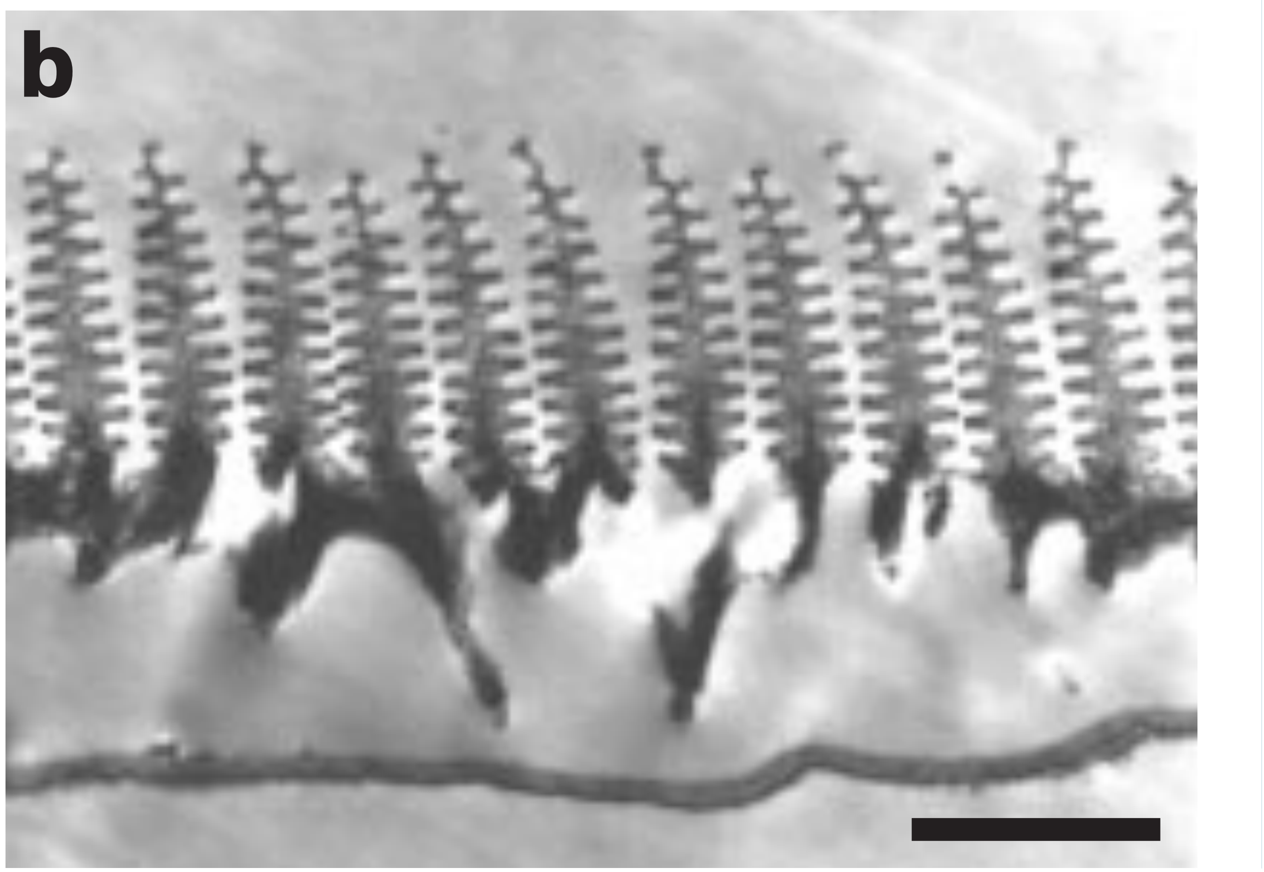
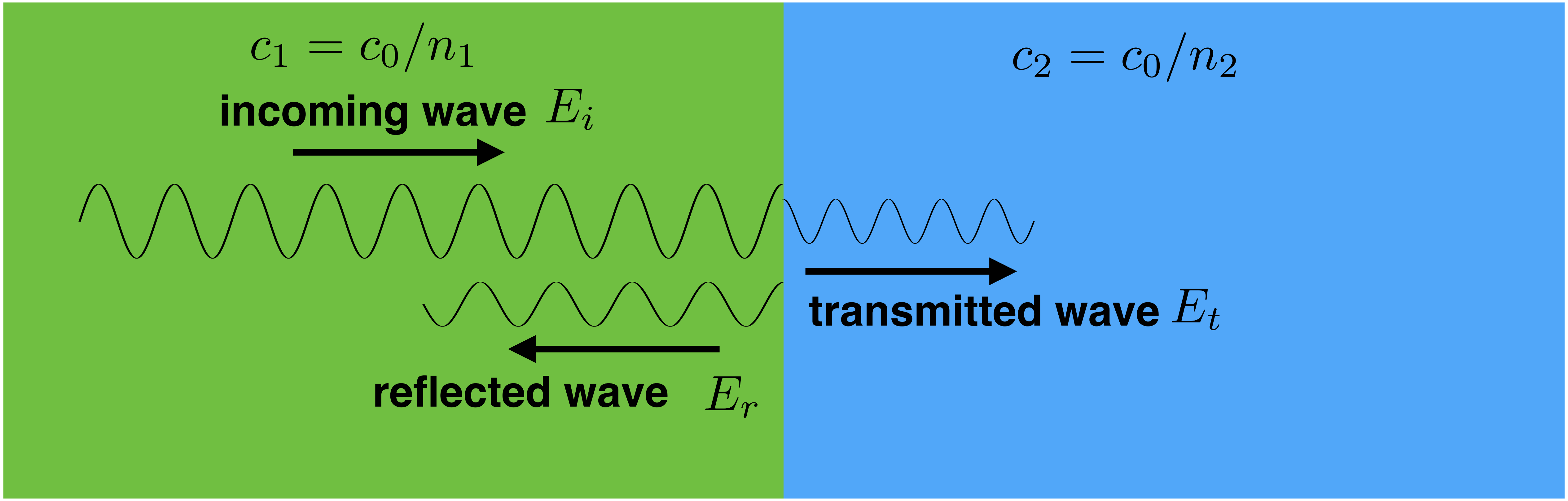
MAE 545: Lecture 3 (2/14)

Structural colors



$1.7 \mu m$

Reflection of light at the interface between two media



boundary conditions for incident waves normal to the interface:

$$E_1 = E_2 \qquad \frac{\partial E_1}{\partial x} = \frac{\partial E_2}{\partial x}$$

amplitude of reflected electric field

$$r \equiv \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

amplitude of transmitted electric field

$$t \equiv \frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2}$$

energy density of electromagnetic waves

$$\propto n|E|^2$$

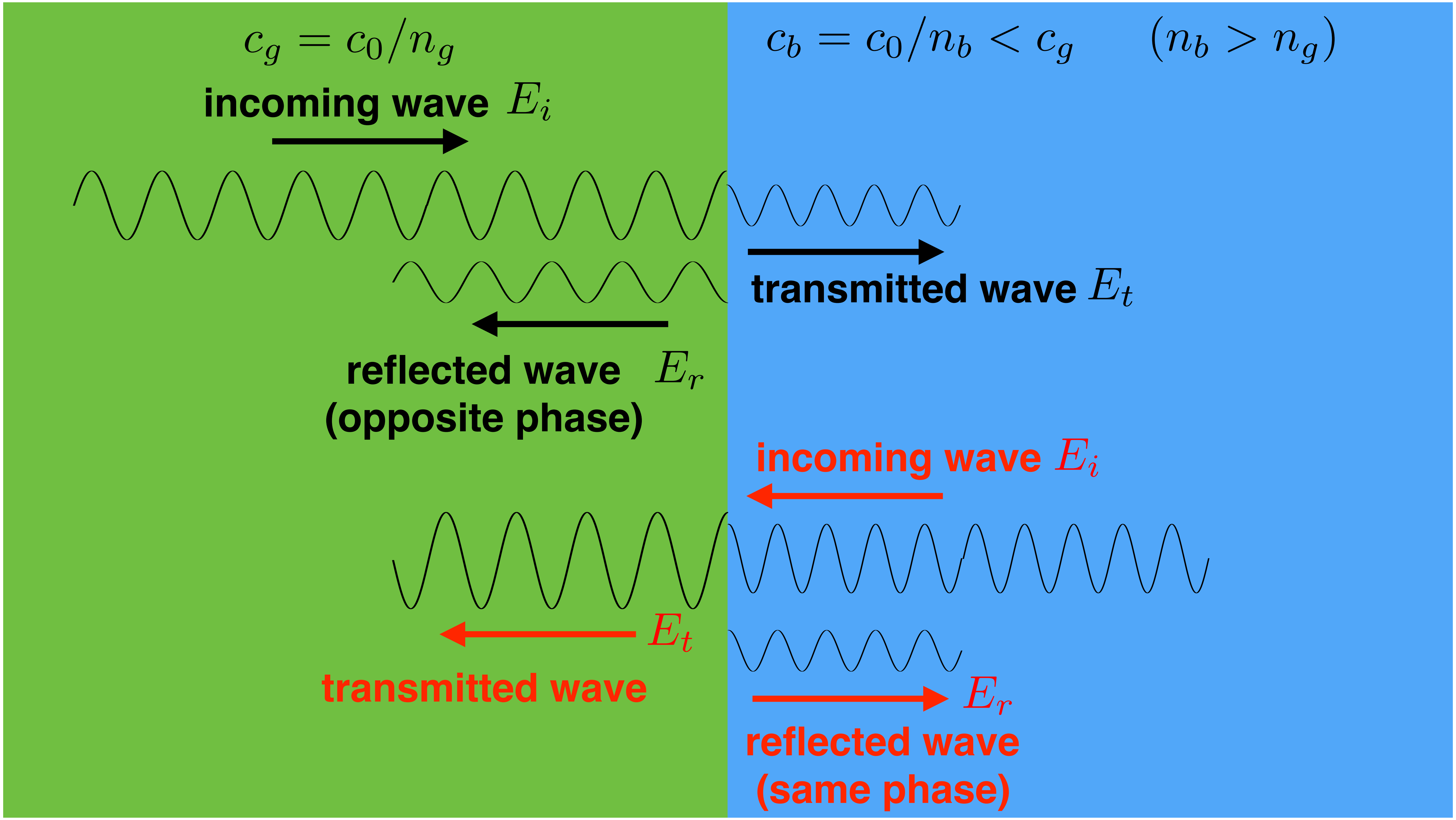
reflectance

$$R \equiv \frac{n_1|E_r|^2}{n_1|E_i|^2} = |r|^2$$

transmittance

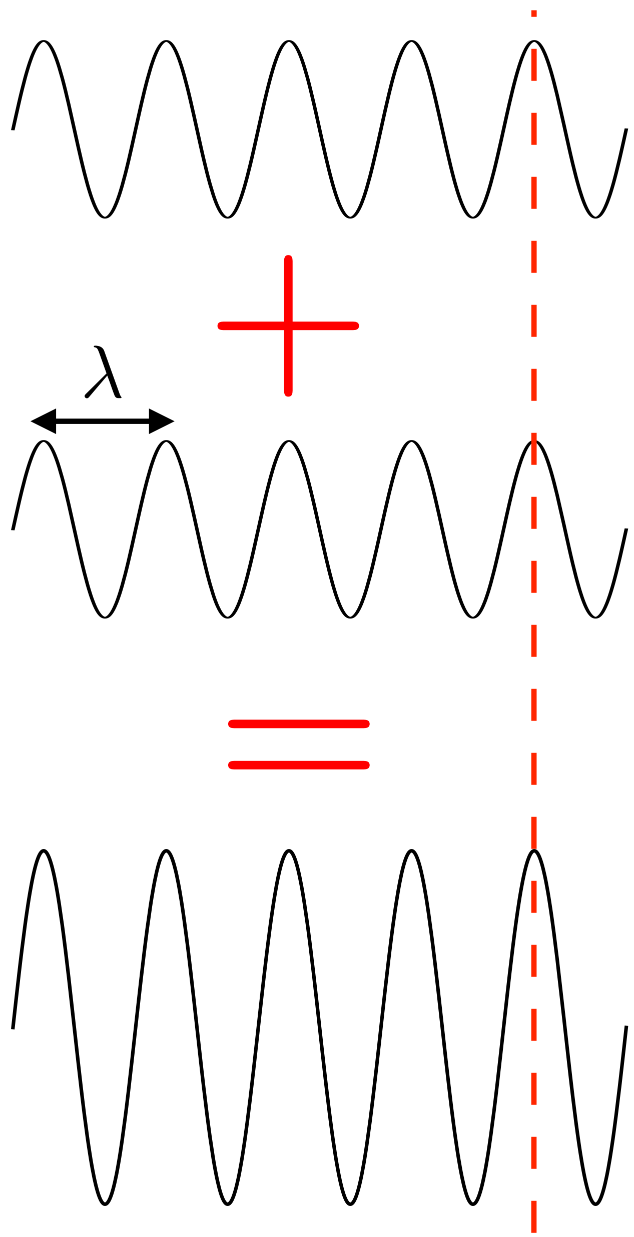
$$T \equiv \frac{n_2|E_t|^2}{n_1|E_i|^2} = |t|^2 \frac{n_2}{n_1} = 1 - R$$

Reflection of light at the interface between two media

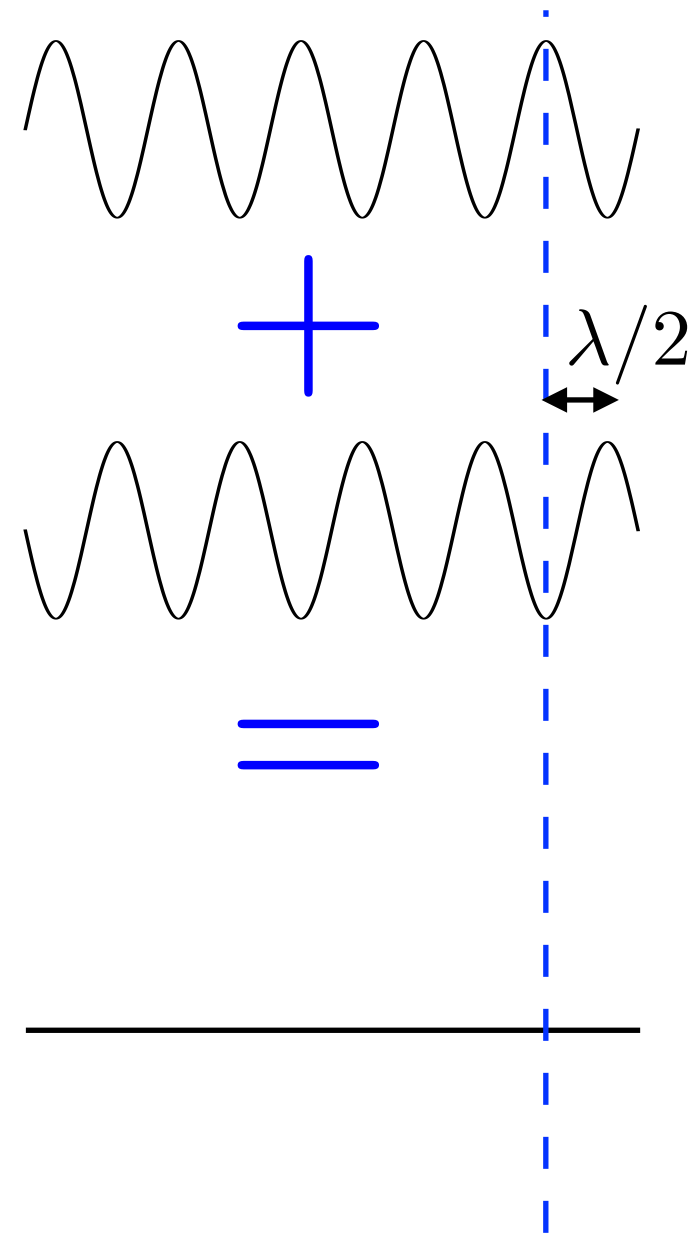


Interference

constructive interference



destructive interference



Constructive interference occurs when the two waves are in phase: waves offset by $m\lambda$ **,**

$$m = 0, \pm 1, \pm 2, \ldots$$

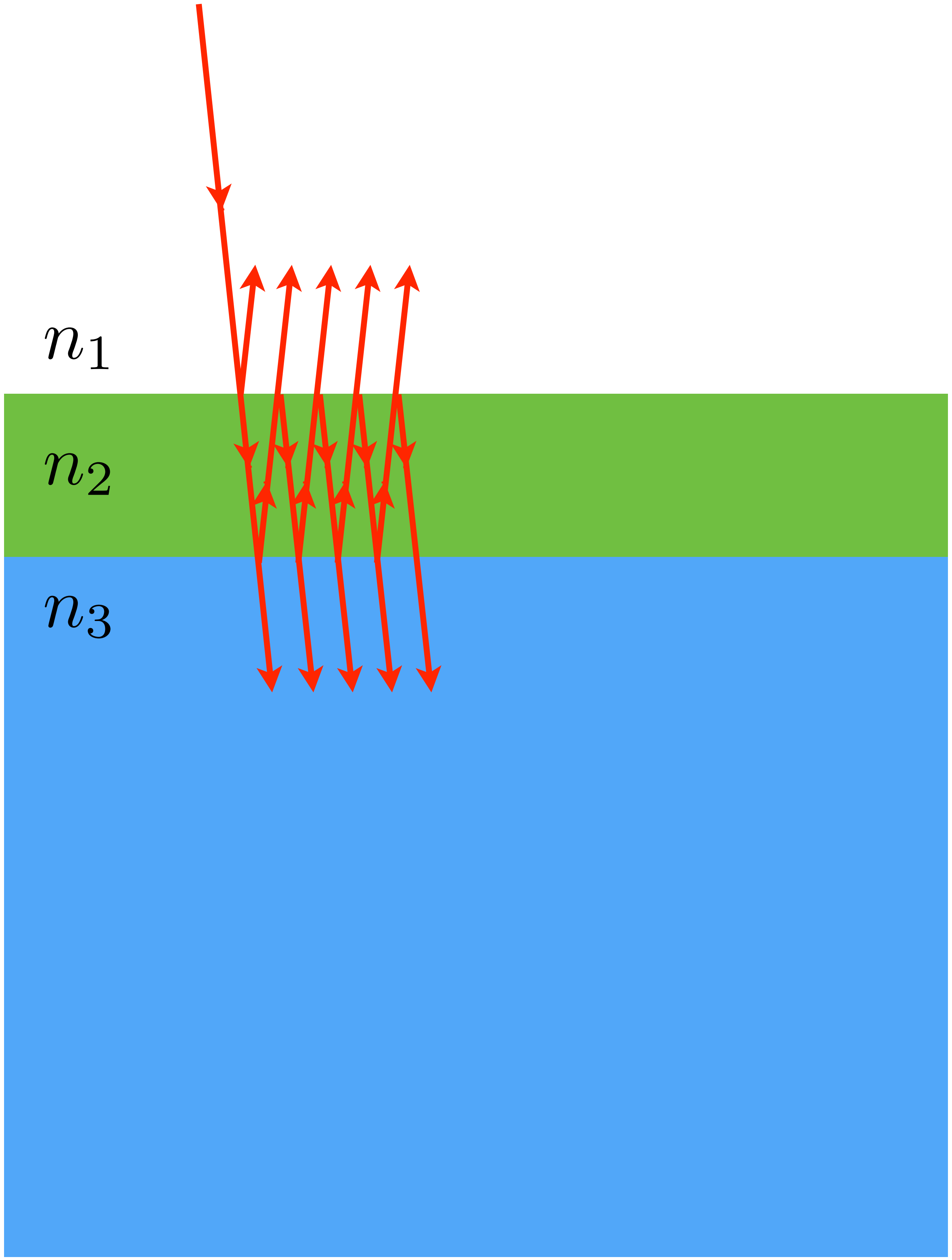
$$e^{ikm\lambda} = e^{i2\pi m} = +1$$

Destructive interference occurs when the two waves are out of phase: waves offset by $(m + 1/2)\lambda$ **,**

