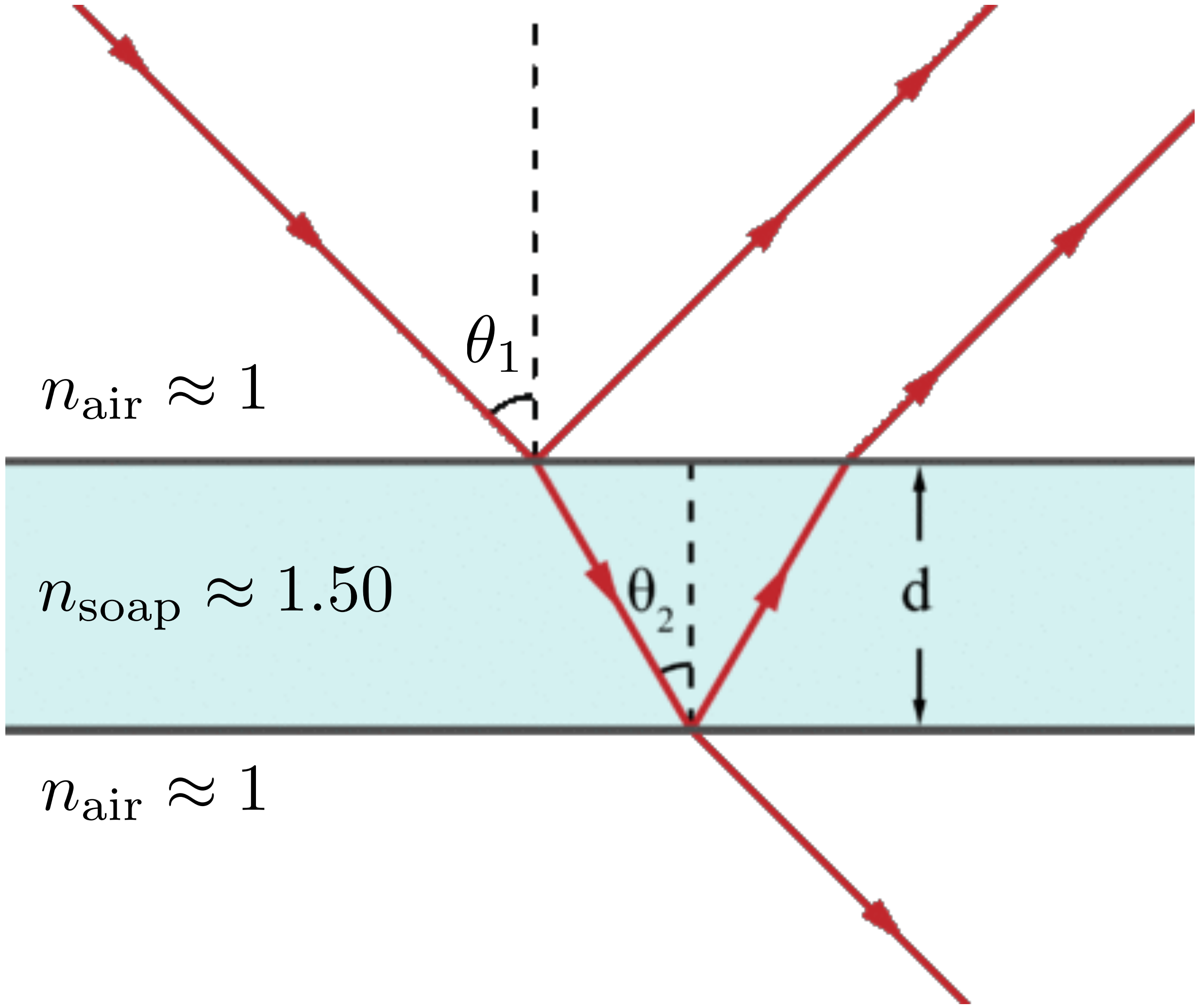
constructive interference

$$OPD = (m + 1/2)\lambda$$

destructive interference

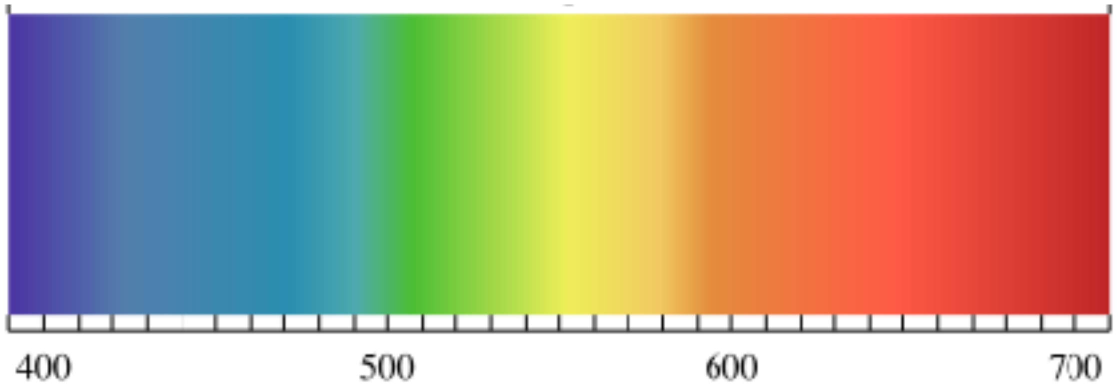
$$OPD = m\lambda$$

Interference on soap bubbles



soap bubble

visible spectrum



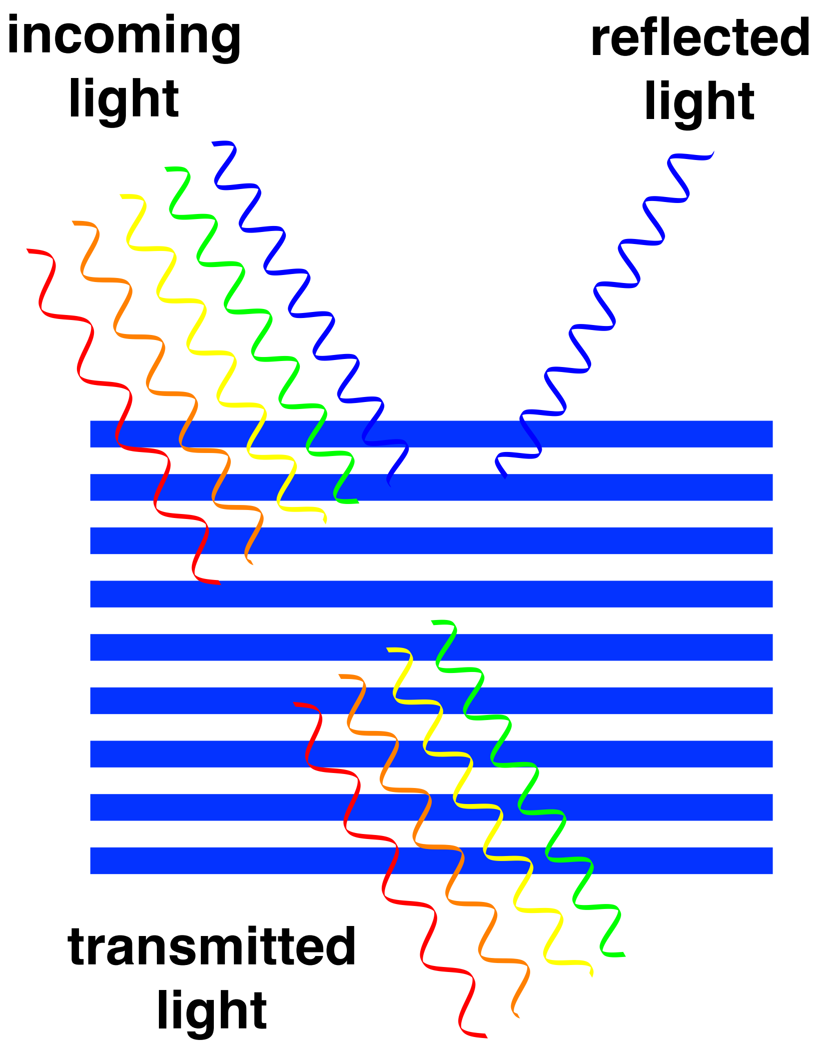