$$m = 0, \pm 1, \pm 2, \ldots$$

$$e^{ik(m+1/2)\lambda} = e^{i(2\pi m + \pi)} = -1$$

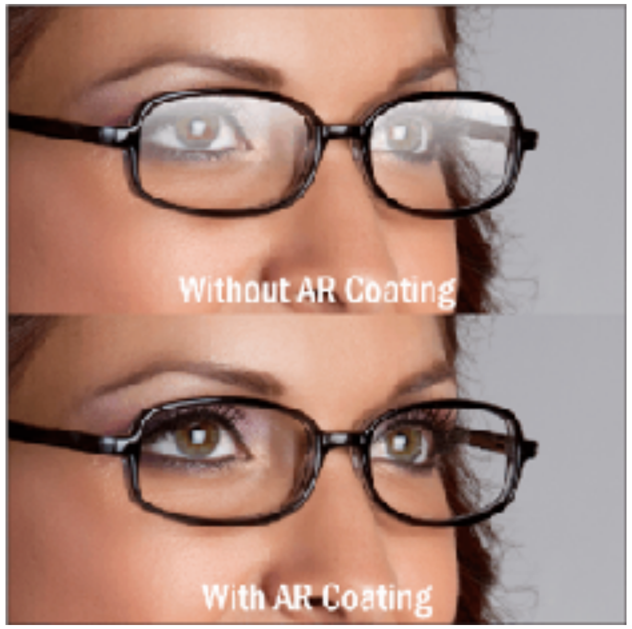
Interference on thin films



Constructive interference of reflected rays results in strongly reflected rays with very little transmission.

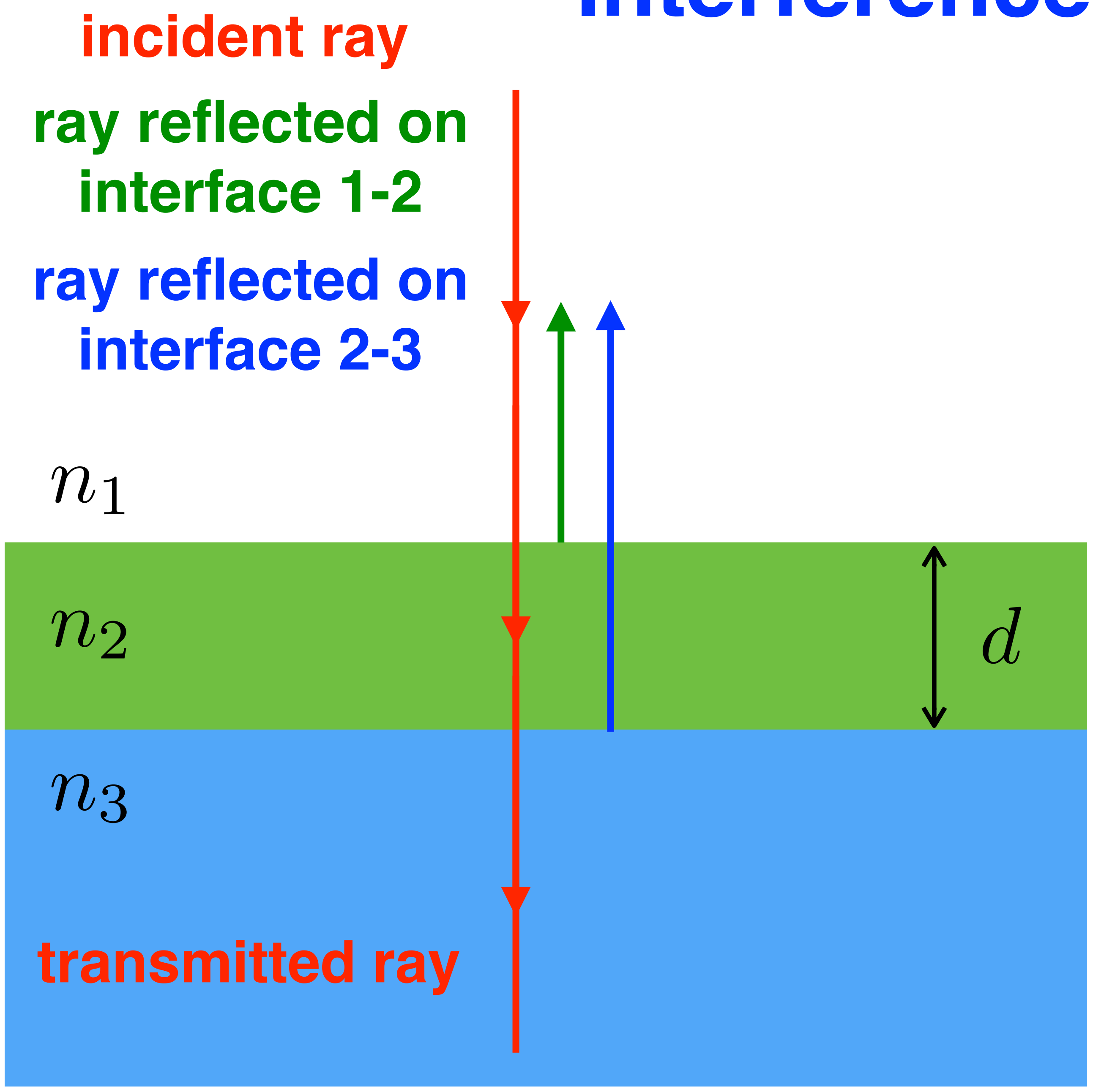
mirrors

Deconstructive interference of reflected rays results in almost perfectly transmitted rays with very little reflection.



antireflective coatings

Interference on thin films



difference between optical path lengths of the two reflected rays

$$OPD = 2n_2d$$

no additional phase difference due to reflections

$n_1 < n_2 < n_3$ $\quad$ $n_1 > n_2 > n_3$

constructive interference of reflected rays

$$OPD = m\lambda$$

destructive interference of reflected rays

$$OPD = (m + 1/2)\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

additional π phase difference due to reflections

$n_1 < n_2 > n_3$ $\quad$ $n_1 > n_2 < n_3$

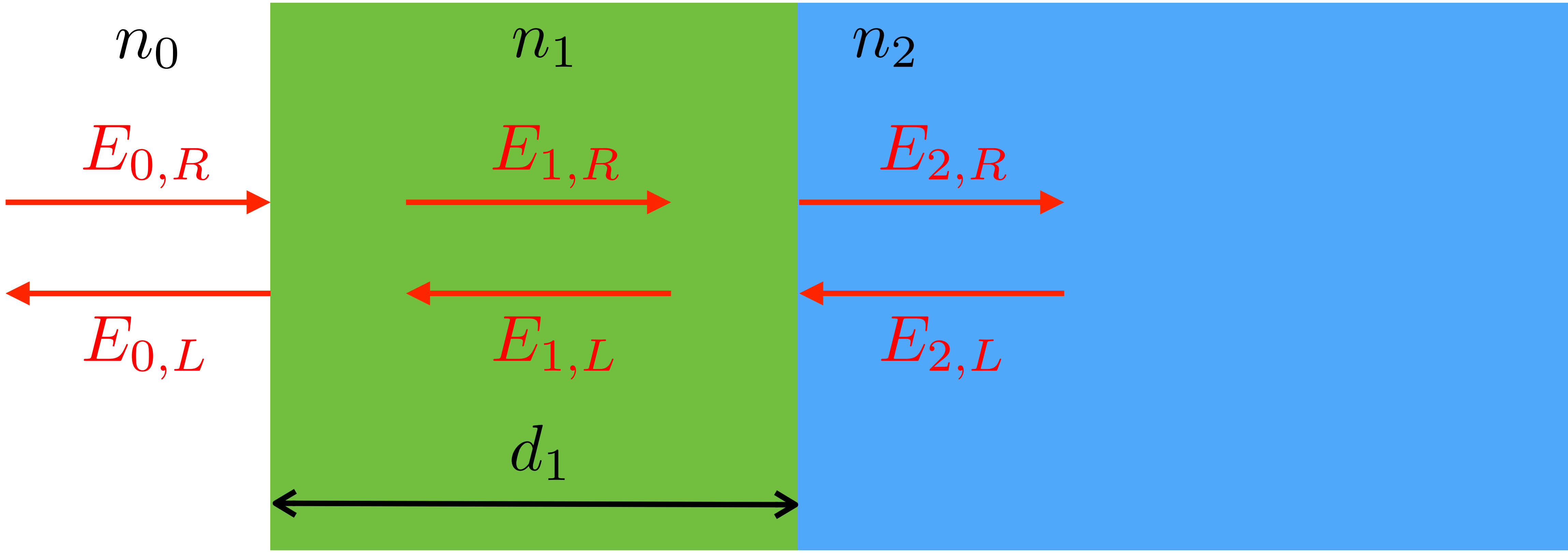
constructive interference of reflected rays

$$OPD = (m + 1/2)\lambda$$

destructive interference of reflected rays

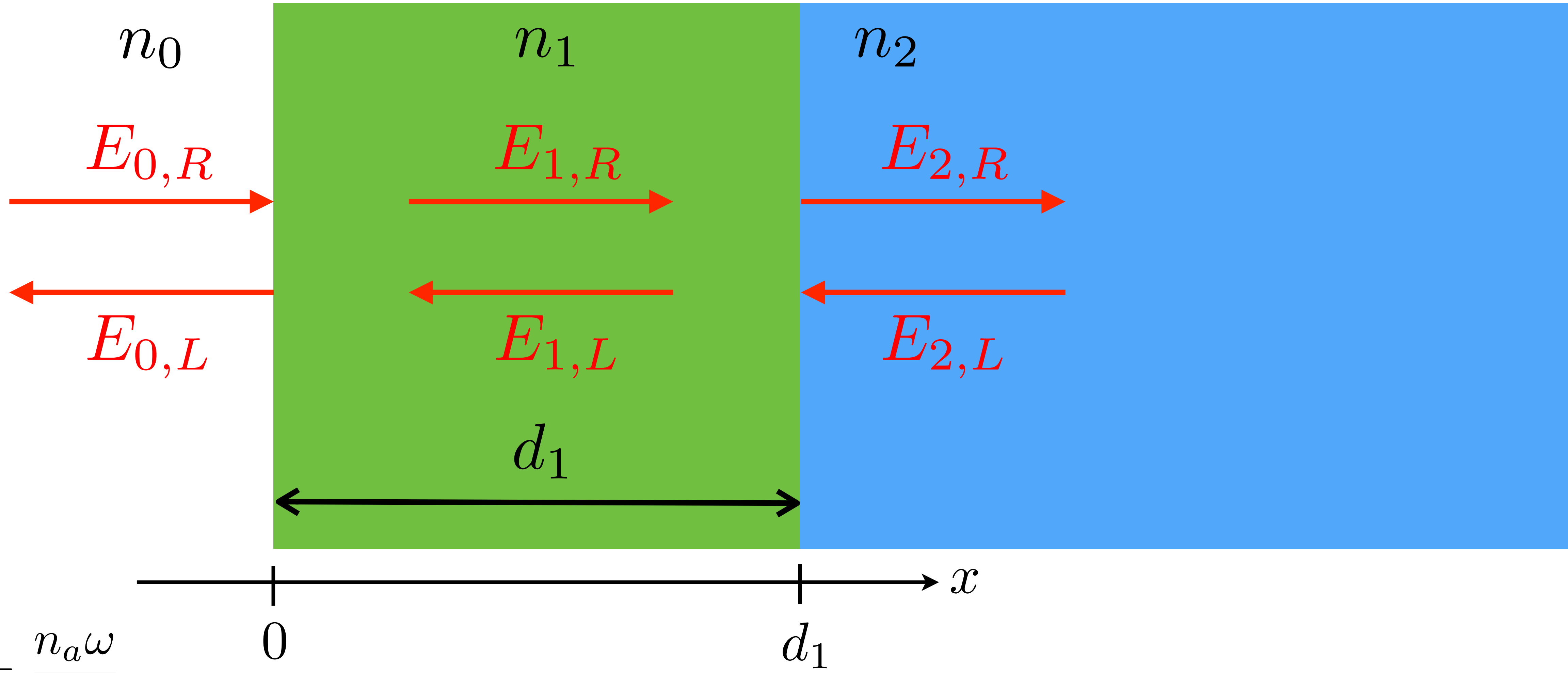
$$OPD = m\lambda$$

Transfer matrices



How can we relate the amplitudes of electromagnetic waves in the region 0 (white) to the amplitudes of electromagnetic waves in the region 2 (blue)?

Transfer matrices



$$k_a = \frac{2\pi n_a}{\lambda} = \frac{n_a \omega}{c_0}$$

Electromagnetic waves in different regions:

$$E_0(x,t) = E_{0,R} e^{i(k_0 x - \omega t)} + E_{0,L} e^{i(-k_0 x - \omega t)}$$

$$E_1(x,t) = E_{1,R} e^{i(k_1 x - \omega t)} + E_{1,L} e^{i(-k_1 x - \omega t)}$$

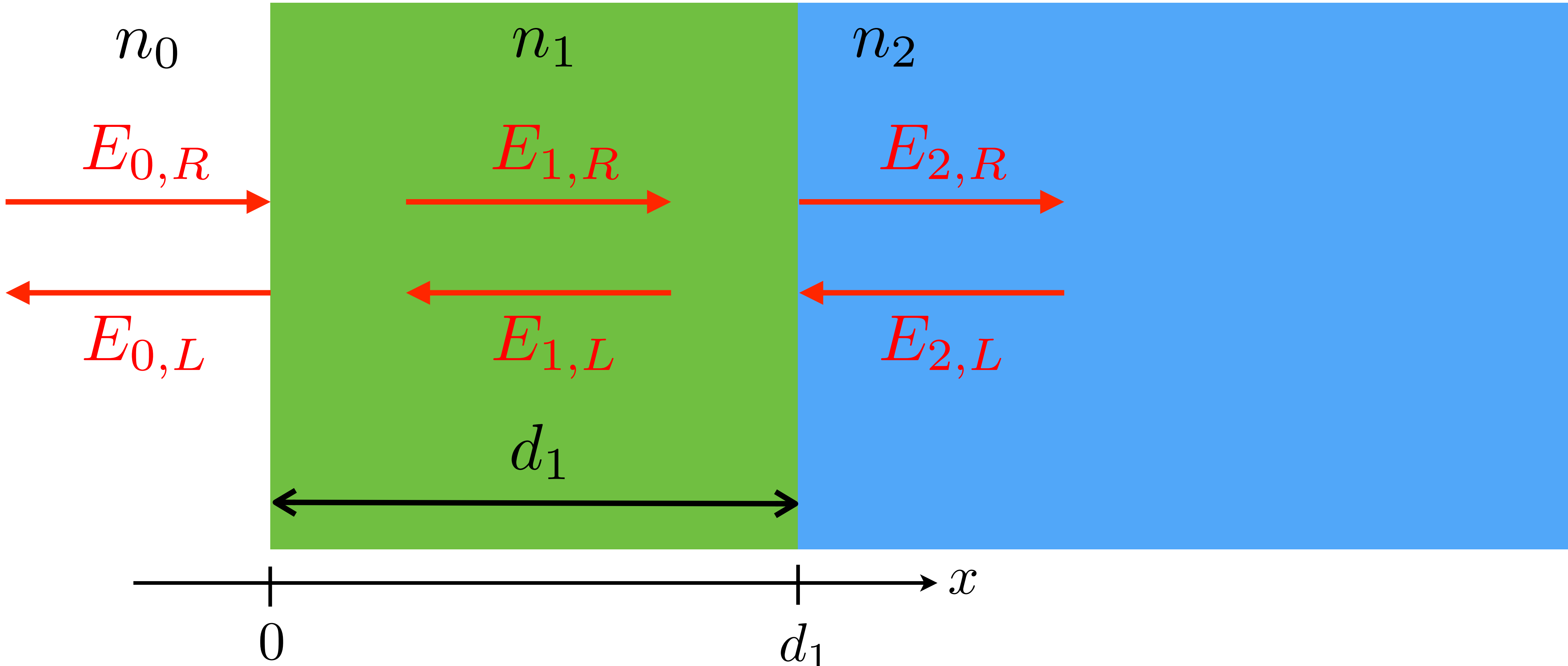
$$E_2(x,t) = E_{2,R} e^{i(k_2 x - \omega t)} + E_{2,L} e^{i(-k_2 x - \omega t)}$$