constructive interference for different colors happens at different angles

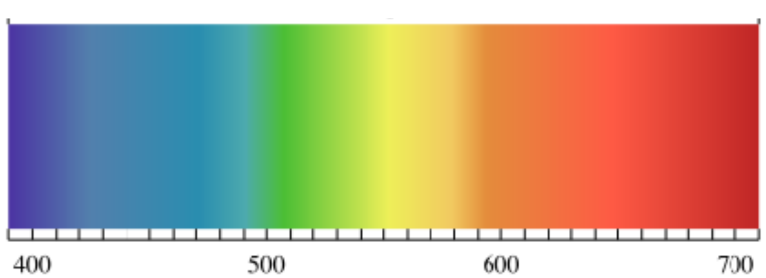
$$2dn_{\text{soap}}\cos(\theta_2) = (m + 1/2)\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Single structural color

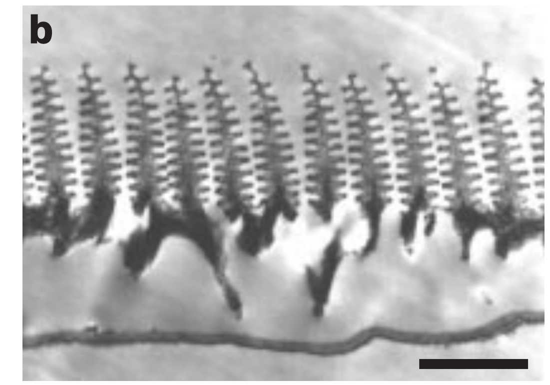
Single reflected color on structures with uniform spacing





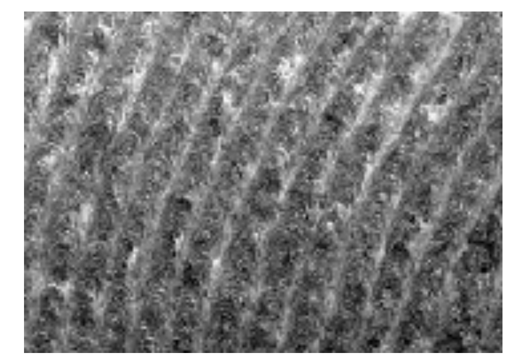
wavelength $\lambda[\mathrm{nm}]$

Morpho butterfly



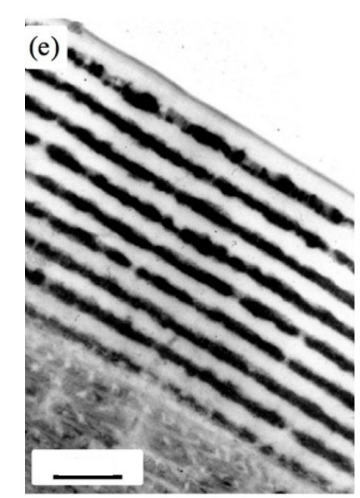
$1.7\mu\mathrm{m}$

Marble berry



$250\mathrm{nm}$

Chrysochroa raja bettle



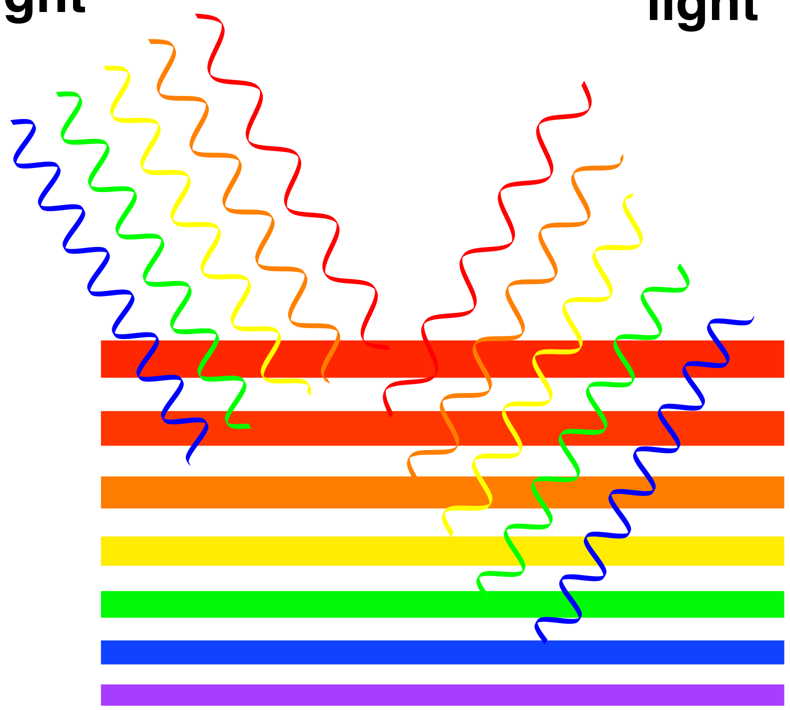
$1\mu\mathrm{m}$

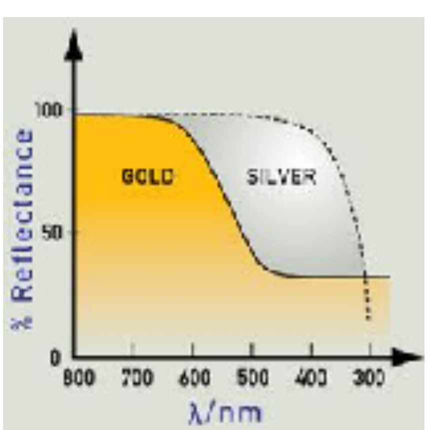
Silver and gold structural colors

Many colors reflected on structures with varying spacing

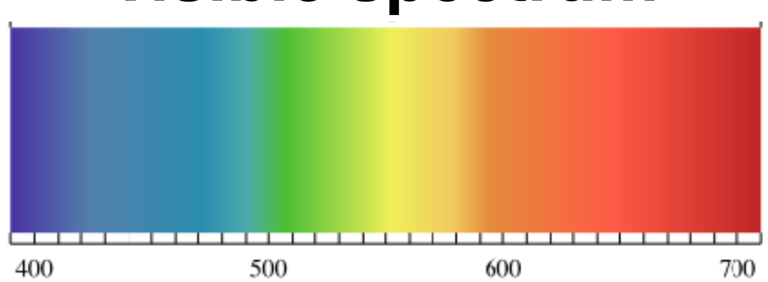
incoming light

reflected light





visible spectrum

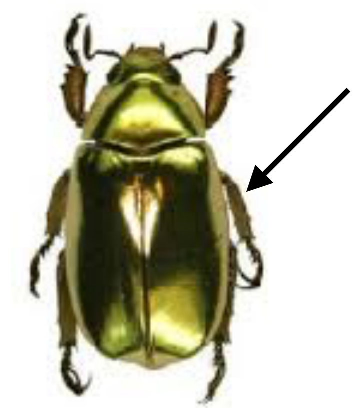


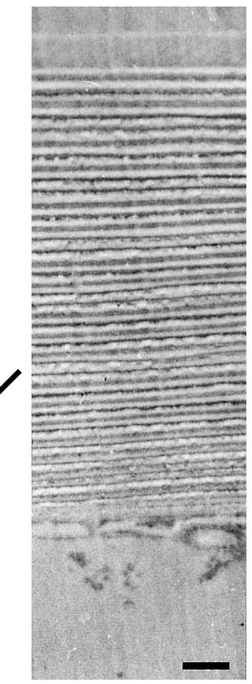