Boundary conditions:

$$E_0(0,t) = E_1(0,t) \qquad E_1(d_1,t) = E_2(d_1,t)$$

$$\frac{\partial E_0}{\partial x}(0,t) = \frac{\partial E_1}{\partial x}(0,t) \qquad \frac{\partial E_1}{\partial x}(d_1,t) = \frac{\partial E_2}{\partial x}(d_1,t)$$

Transfer matrices



Boundary conditions:

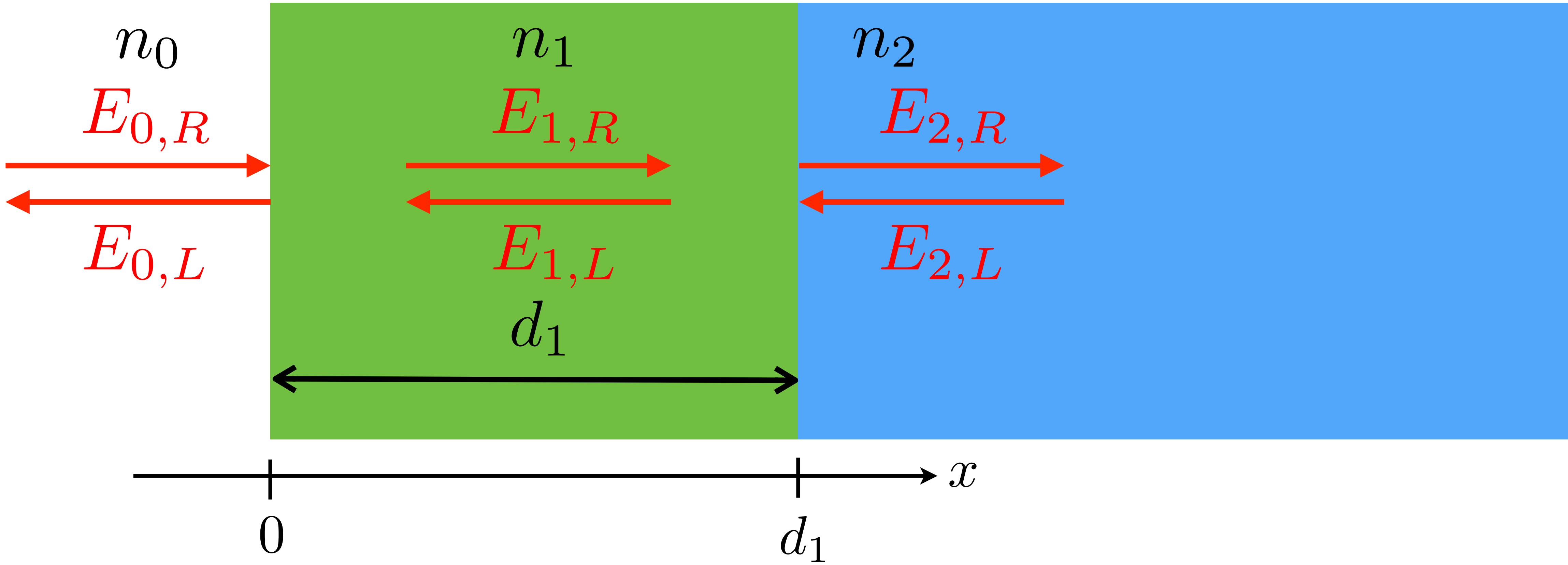
$$E_0(0,t) = E_1(0,t) \qquad E_1(d_1,t) = E_2(d_1,t)$$

$$\frac{\partial E_0}{\partial x}(0,t) = \frac{\partial E_1}{\partial x}(0,t) \qquad \frac{\partial E_1}{\partial x}(d_1,t) = \frac{\partial E_2}{\partial x}(d_1,t)$$

We would like to relate boundary conditions at two different interfaces via a transfer matrix *M_1*:

$$\begin{pmatrix} E_2(d_1,t) \\ \frac{\partial E_2}{\partial x}(d_1,t) \end{pmatrix} = M_1 \begin{pmatrix} E_0(0,t) \\ \frac{\partial E_0}{\partial x}(0,t) \end{pmatrix}$$

Transfer matrices



Electromagnetic waves in regions 1:

$$E_1(x,t) = E_{1,R}e^{i(k_1 x - \omega t)} + E_{1,L}e^{i(-k_1 x - \omega t)}$$

Relation between boundary conditions:

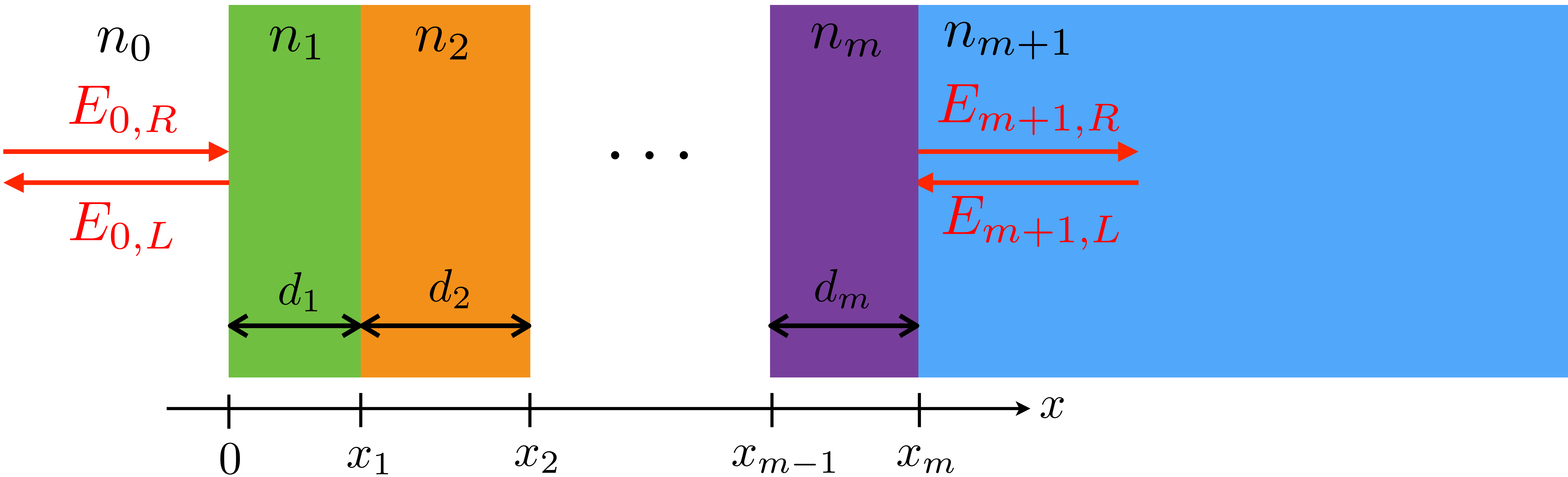
$$\begin{pmatrix} E_1(d_1,t) \\ \frac{\partial E_1}{\partial x}(d_1,t) \end{pmatrix} = M_1 \begin{pmatrix} E_1(0,t) \\ \frac{\partial E_1}{\partial x}(0,t) \end{pmatrix}$$

$$\begin{pmatrix} E_{1,R}e^{ik_1 d_1} + E_{1,L}e^{-ik_1 d_1} \\ ik_1 E_{1,R}e^{ik_1 d_1} - ik_1 E_{1,L}e^{-ik_1 d_1} \end{pmatrix} = M_1 \begin{pmatrix} E_{1,R} + E_{1,L} \\ ik_1 E_{1,R} - ik_1 E_{1,L} \end{pmatrix}$$

Transfer matrix M_1 can be obtained by solving equations above:

$$M_1 = \begin{pmatrix} \cos(k_1 d_1), & \frac{\sin(k_1 d_1)}{k_1} \\ -k_1 \sin(k_1 d_1), & \cos(k_1 d_1) \end{pmatrix}$$

Transfer matrices



Transfer matrix for *m* layers:

$$\begin{pmatrix} E_{m+1}(x_m, t) \\ \frac{\partial E_{m+1}}{\partial x}(x_m, t) \end{pmatrix} = M \begin{pmatrix} E_0(0, t) \\ \frac{\partial E_0}{\partial x}(0, t) \end{pmatrix}$$

$$M = M_m \cdot \ldots \cdot M_2 \cdot M_1$$

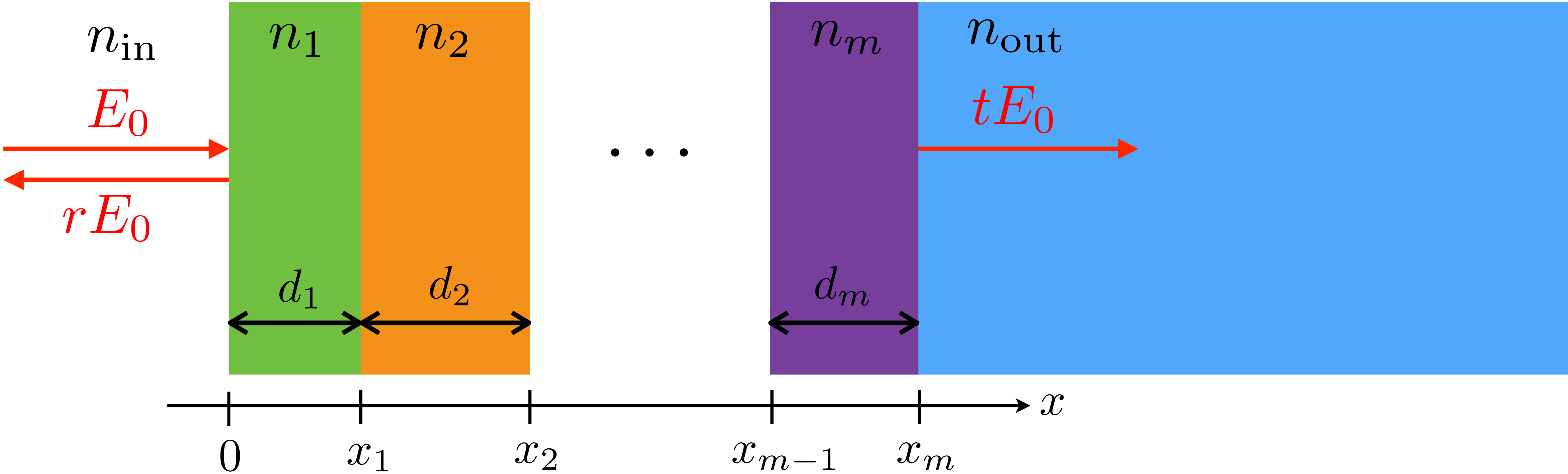
$$M_a = \begin{pmatrix} \cos(k_a d_a), & \frac{\sin(k_a d_a)}{k_a} \\ -k_a \sin(k_a d_a), & \cos(k_a d_a) \end{pmatrix}$$

Note:

$$\det(M) = \det(M_a) = 1$$

$$k_a = \frac{2\pi n_a}{\lambda} = \frac{n_a \omega}{c_0}$$

Transfer matrices



Incoming and outgoing electromagnetic waves:

$$E_{\rm in}(x,t) = E_0 e^{i(k_{\rm in}x-\omega t)} + rE_0 e^{i(-k_{\rm in}x-\omega t)}$$

$$E_{\rm out}(x,t) = tE_0 e^{i(k_{\rm out}x-\omega t)}$$

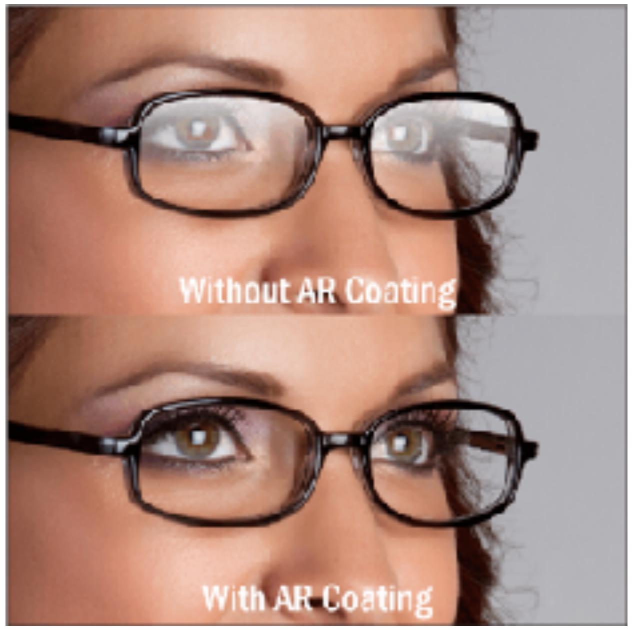
$$\begin{pmatrix} E_{\rm out}(x_m,t) \\ \frac{\partial E_{\rm out}}{\partial x}(x_m,t) \end{pmatrix} = \begin{pmatrix} M_{11}, & M_{12} \\ M_{21}, & M_{22} \end{pmatrix} \begin{pmatrix} E_{\rm in}(0,t) \\ \frac{\partial E_{\rm in}}{\partial x}(0,t) \end{pmatrix}$$

Amplitudes of reflected and transmitted waves:

$$r = \frac{(M_{21}+k_{\rm in}k_{\rm out}M_{12}) + i(k_{\rm in}M_{22} - k_{\rm out}M_{11})}{(-M_{21}+k_{\rm in}k_{\rm out}M_{12}) + i(k_{\rm out}M_{11} + k_{\rm in}M_{22})}$$

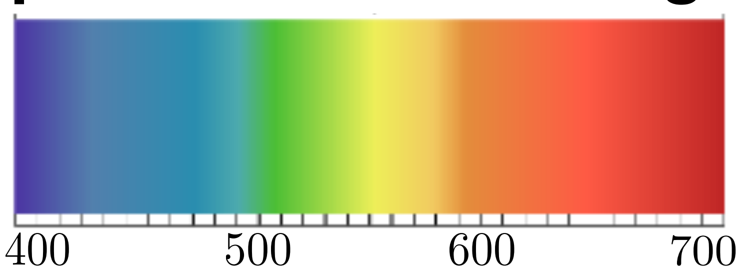
$$t = \frac{2ik_{\rm in}e^{-ix_m k_{\rm out}}}{(-M_{21}+k_{\rm in}k_{\rm out}M_{12}) + i(k_{\rm out}M_{11} + k_{\rm in}M_{22})}$$

Example: antireflective coating

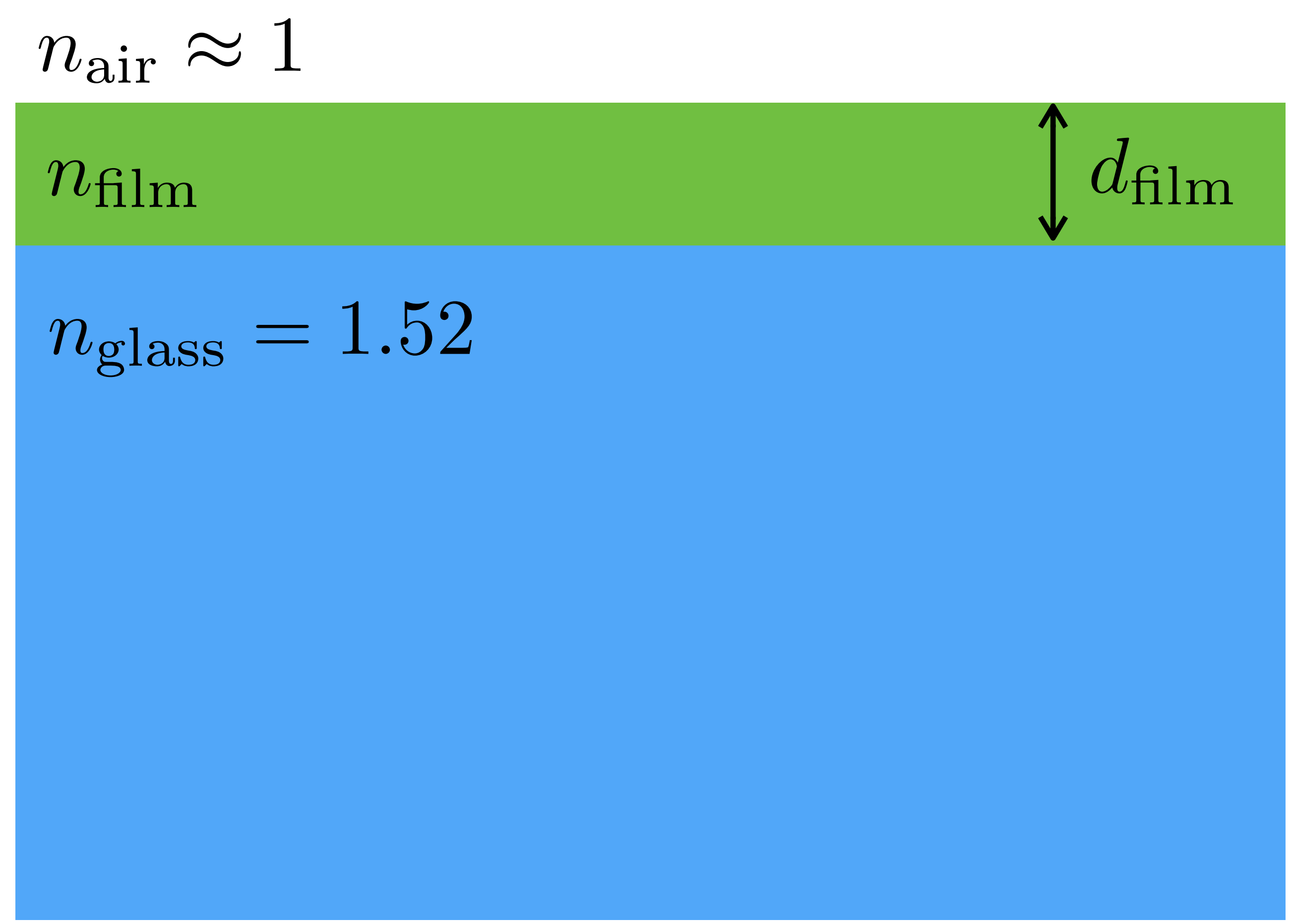


We would like to design a thin film coating for glasses that minimizes reflection of visible light.

spectrum of visible light



wavelength [nm]



Assume that thin film is made of MgF_2 that can be easily applied with physical vapor deposition:

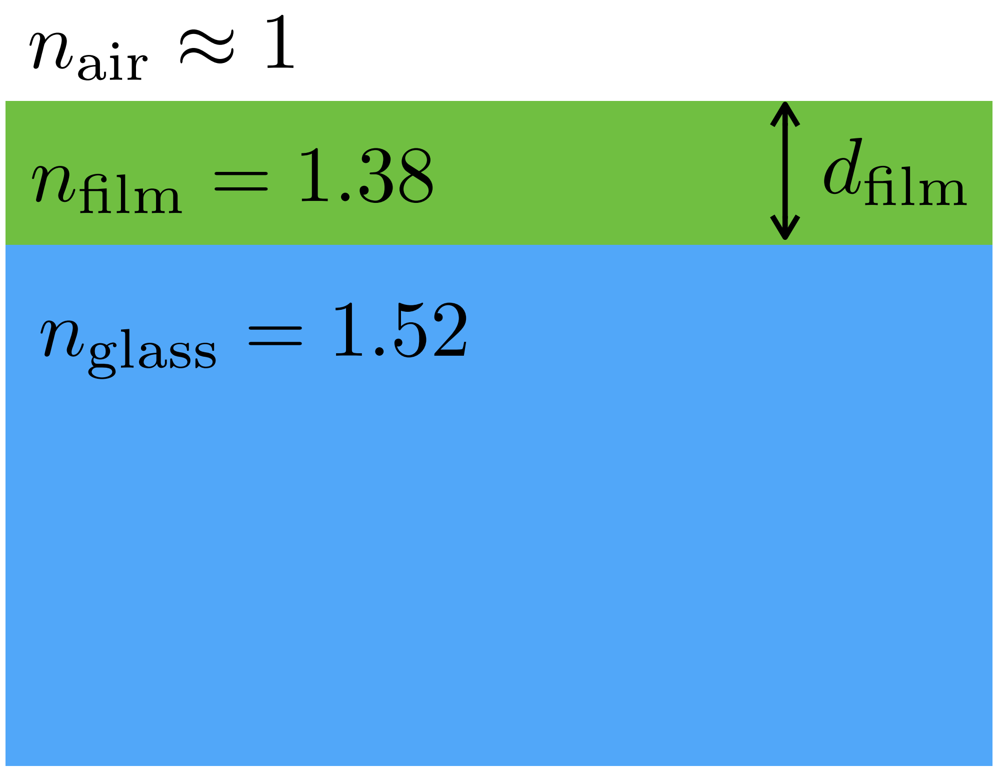
$$n_{\text{film}} = 1.38$$

Note: the condition for deconstructive interference of reflected rays can be satisfied only for discrete set of wavelengths λ_0 :

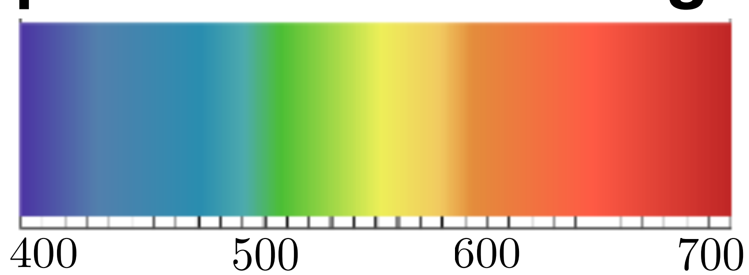
$$2d_{\text{film}}n_{\text{film}} = \left(m + \frac{1}{2}\right)\lambda_0$$

$$m = 0, 1, 2, \ldots$$

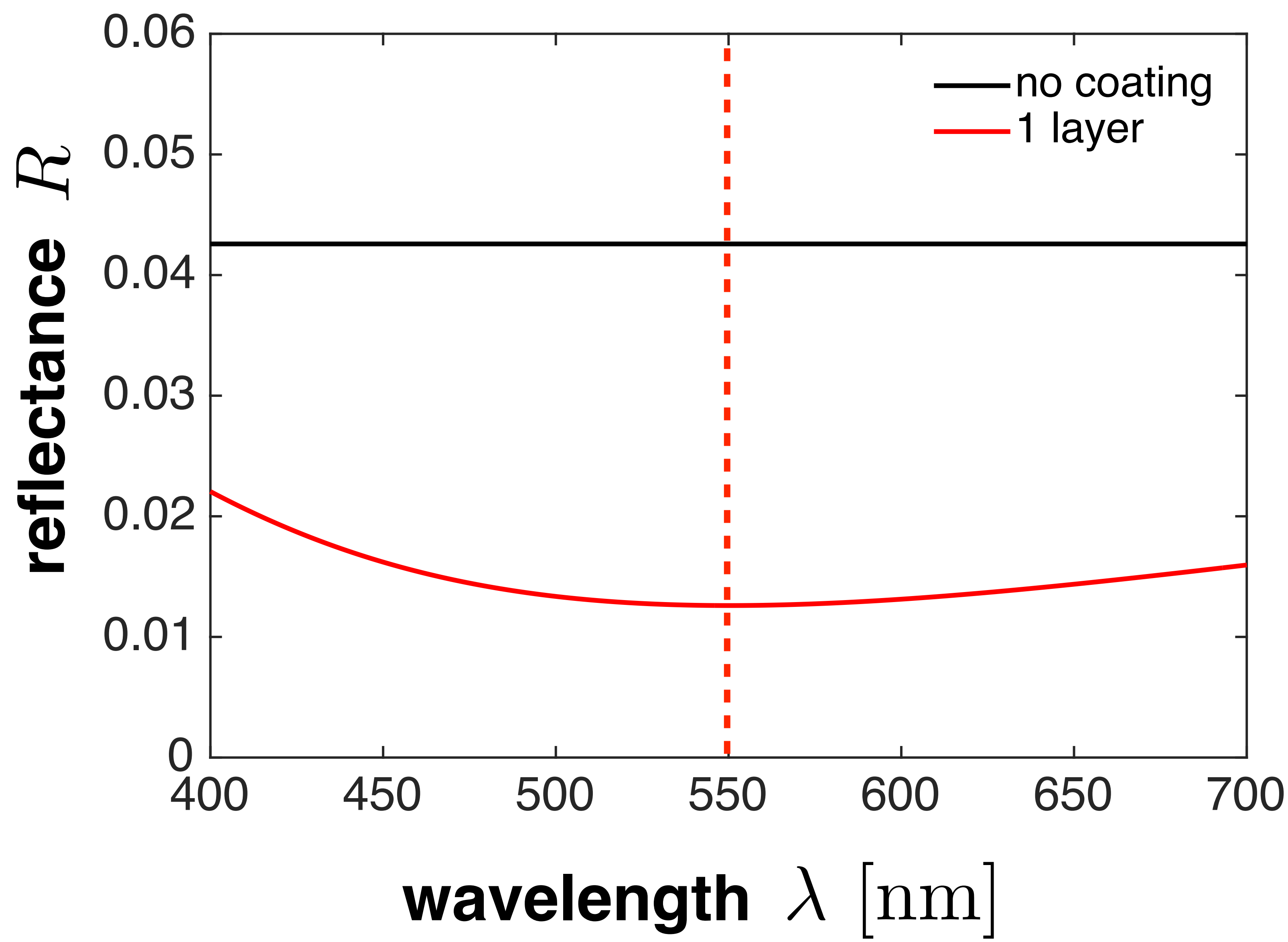
Example: antireflective coating



spectrum of visible light



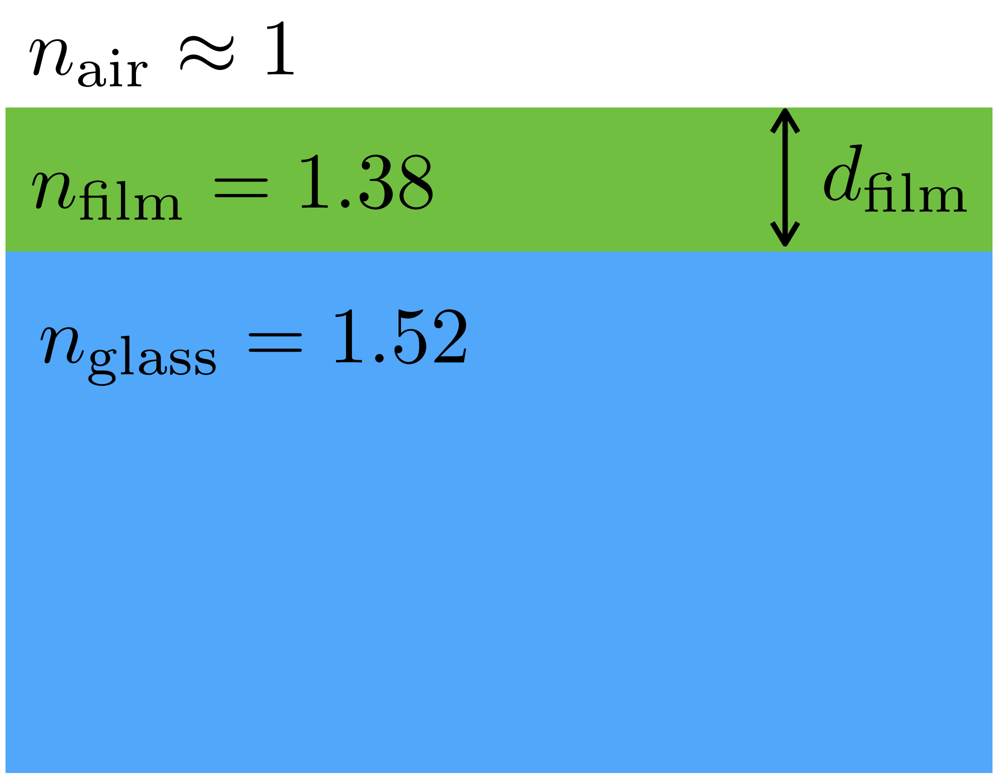
wavelength $[\mathrm{nm}]$

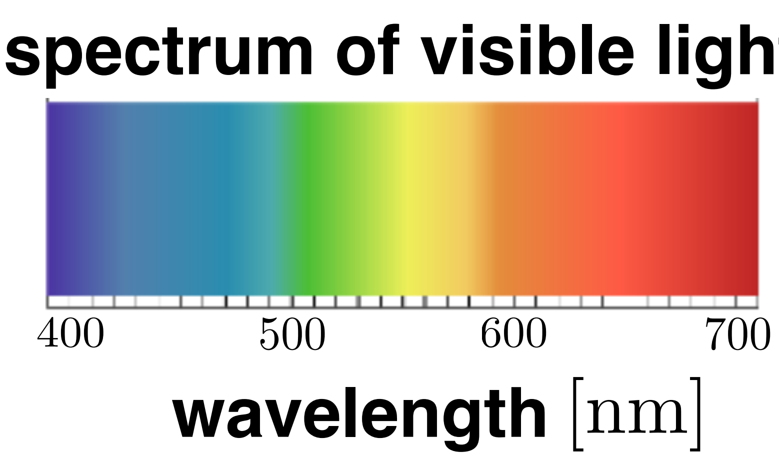


Use film thickness that corresponds to the destructive interference for the wavelength in the middle of the visible spectrum $\lambda_{\mathrm{target}} = 550\,\mathrm{nm}$:

$$d_{\mathrm{film}} = \frac{\lambda_{\mathrm{target}}}{4n_{\mathrm{film}}} = 100\,\mathrm{nm}$$

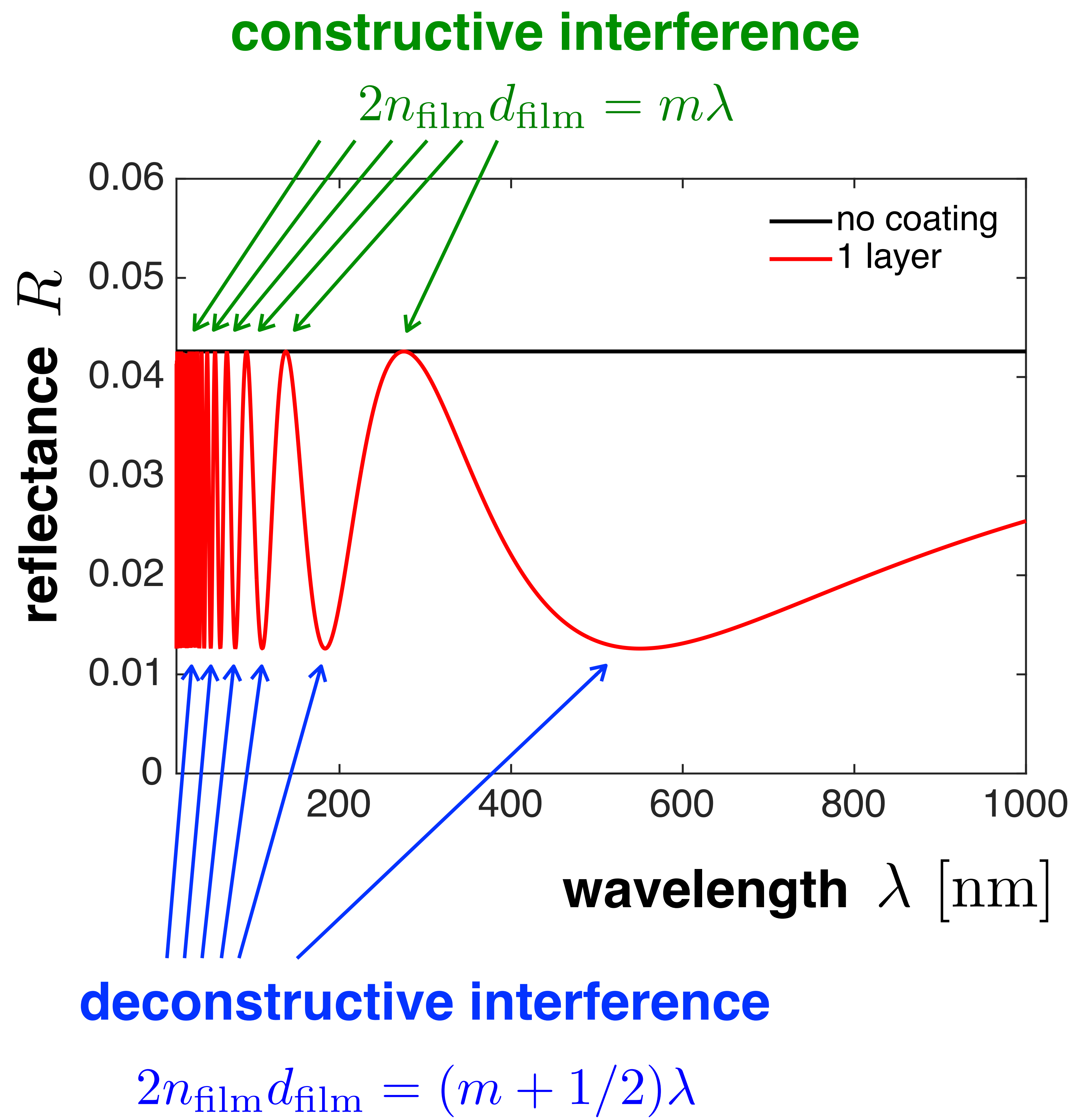
Example: antireflective coating



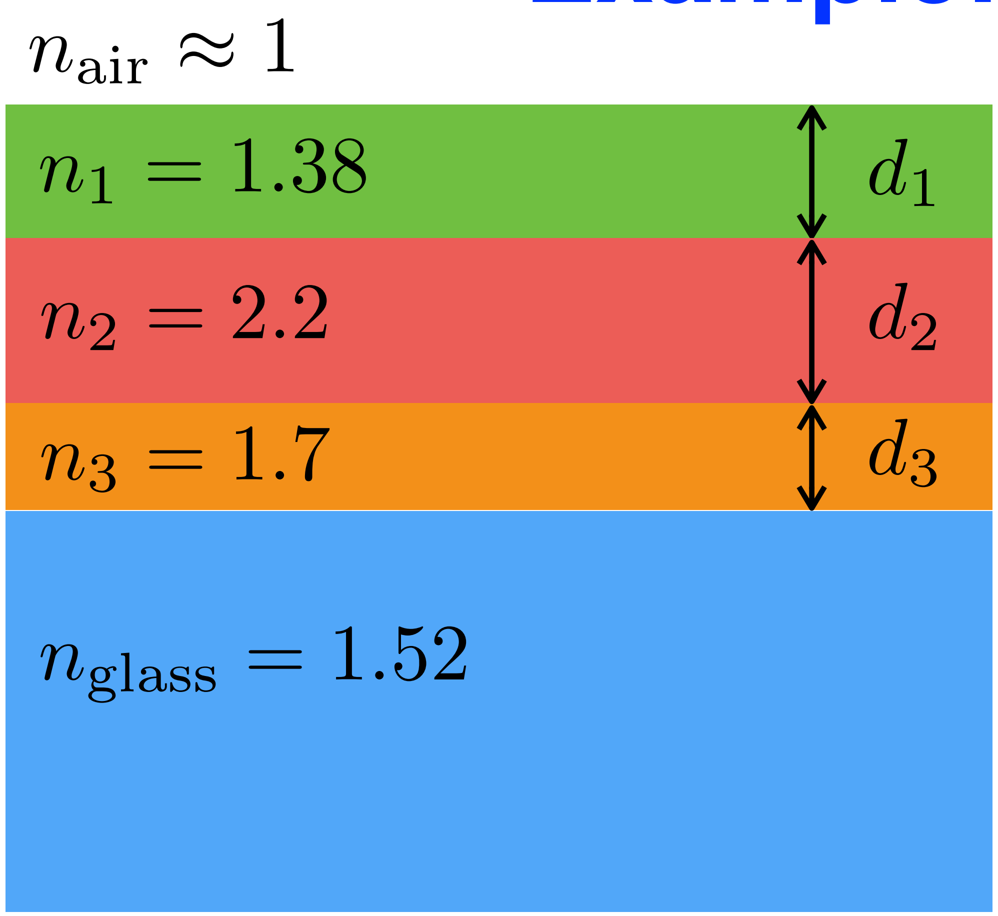


$\lambda_{\mathrm{target}} = 550\,\mathrm{nm}$

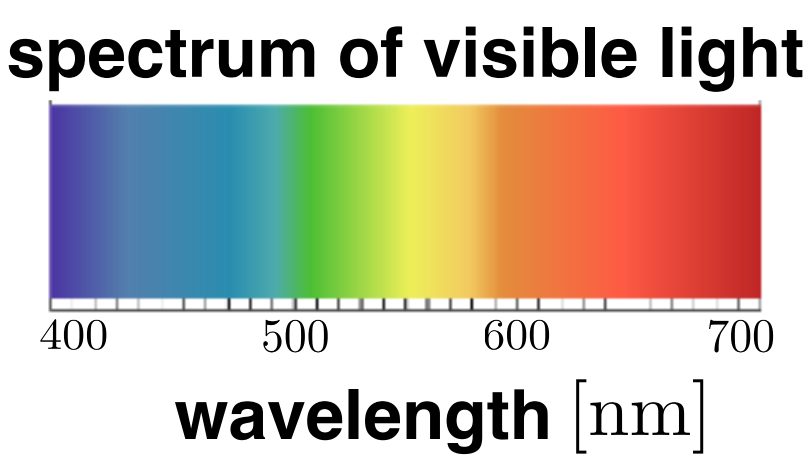
$$d_{\mathrm{film}} = \frac{\lambda_{\mathrm{target}}}{4 n_{\mathrm{film}}} = 100\,\mathrm{nm}$$