wavelength $\lambda[\mathrm{nm}]$

chirped structure

Chrysina limbata beetle

Chrysina aurigans beetle



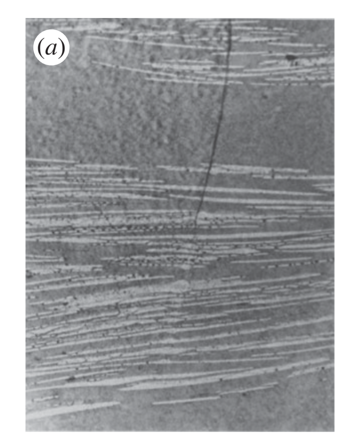


$1\mu\mathrm{m}$

thicker → thinner

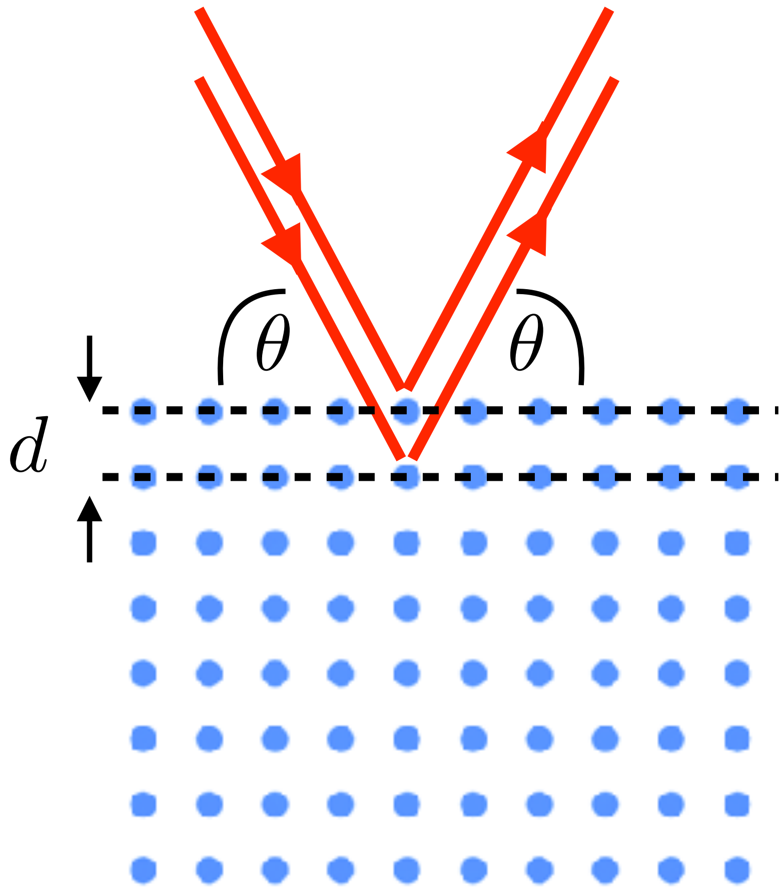
disordered layer spacing

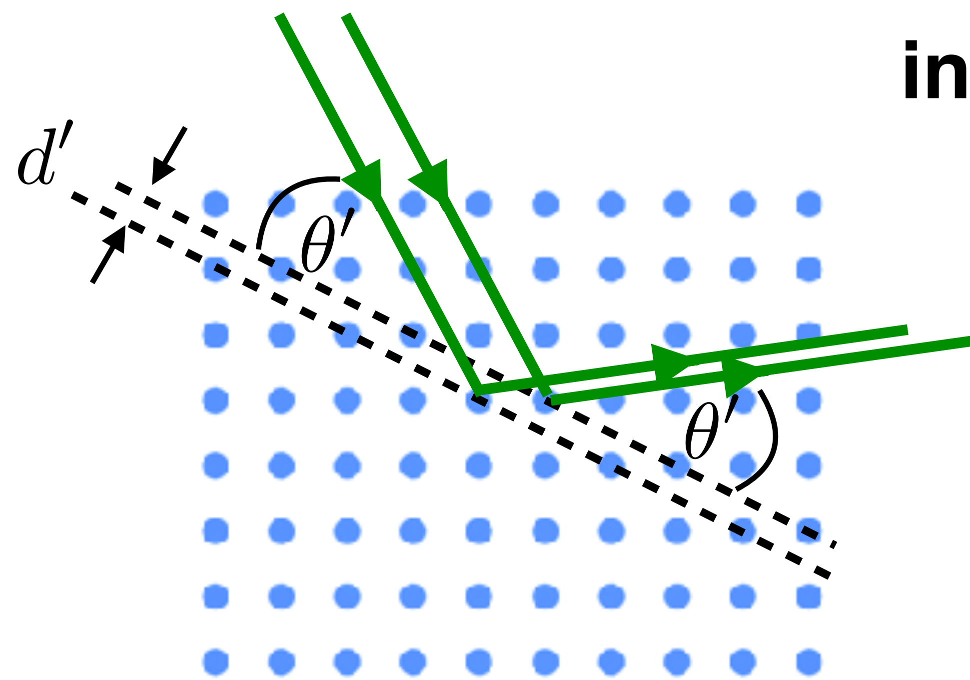
bleak fish



$1\mu\mathrm{m}$

Bragg scattering on crystal layers





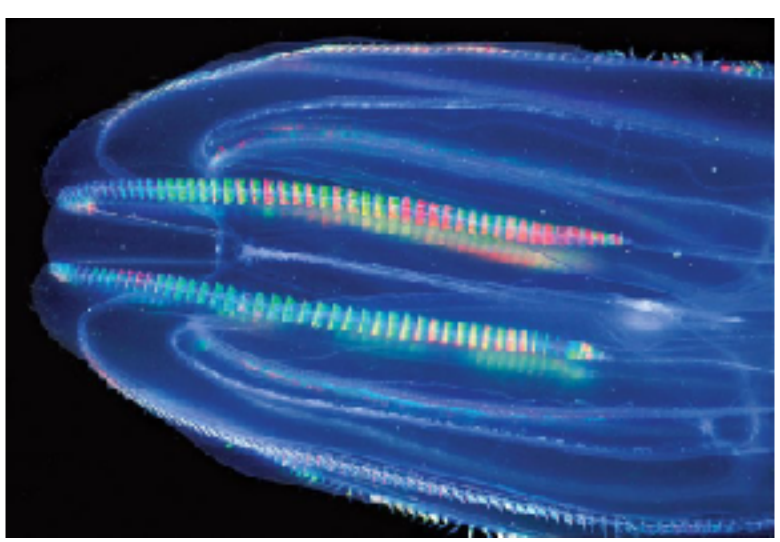
constructive interference condition

$$2d \sin\theta = m\lambda$$

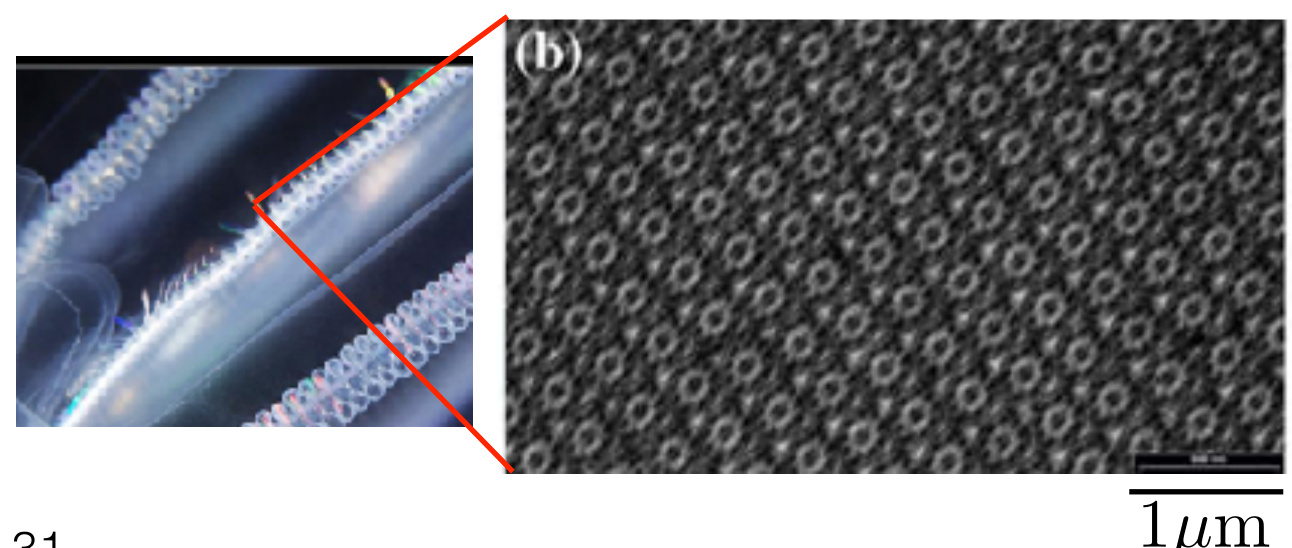
$$2d' \sin\theta' = m\lambda'$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Comb jelly

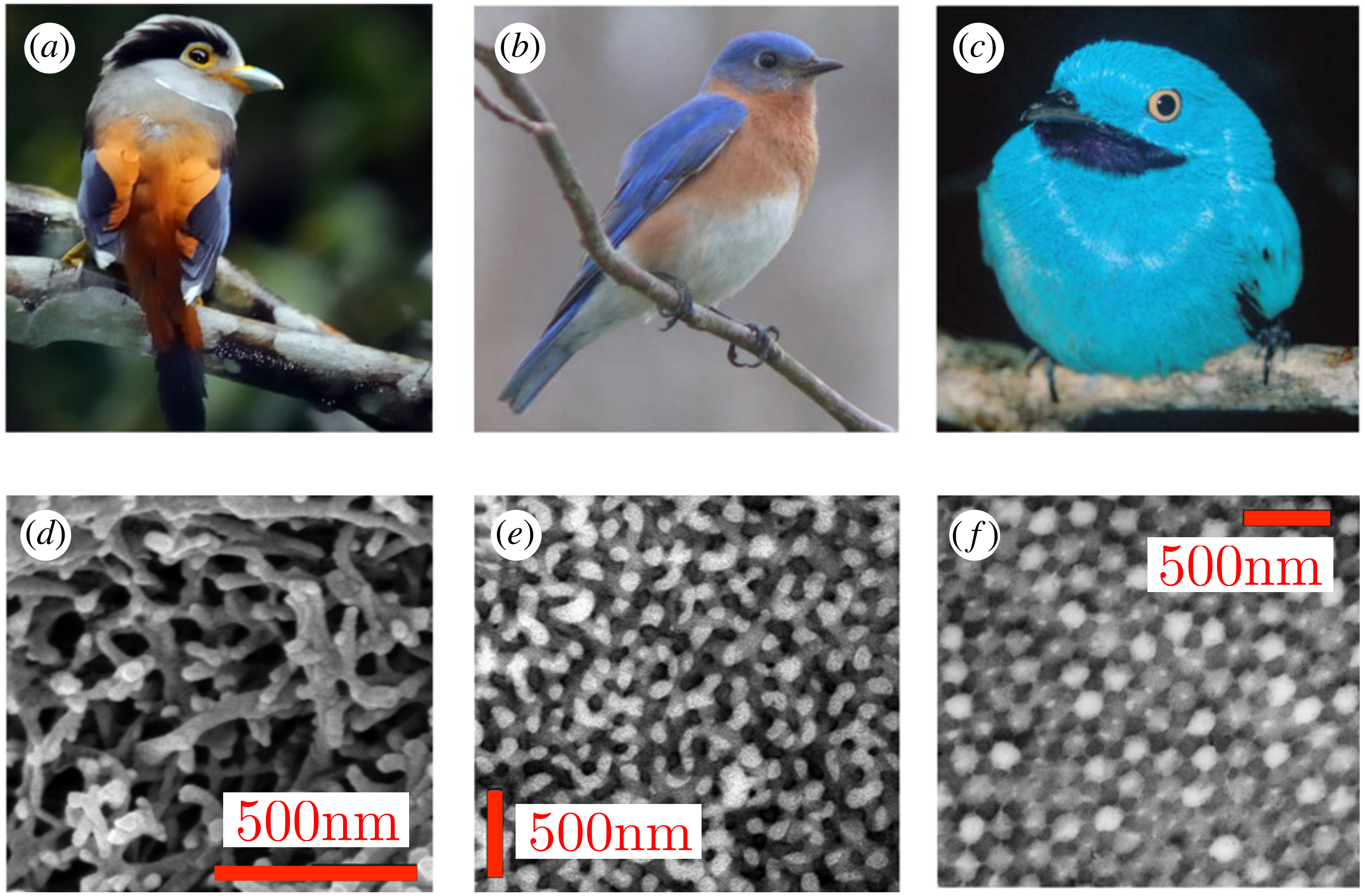


Beating cilia are changing crystal orientation



Scattering on disordered structures

Silver-breasted Broadbill | Eastern bluebird | Plum-throated Cotinga



Disordered structures with a characteristic length scale.

This length scale determines what light wavelengths are preferentially scattered.

This gives rise to blue colors in birds above.

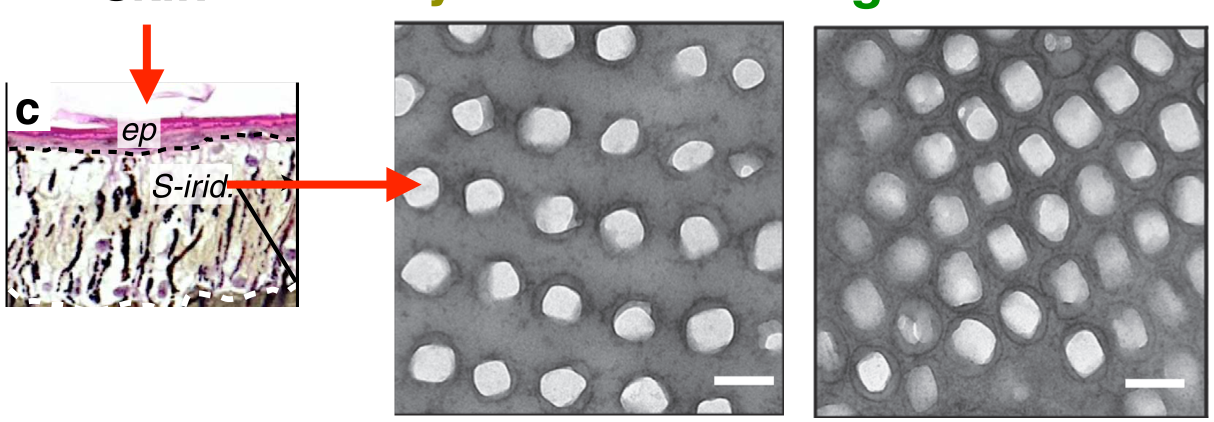
Dynamic structural colors

Chameleon (speed 8x)

J. Teyssier et al., Nat. Comm. 6, 6368 (2015)

Changes in osmotic concentration lead to the swelling of cells in excited chameleon. This changes the spacing of periodic structure from which the ambient light is reflected.

skin **yellow color** **green color**

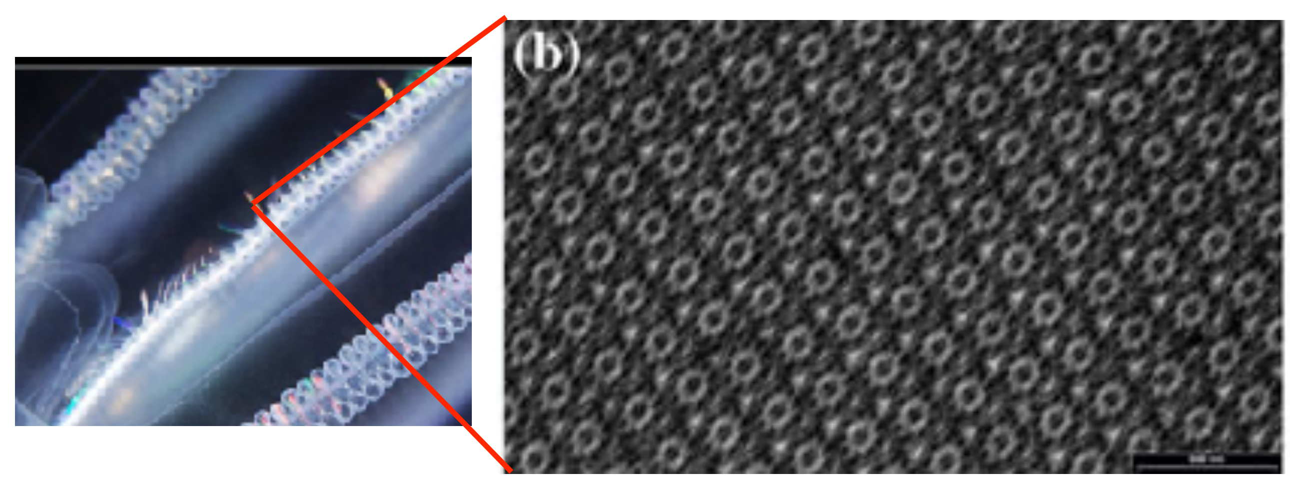


200nm 200nm

Comb Jelly (real time)