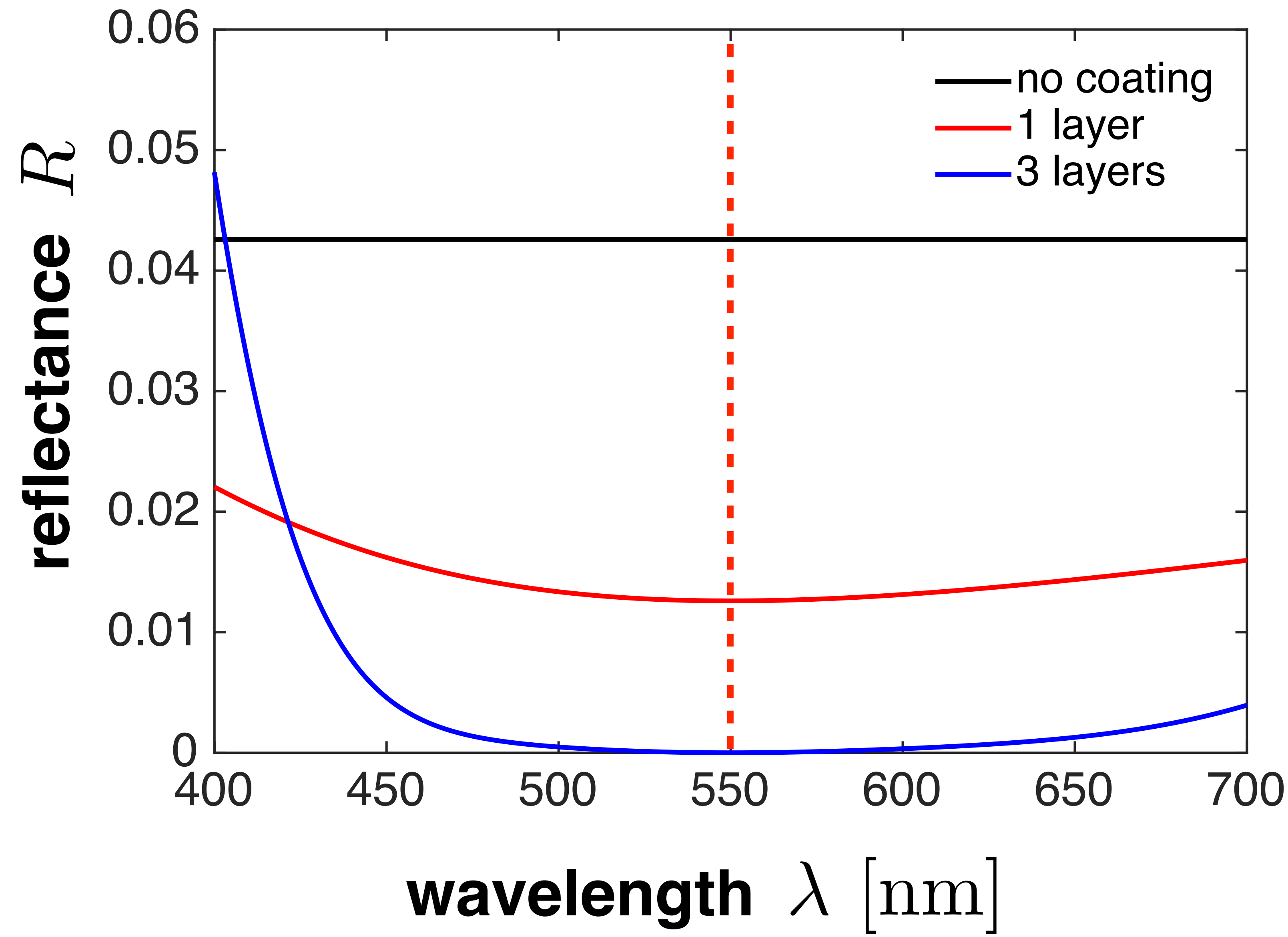


Example: antireflective coating



Multiple layers of coating significantly reduce the reflectance of visible spectrum!



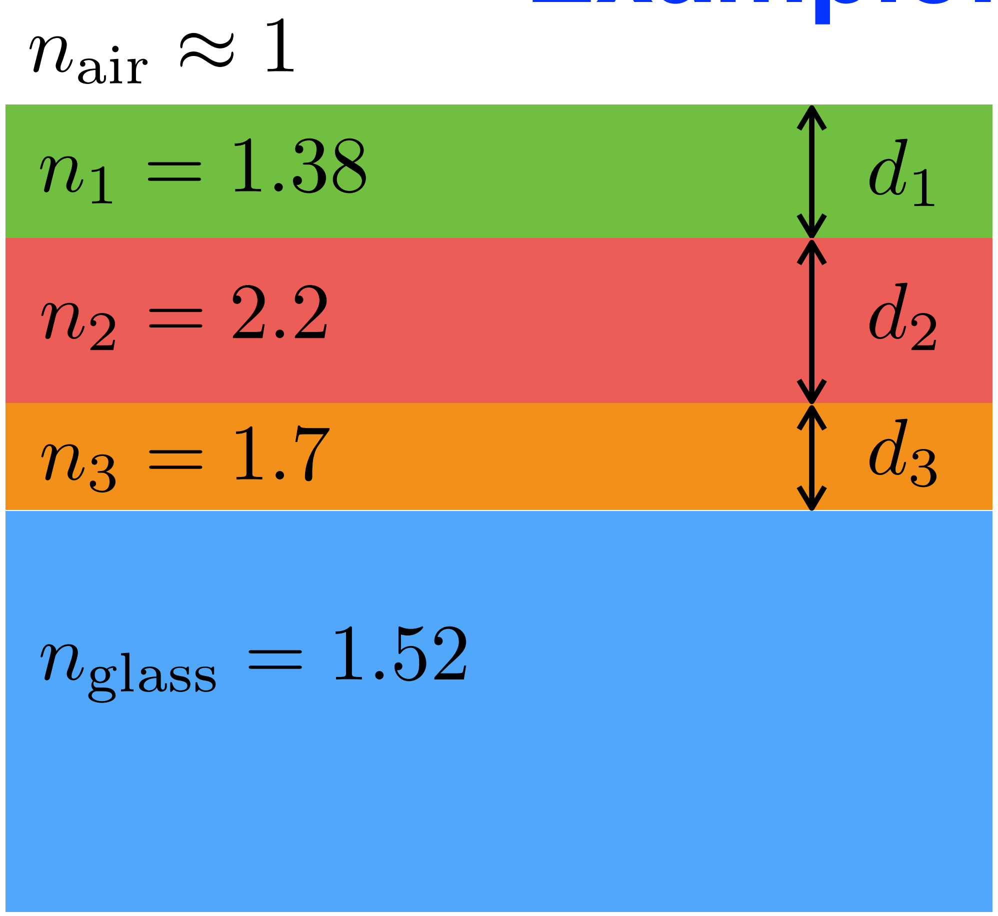


Use film thicknesses that correspond to the destructive interference for the wavelength in the middle of the visible spectrum $\lambda_{\text{target}} = 550\,\text{nm}$:

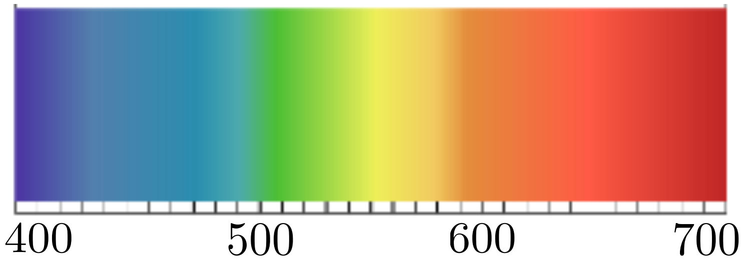
$$d_1 = \lambda_{\text{target}}/(4n_1)$$
$$d_2 = \lambda_{\text{target}}/(2n_2)$$
$$d_3 = \lambda_{\text{target}}/(4n_3)$$

note the additional phase difference!

Example: antireflective coating



spectrum of visible light



wavelength [nm]

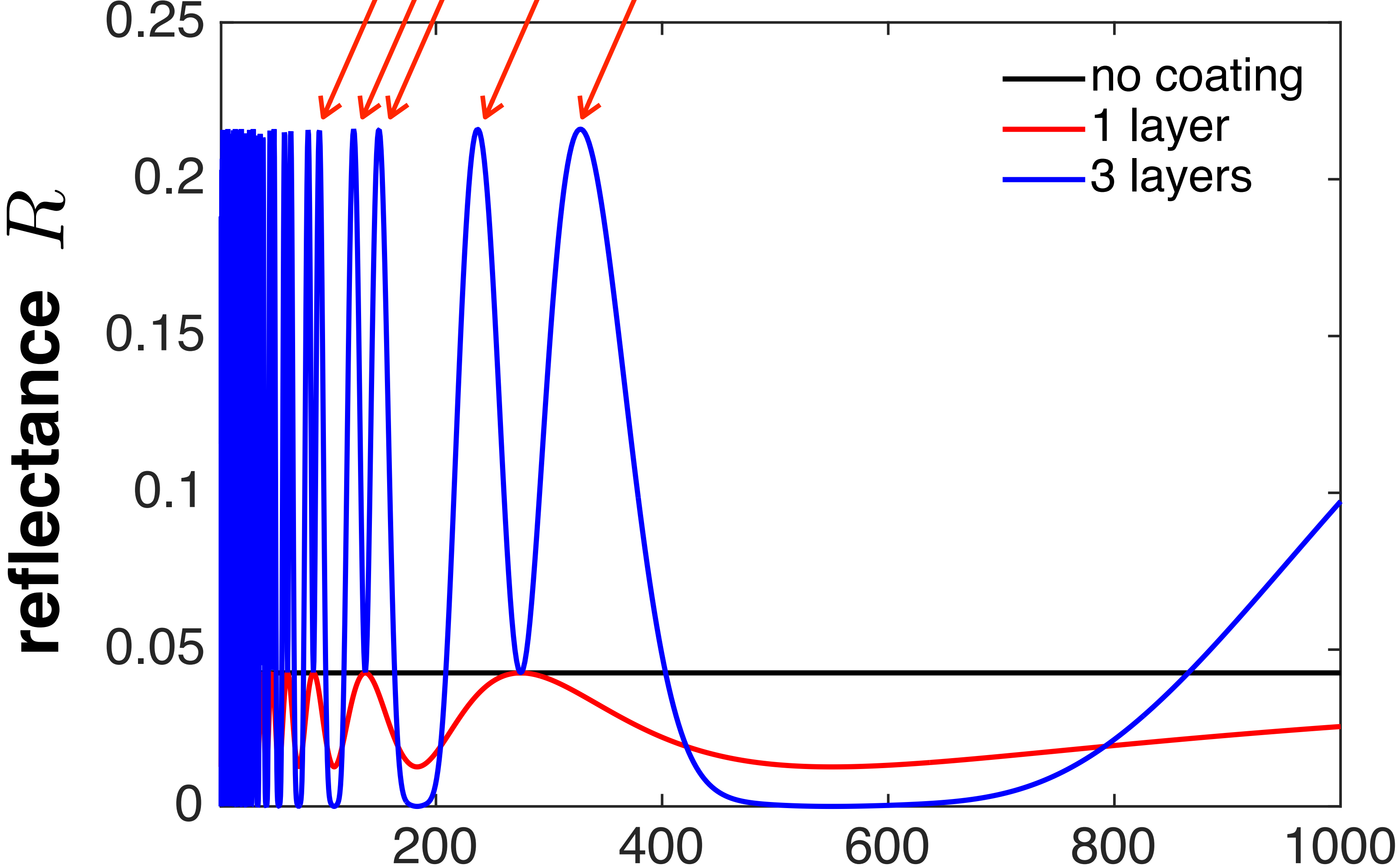
$\lambda_{\mathrm{target}} = 550\,\mathrm{nm}$

$d_1 = \lambda_{\mathrm{target}}/(4n_1)$

$d_2 = \lambda_{\mathrm{target}}/(2n_2)$

$d_3 = \lambda_{\mathrm{target}}/(4n_3)$

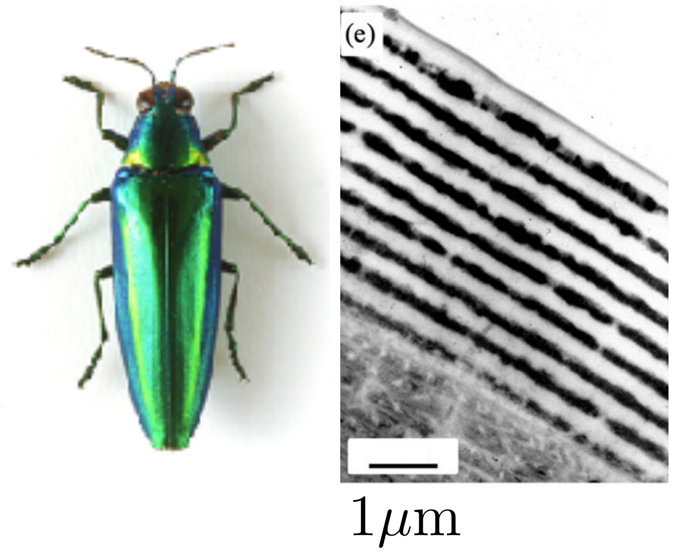
Multiple layers of coating significantly enhance reflectance of certain wavelengths outside the visible spectrum!

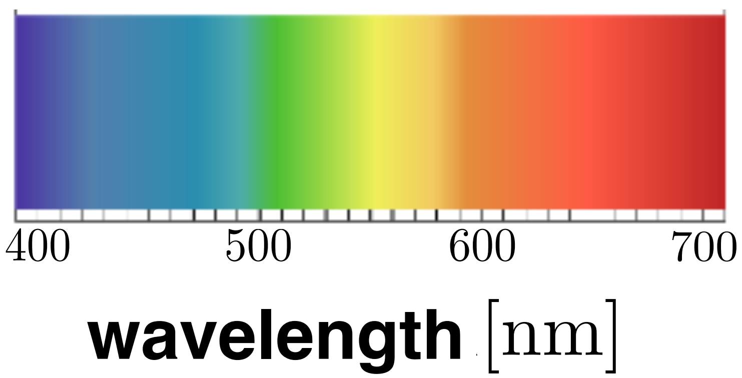


Additional peaks (minima) correspond to the constructive (deconstructive) interference for rays scattered on different combination of interfaces.

Example: structural color

Chrysochroa raja bettle





Typical refraction indices:

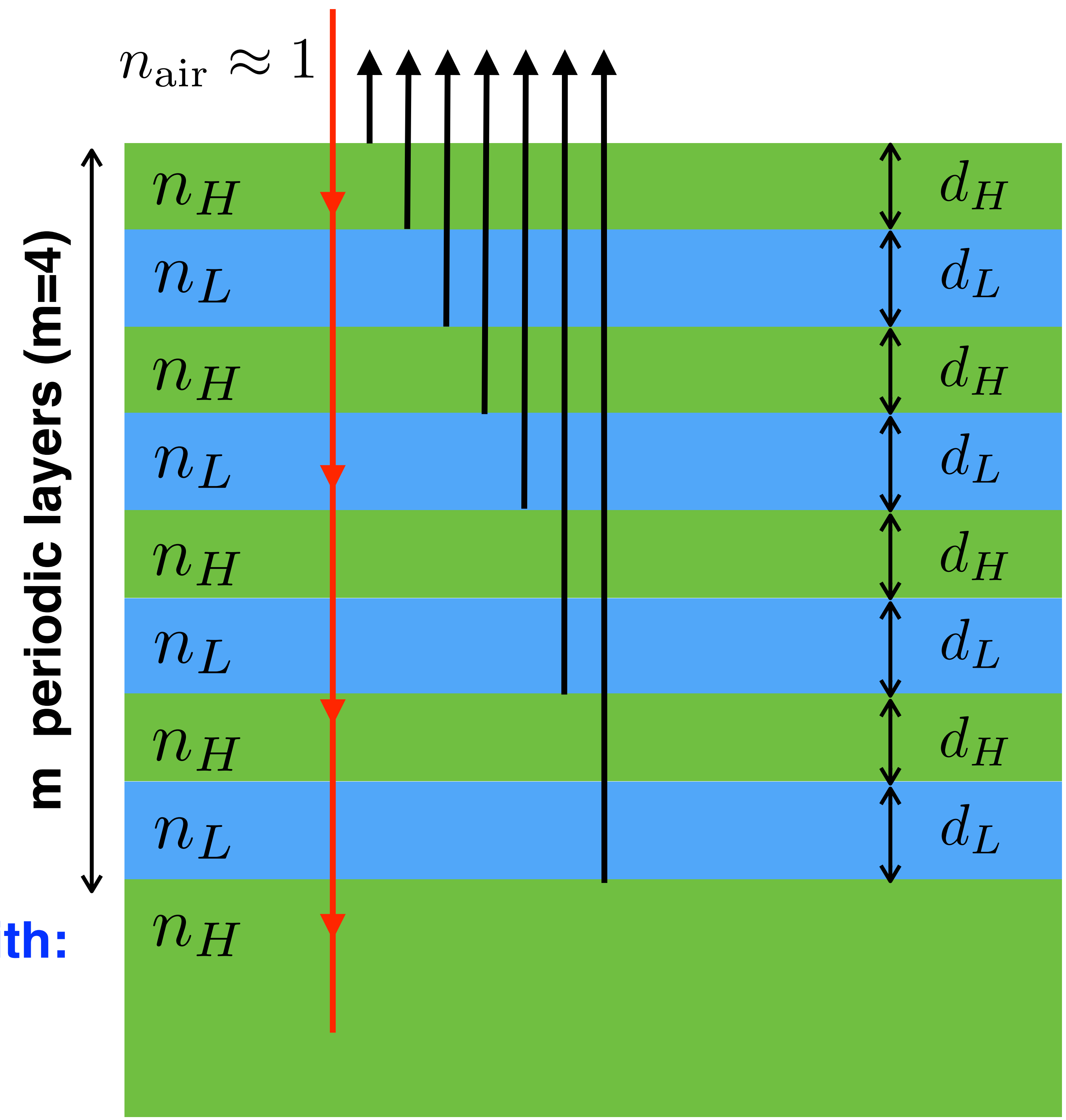
$$n_H = 1.69 \qquad n_L = 1.56$$

Constructive interference of reflected rays can be achieved with:

$$d_H = \frac{\lambda_0}{4n_H} = 74\,\mathrm{nm}$$

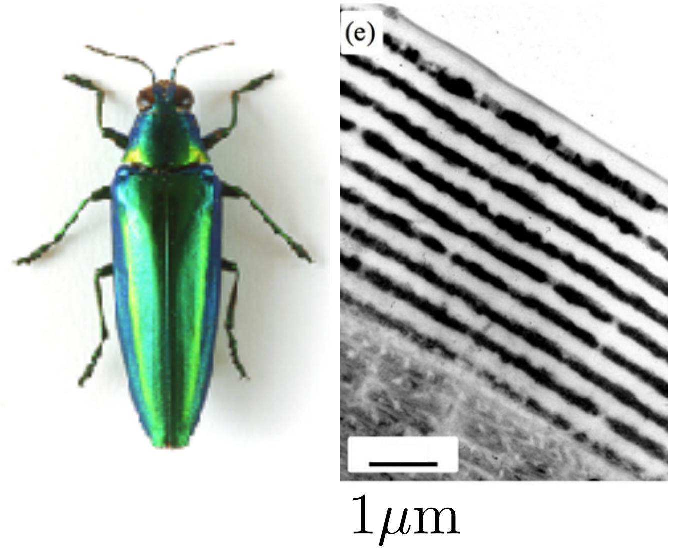
$$d_L = \frac{\lambda_0}{4n_L} = 80\,\mathrm{nm}$$