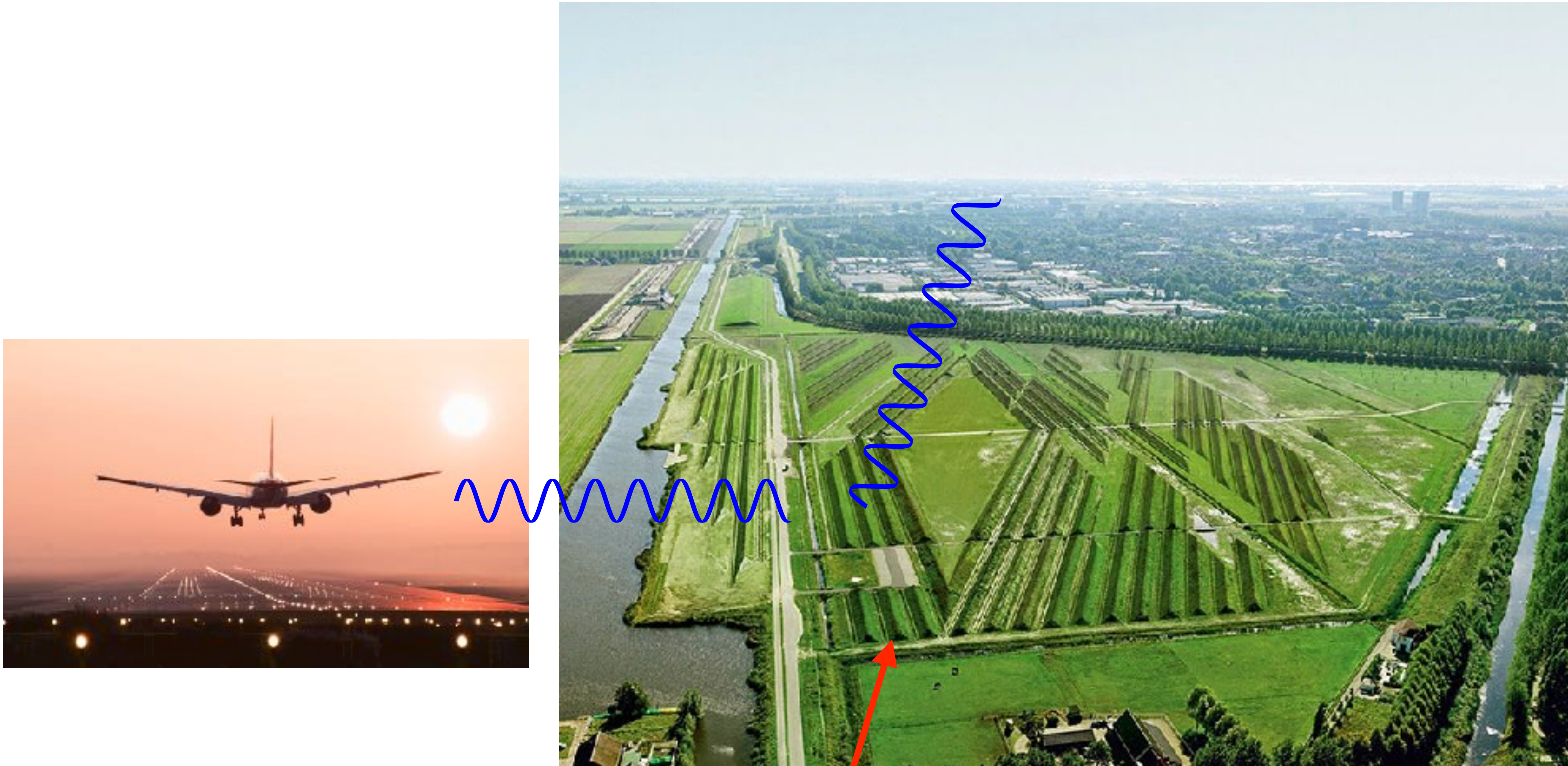
https://www.youtube.com/watch?v=Qy90d0XvJIE

Rainbow color waves are produced by the beating of cilia, which change the orientation of periodic structure from which the ambient light is reflected.



$1\mu m$

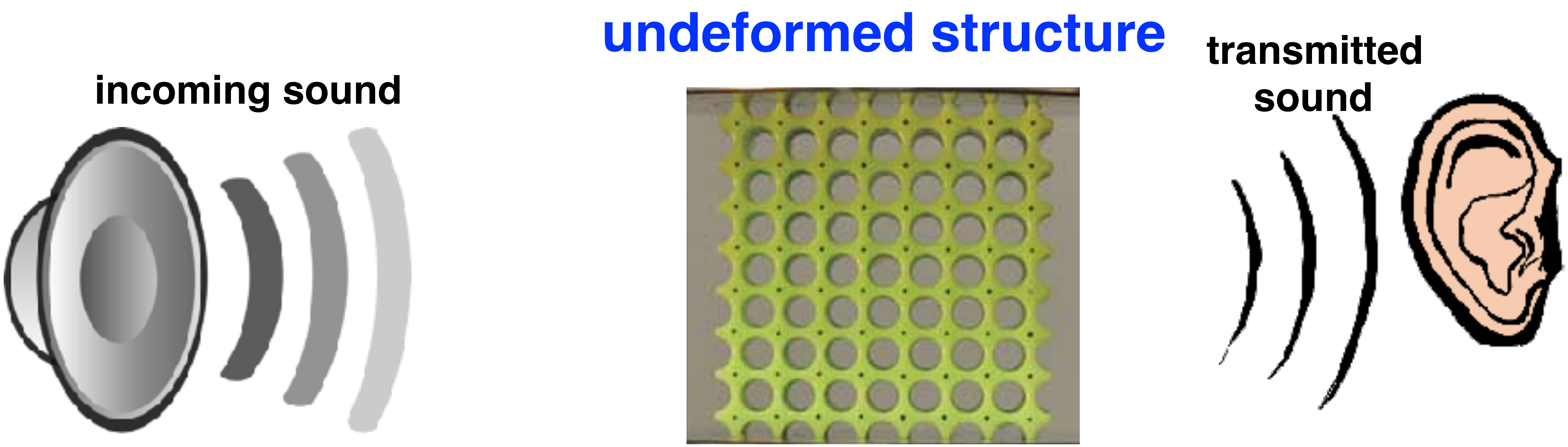
Noise barriers around the Amsterdam airport

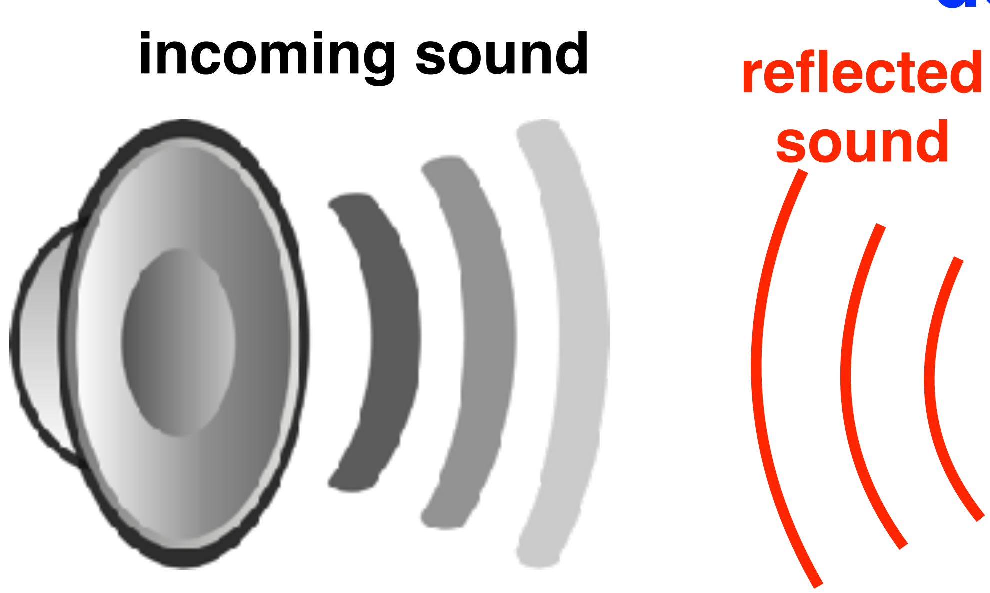


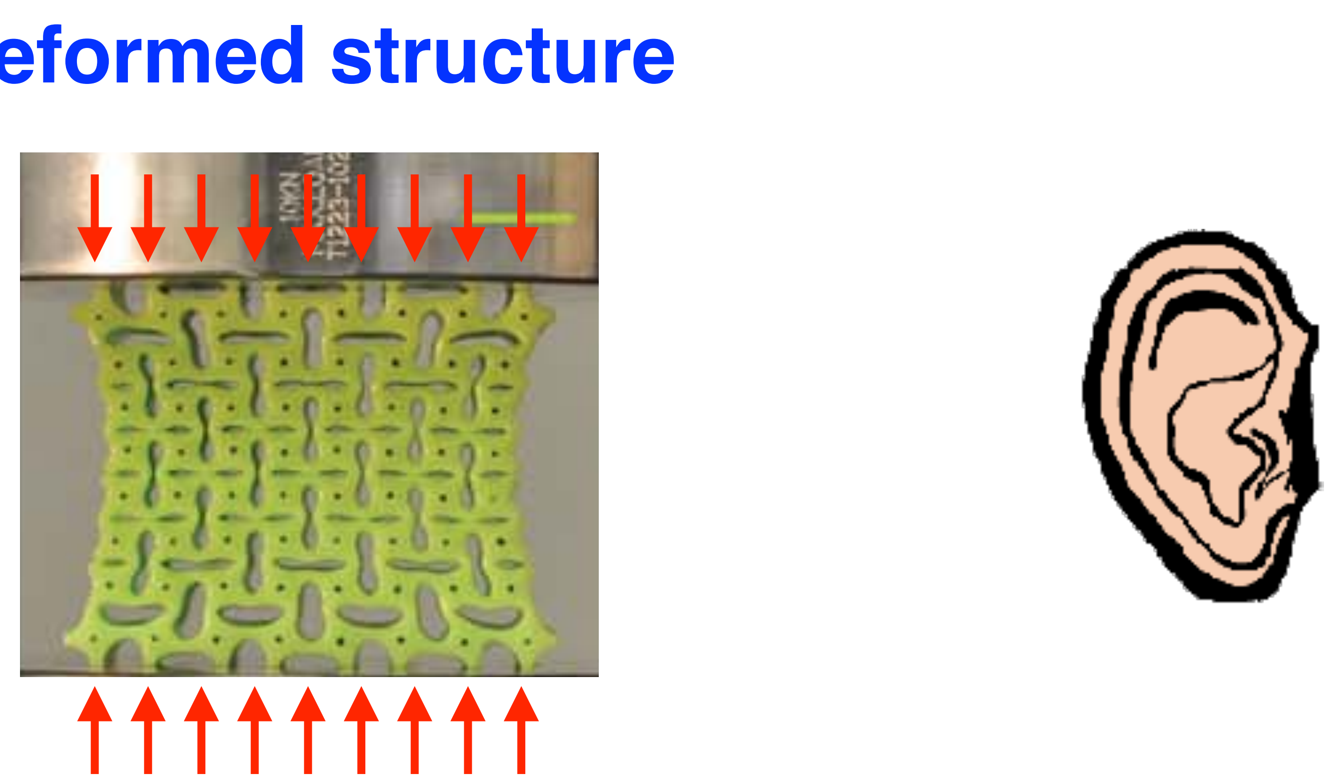
Sound from airplanes that are landing and taking off is reflected from artificial barriers into the atmosphere.

Controllable sound filters

In periodic structures sound waves of certain frequencies (within a "band gap") cannot propagate. The range of "band gap" frequencies depends on material properties, the geometry of structure and the external load. Note: the wavelength of sound waves within the "band gap" is comparable to the size of the periodic unit cell!