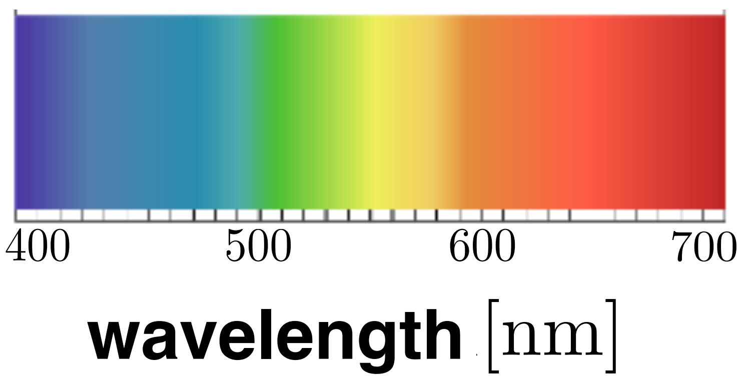
We would like to design periodic structure, which preferentially reflects green color with $\lambda_0 = 500\,\mathrm{nm}$.



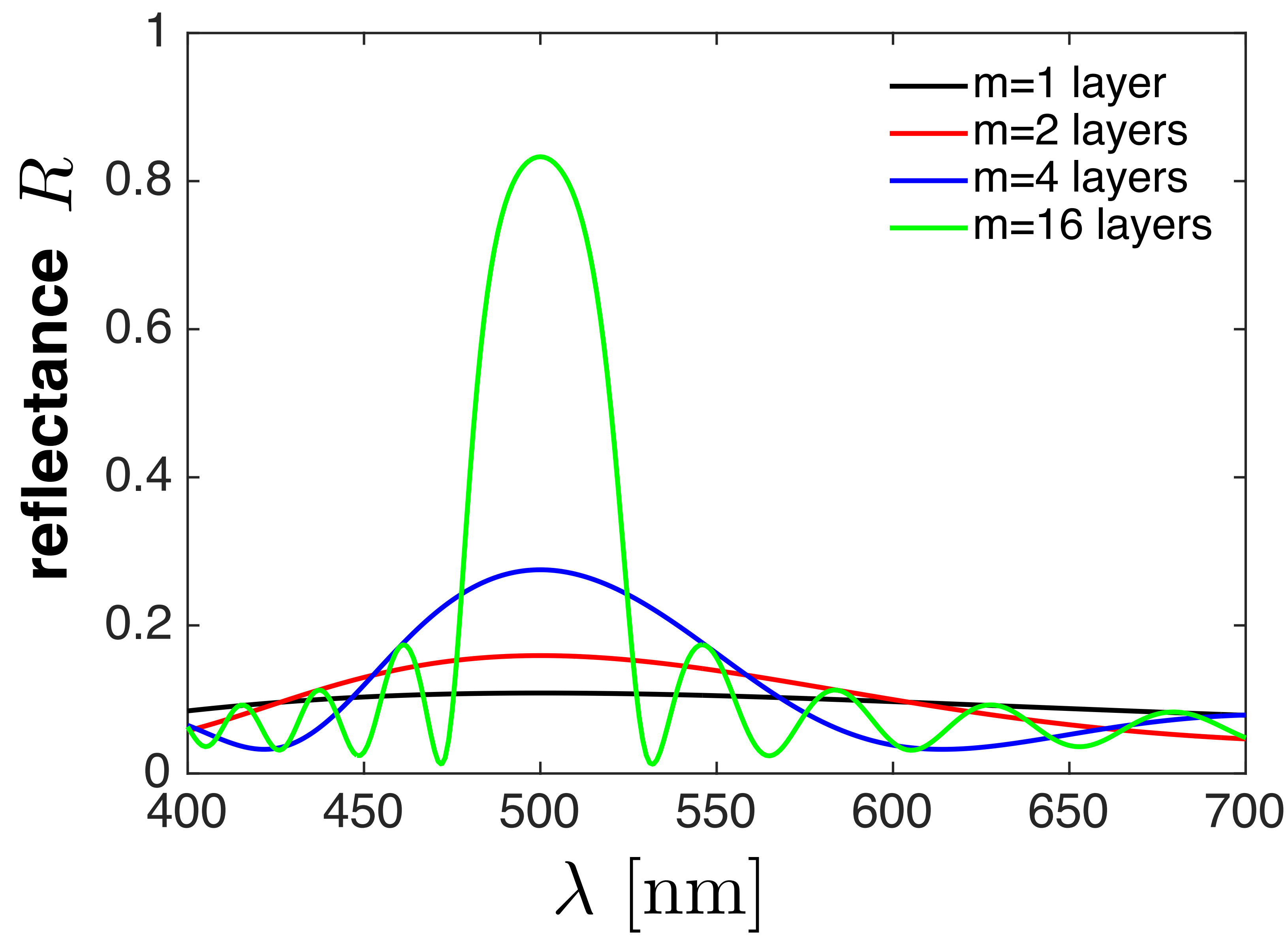
Example: structural color

Chrysochroa raja bettle



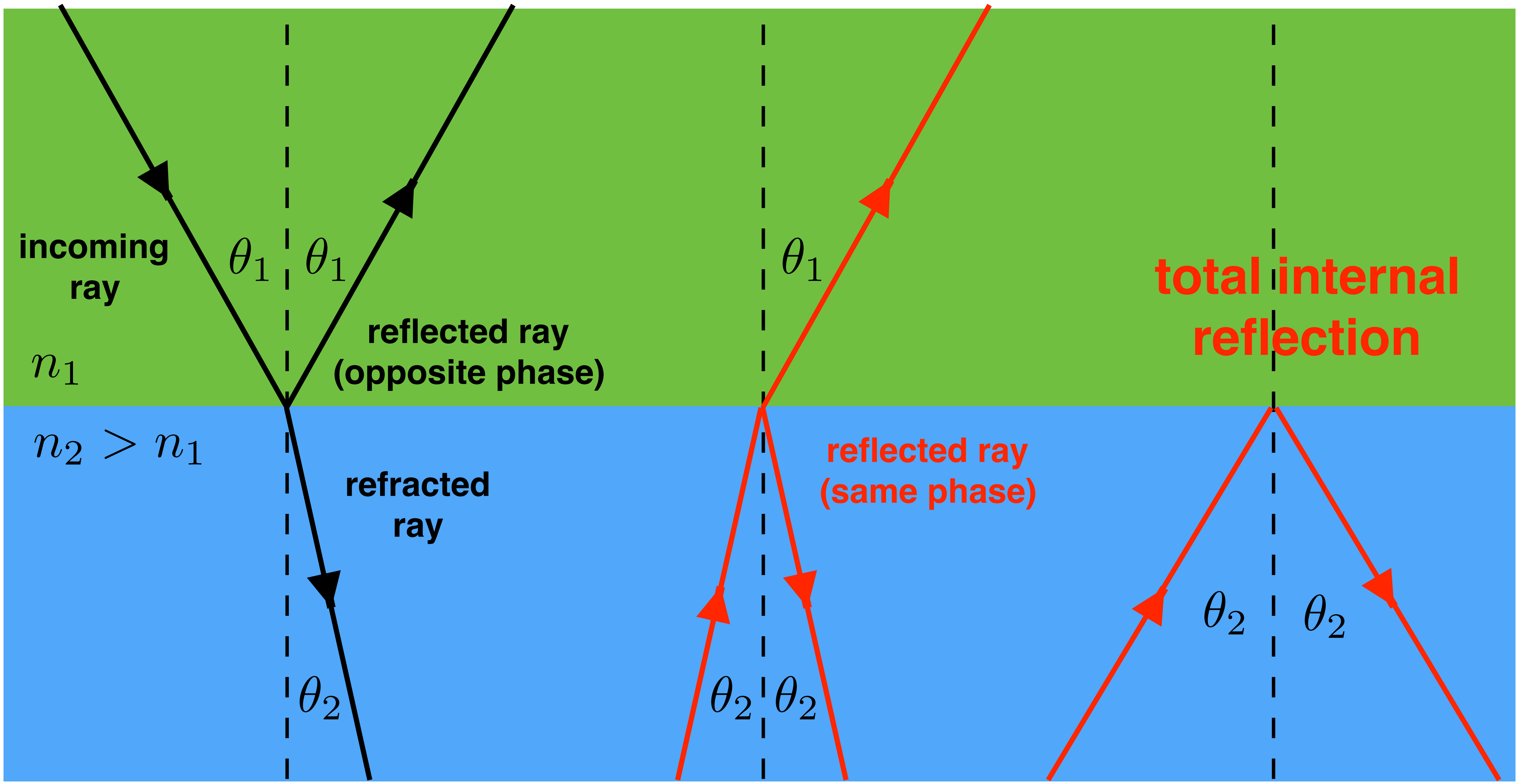


Multiple periodic layers increase the reflectance of target wavelength $\lambda_0 = 500\,\mathrm{nm}$!



In periodic structures high reflectance is achieved for a range of wavelengths around the target wavelength. This range is called band gap.

Refraction of light



Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

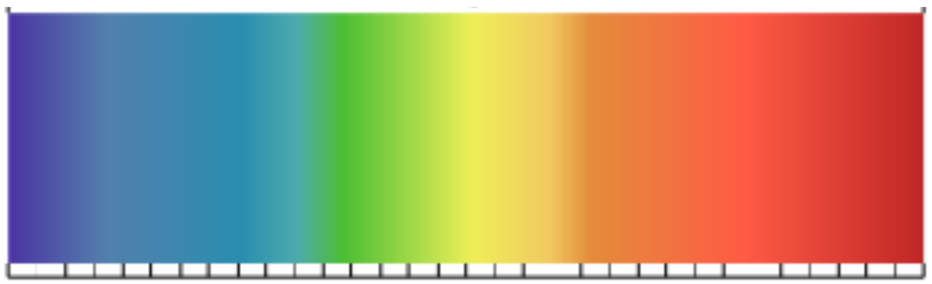
Total internal reflection

$$\theta_2 > \arcsin(n_1/n_2)$$

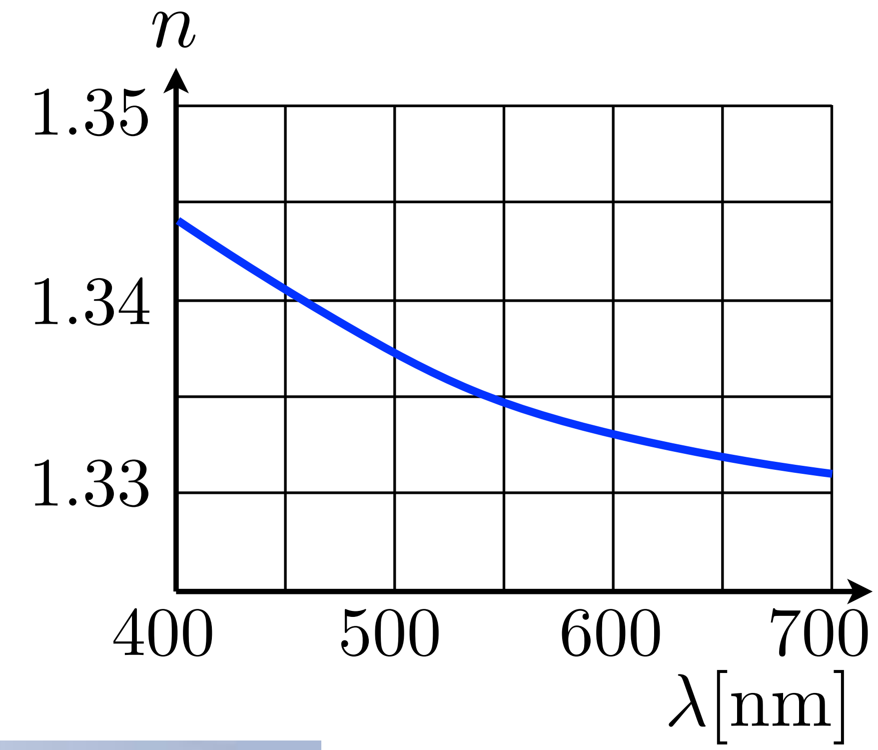
Rainbow

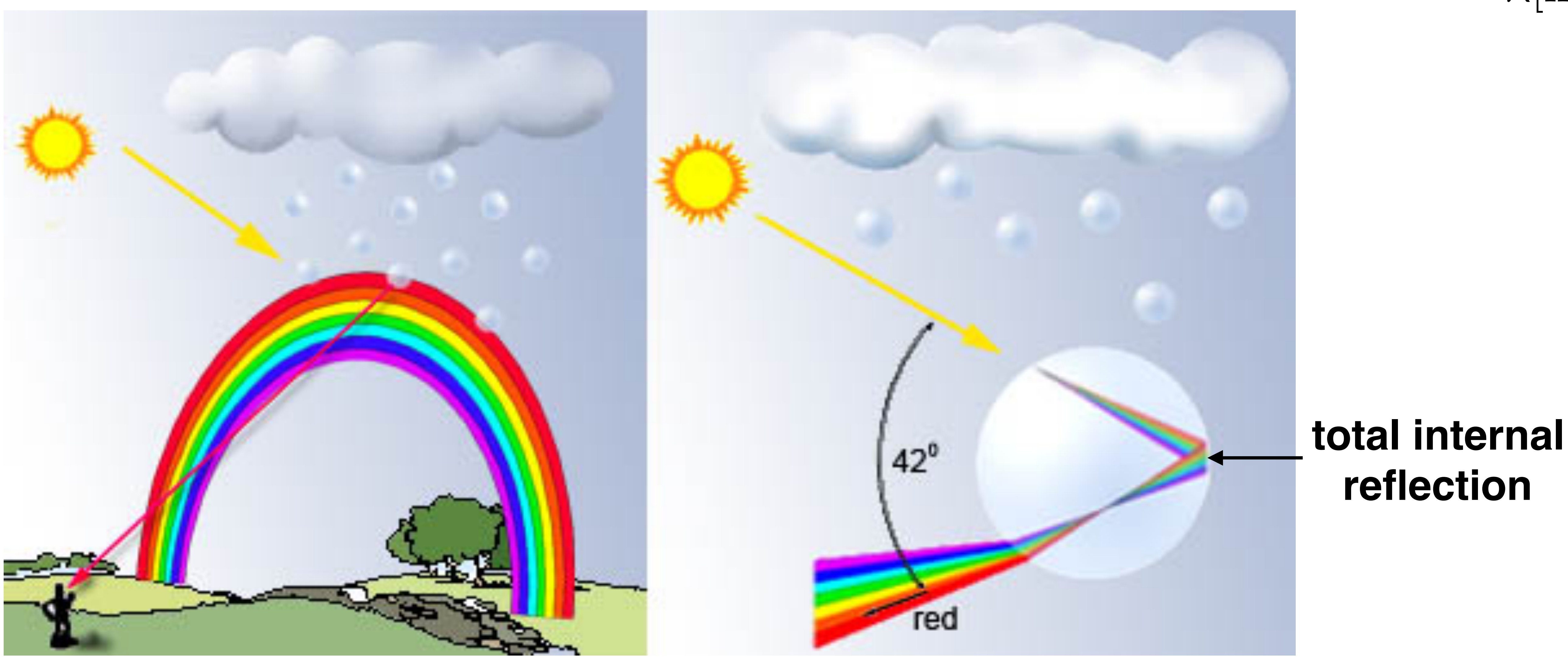
Rainbow forms because refraction index *n* in water droplets depends on the color (wavelength) of light.