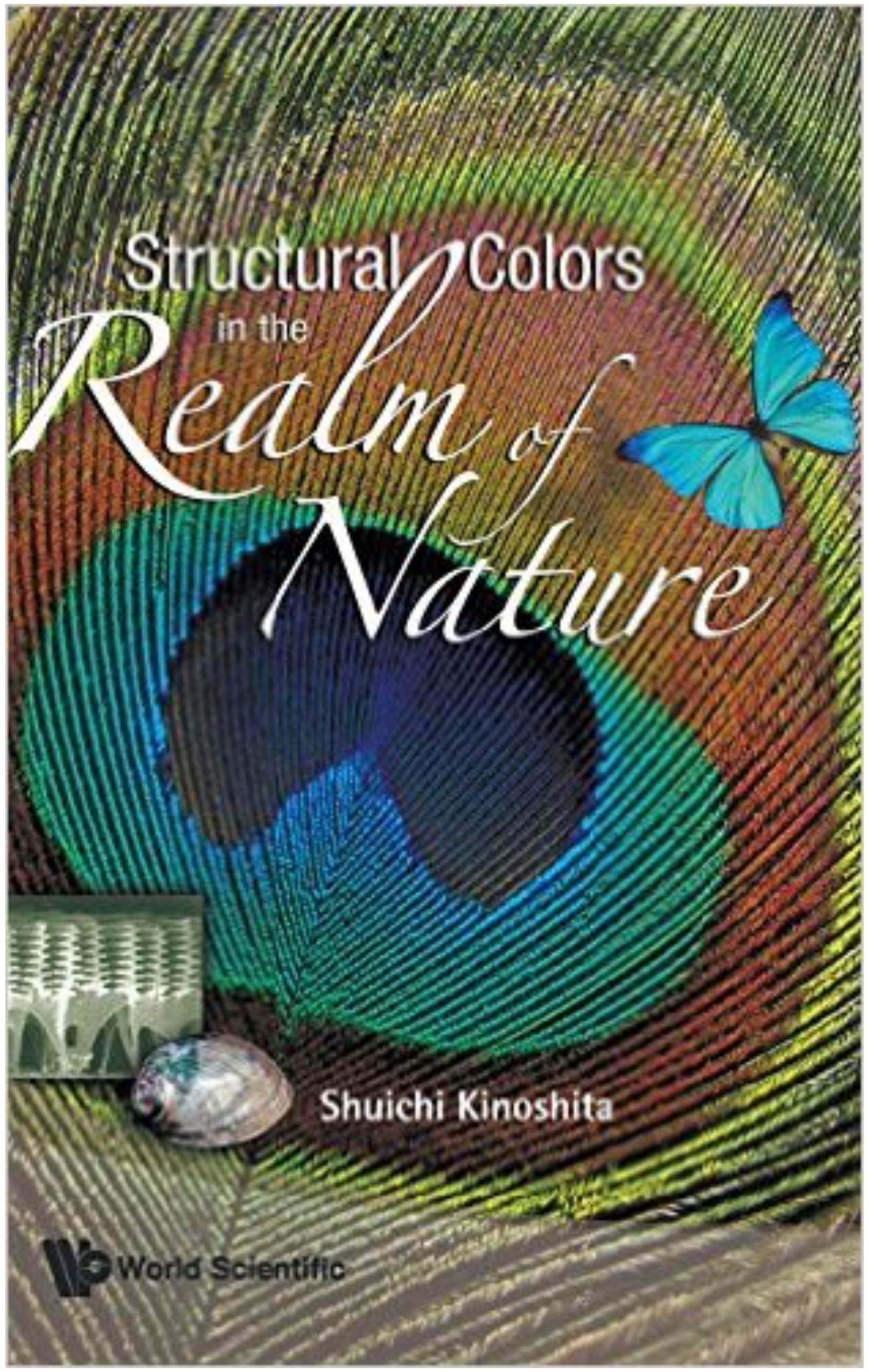


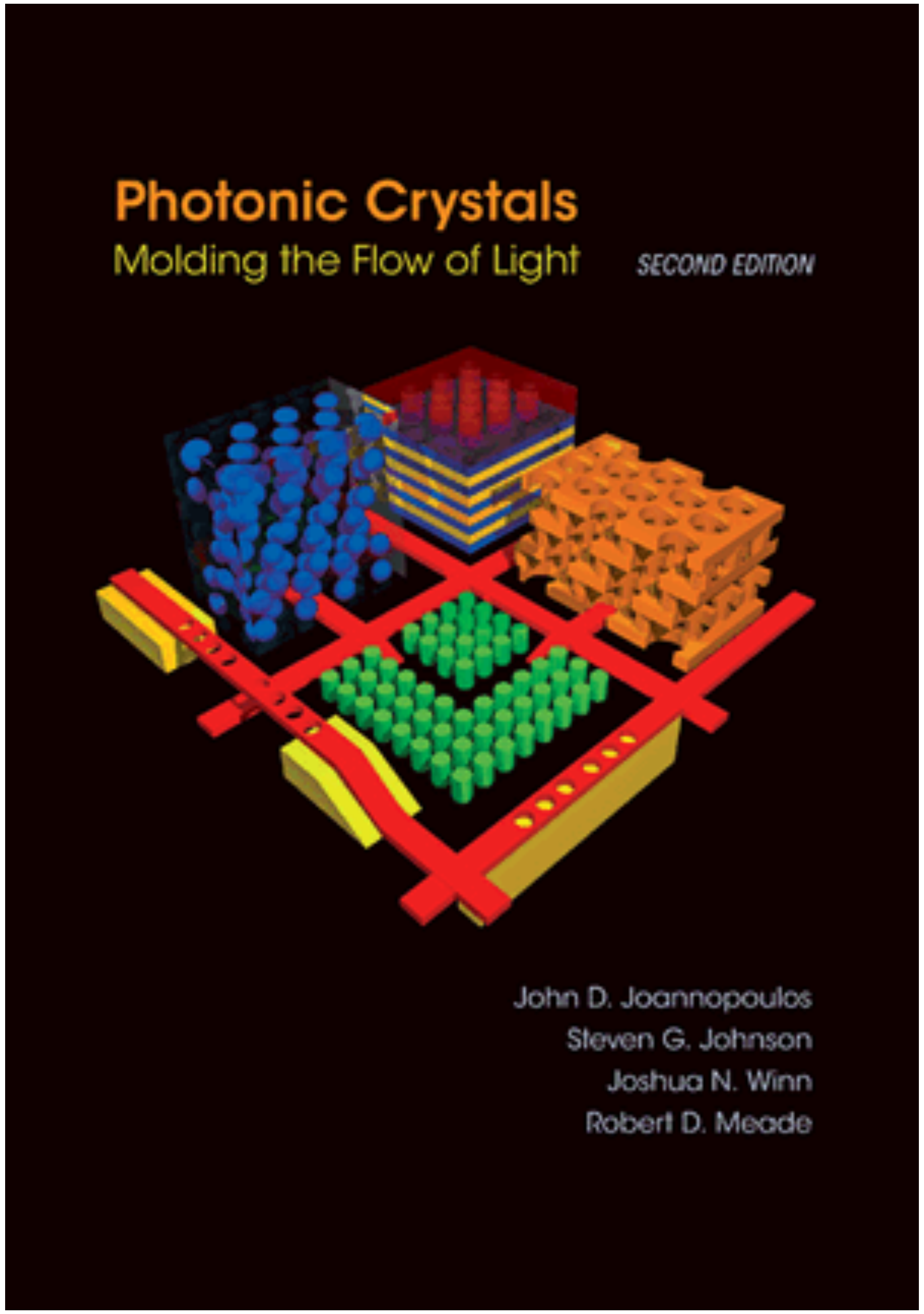




P. Wang, J. Shim and K. Bertoldi, <u>PRB</u> **88**, 014304 (2013)

Further reading





http://ab-initio.mit.edu/book/