$n_{\text{purple}} > n_{\text{blue}} > n_{\text{green}} > n_{\text{yellow}} > n_{\text{orange}} > n_{\text{red}}$

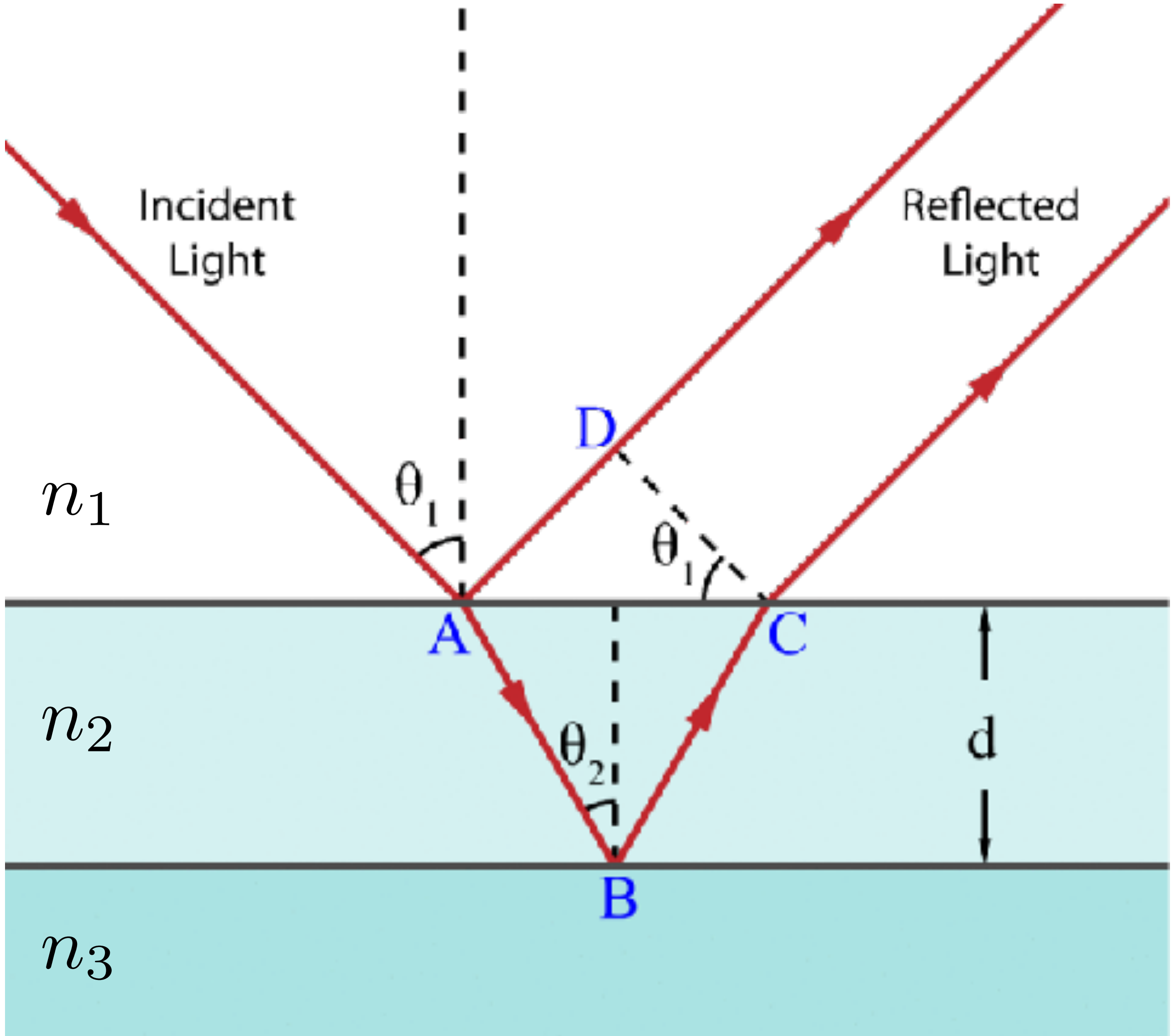


wavelength λ[nm]





Interference on thin films



difference between optical path lengths of the two reflected rays

$$OPD = n_2\left(\overline{AB} + \overline{BC}\right) - n_1\overline{AD}$$

$$OPD = 2n_2 d\cos(\theta_2)$$

no additional phase difference due to reflections

$n_1 < n_2 < n_3$ $n_1 > n_2 > n_3$

constructive interference

$$OPD = m\lambda$$

destructive interference

$$OPD = (m + 1/2)\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

additional π phase difference due to reflections

$n_1 < n_2 > n_3$ $n_1 > n_2 < n_3$

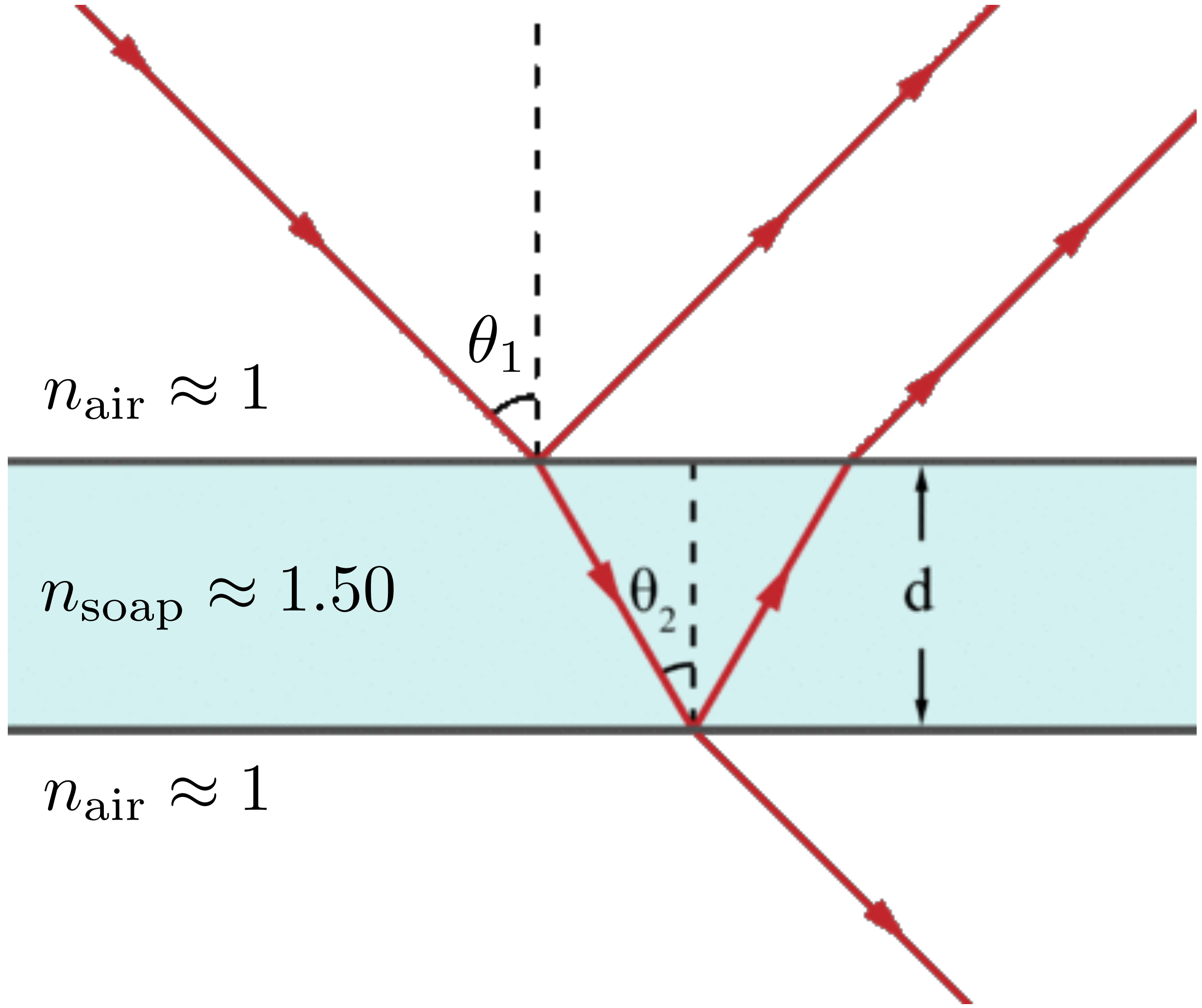
constructive interference

$$OPD = (m + 1/2)\lambda$$

destructive interference

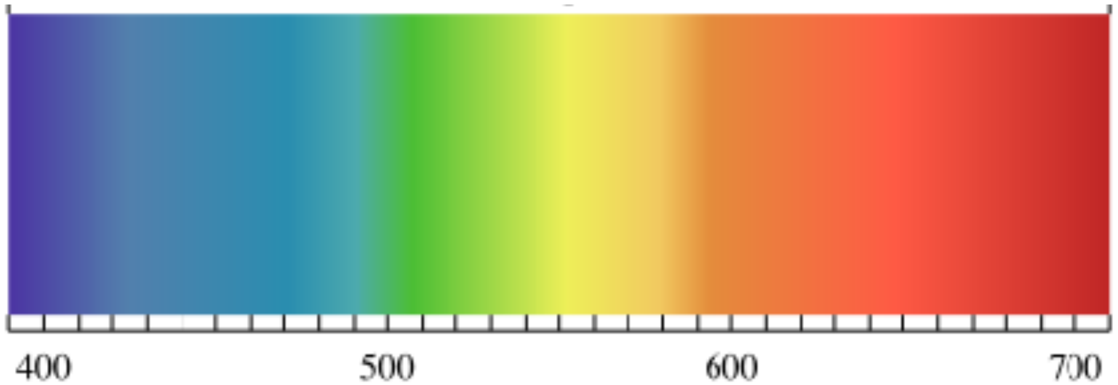
$$OPD = m\lambda$$

Interference on soap bubbles



soap bubble

visible spectrum



wavelength $\lambda[\text{nm}]$

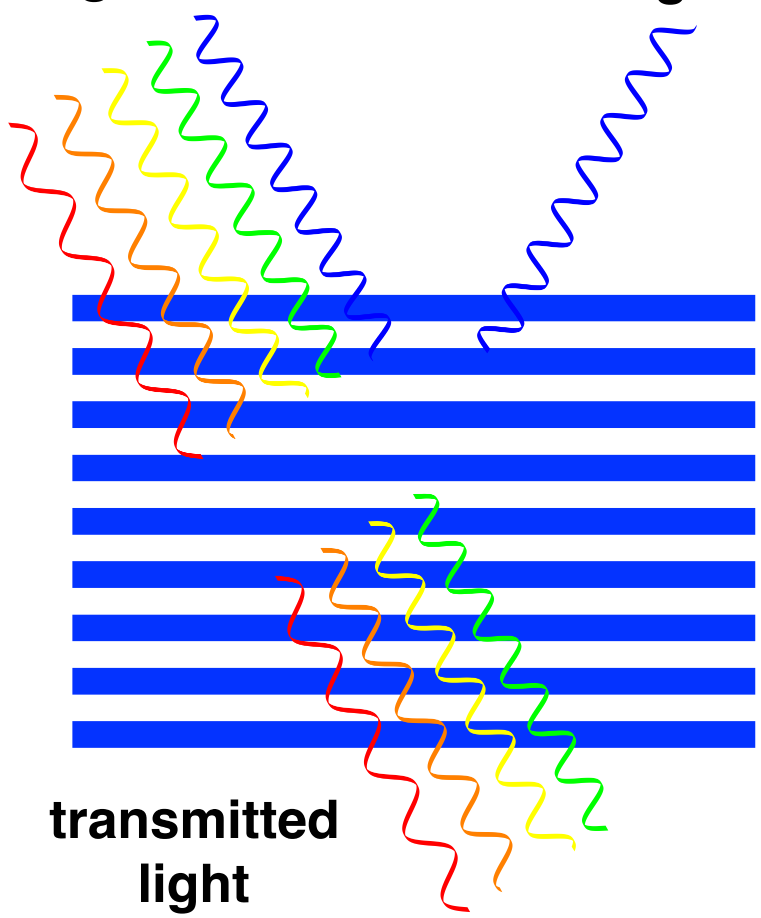
constructive interference for different colors happens at different angles

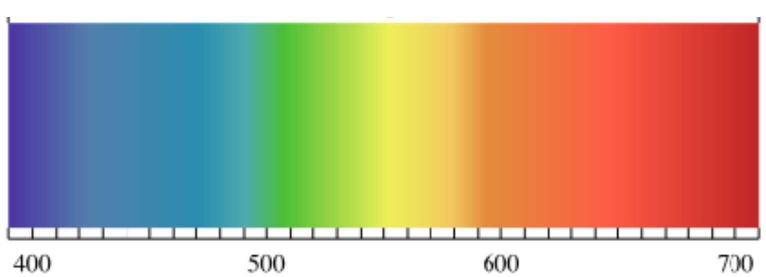
$$2dn_{\text{soap}}\cos(\theta_2) = (m + 1/2)\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Structural colors on periodic structures

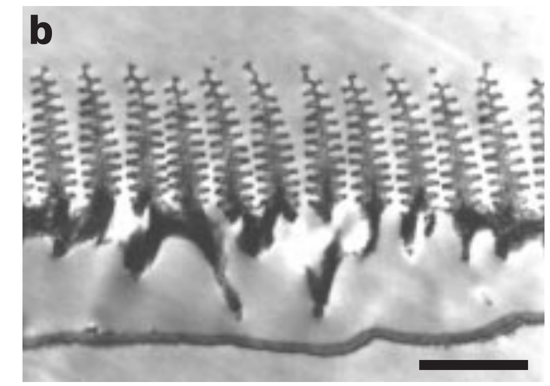
Single reflected color on structures with uniform spacing





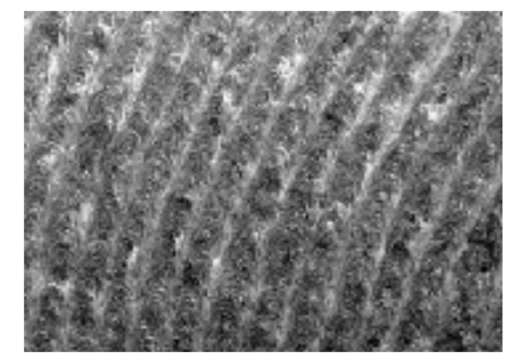
wavelength $\lambda[\mathrm{nm}]$

Morpho butterfly



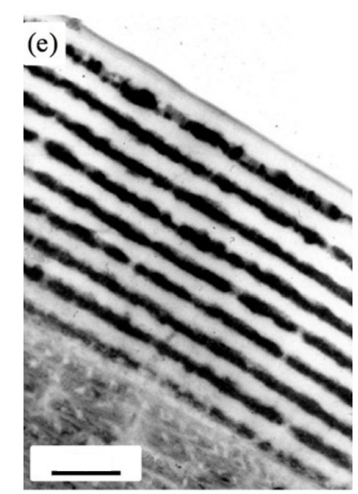
$1.7\mu\mathrm{m}$

Marble berry



$250\mathrm{nm}$

Chrysochroa raja bettle

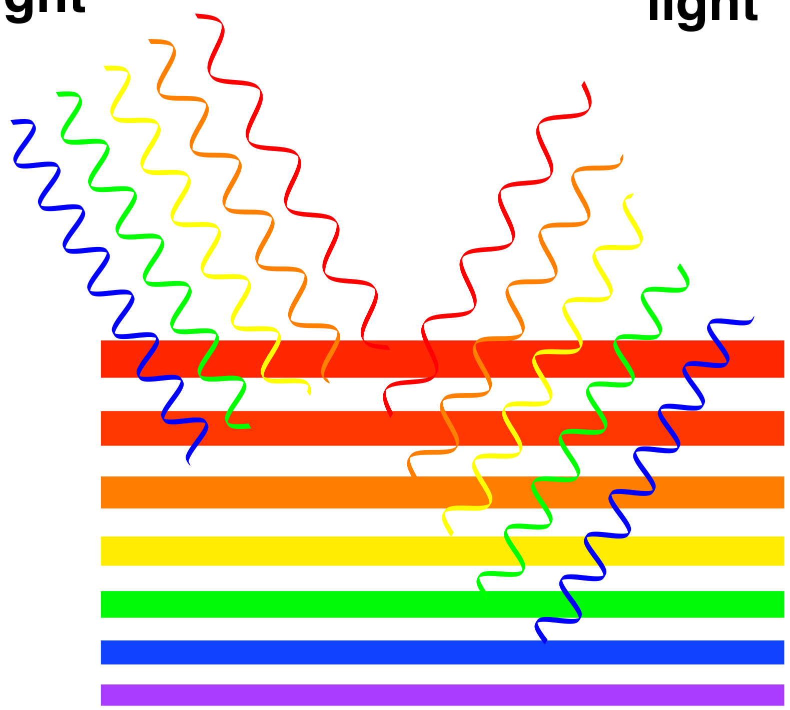


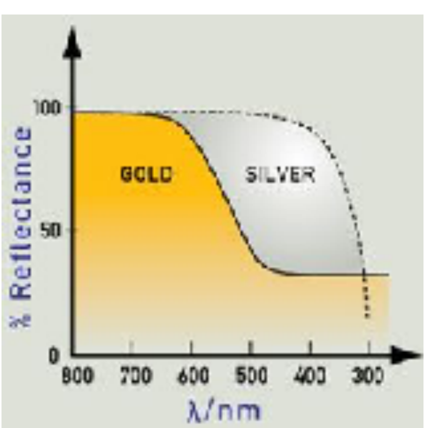
$1\mu\mathrm{m}$

reflected color depends on the viewing angle!

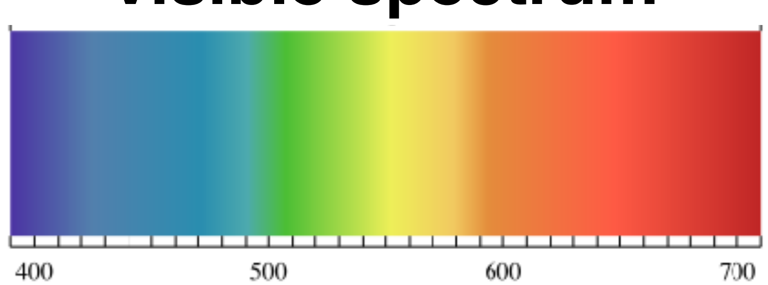
Silver and gold structural colors

Many colors reflected on structures with varying spacing





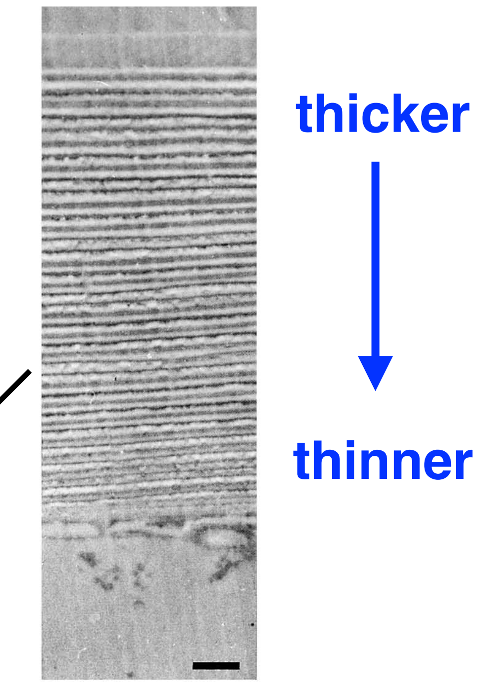
visible spectrum



wavelength $\lambda[\mathrm{nm}]$

chirped structure

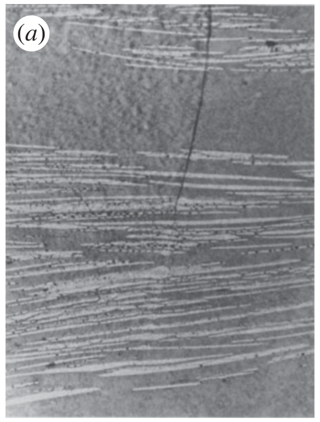
Chrysina limbata beetle **Chrysina aurigans beetle**



$1\mu\mathrm{m}$

disordered layer spacing

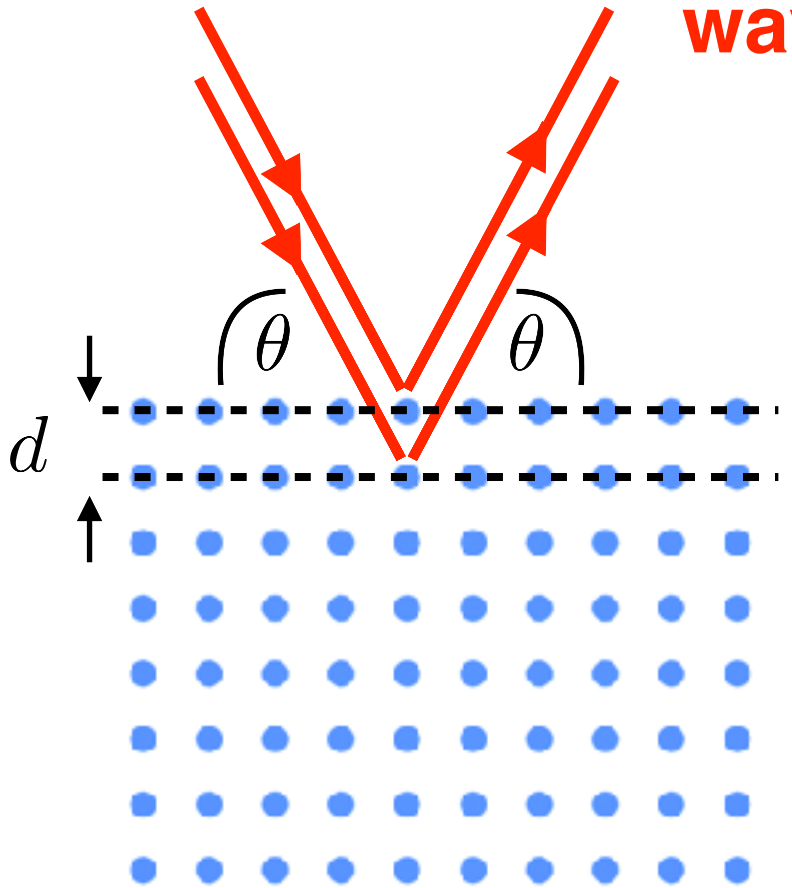
bleak fish

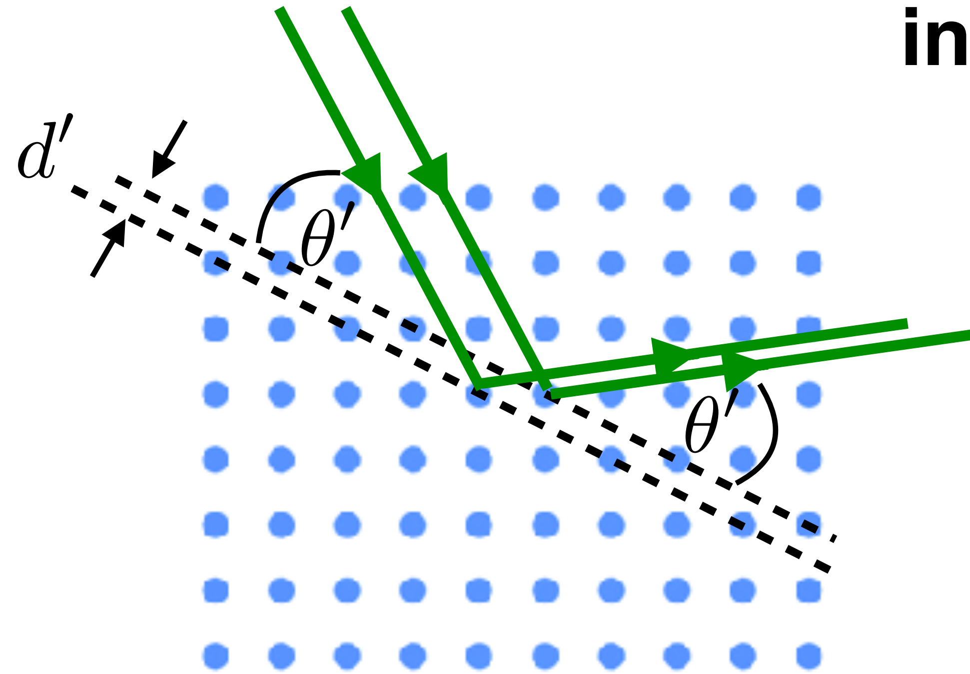


$1\mu\mathrm{m}$

Bragg scattering on crystal layers

Constructive interference for waves with different wavelengths occurs in different crystal planes!





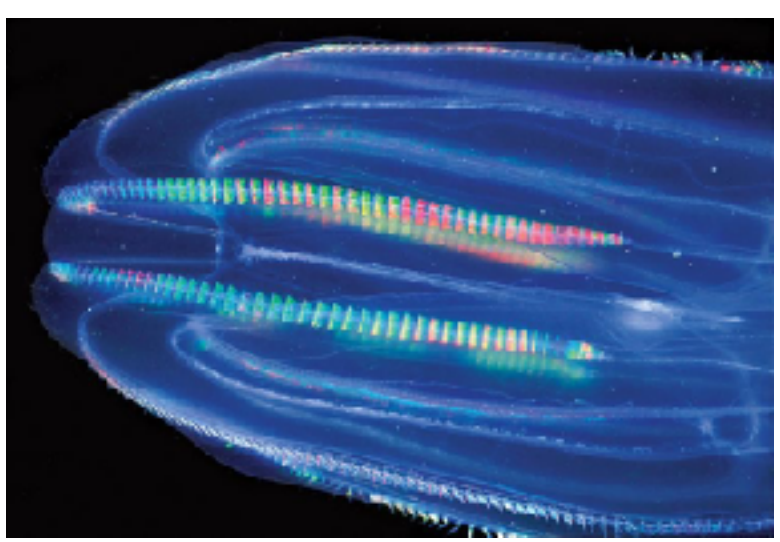
constructive interference condition

$$2d\sin\theta = m\lambda$$

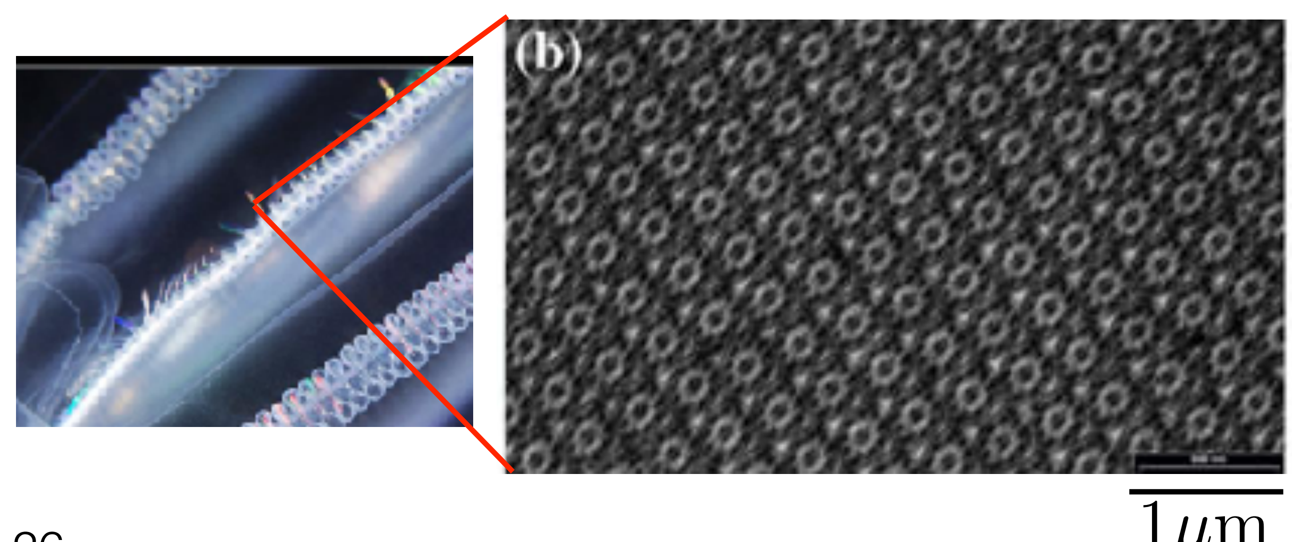
$$2d'\sin\theta' = m\lambda'$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Comb jelly

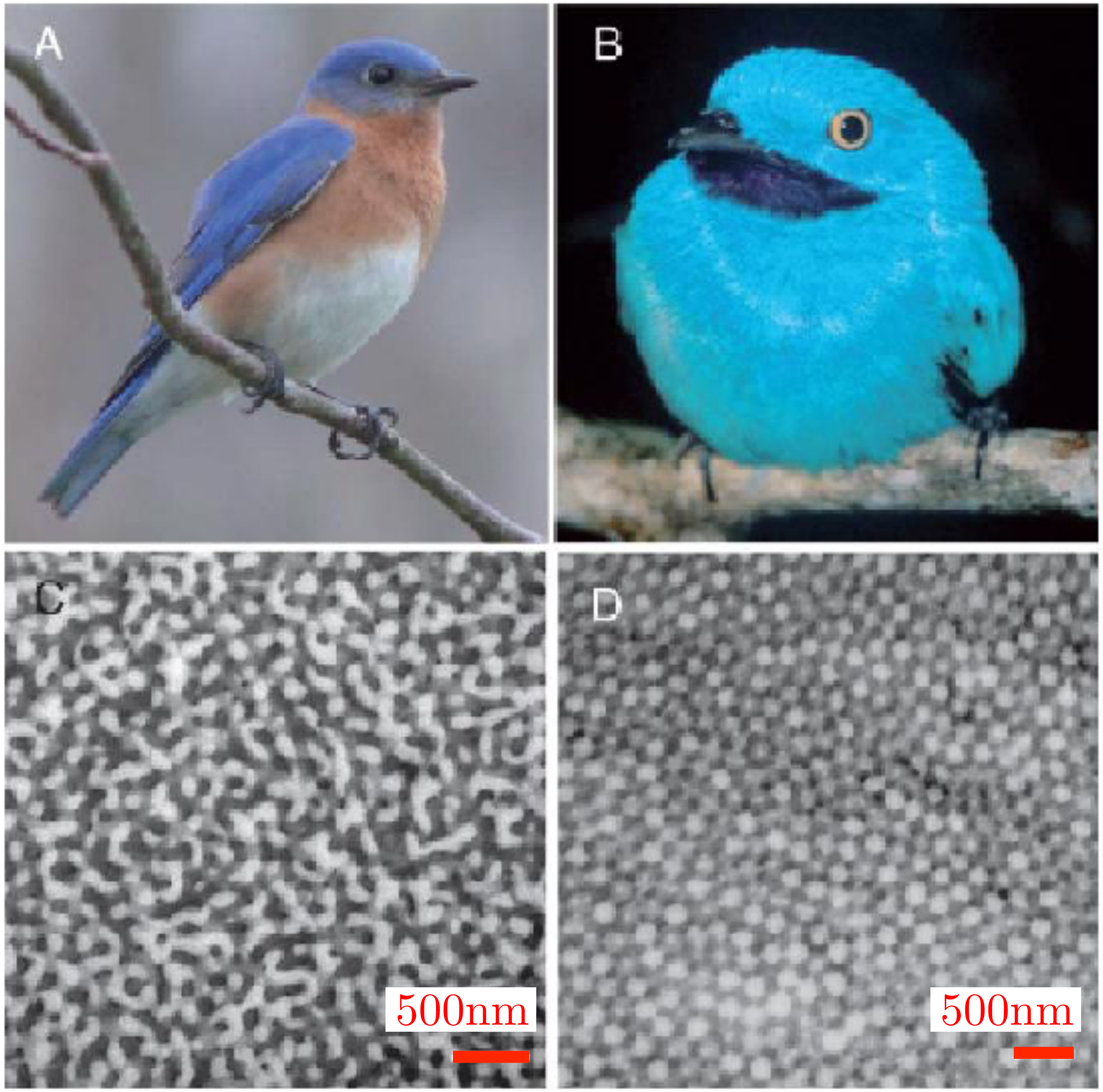


Beating cilia are changing crystal orientation



Scattering on disordered structures

Eastern bluebird | **Plum-throated Cotinga**



V. Saranathan et al.,
J. R. Soc. Interface 9, 2563 (2012)

Disordered structures with a characteristic length scale.

This length scale determines what light wavelengths are preferentially scattered.

The selectively reflected wavelengths are the same in all directions!

This gives rise to blue colors in these birds.

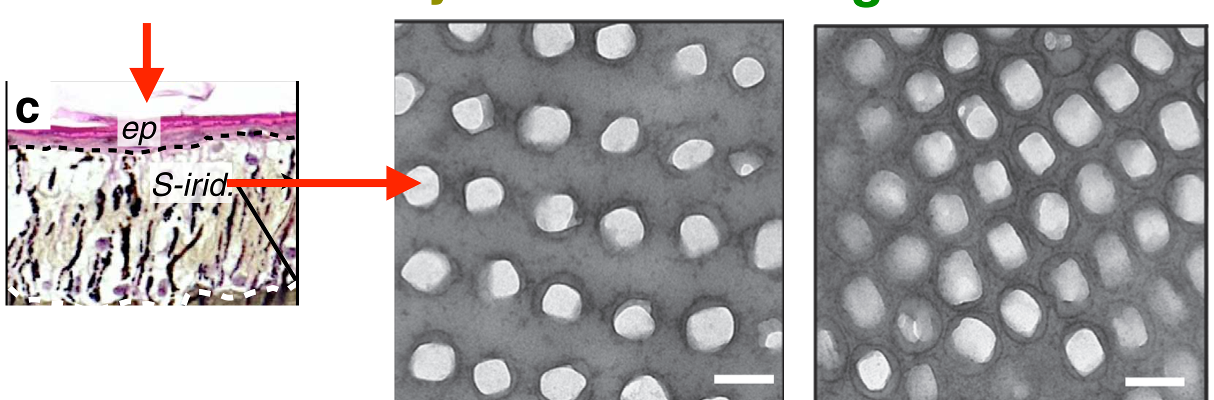
Dynamic structural colors

Chameleon (speed 8x)

J. Teyssier et al., Nat. Comm. 6, 6368 (2015)

Changes in osmotic concentration lead to the swelling of cells in excited chameleon. This changes the spacing of periodic structure from which the ambient light is reflected.

skin **yellow color** **green color**

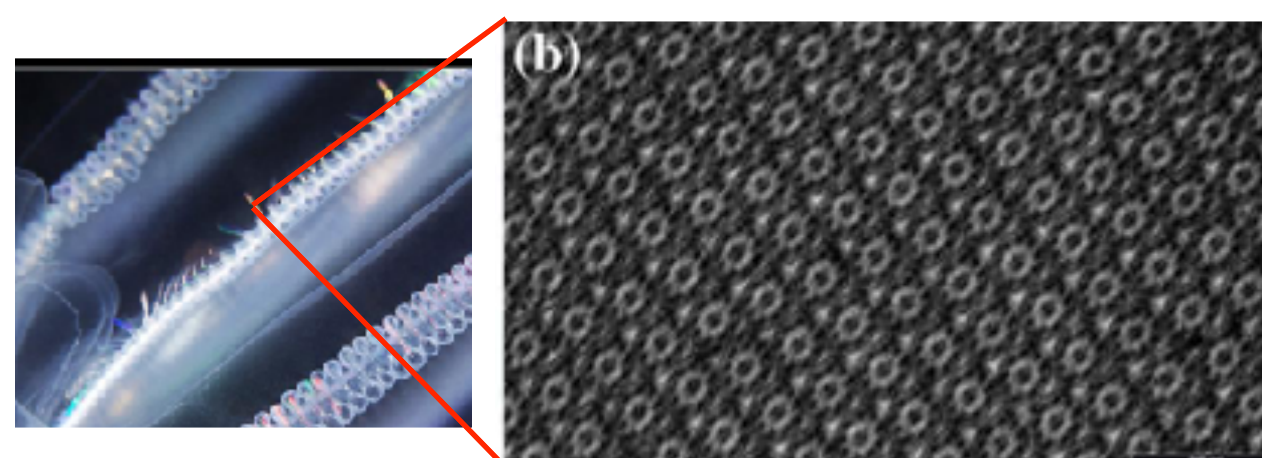


200nm 200nm

Comb Jelly (real time)

https://www.youtube.com/watch?v=Qy90d0XvJIE

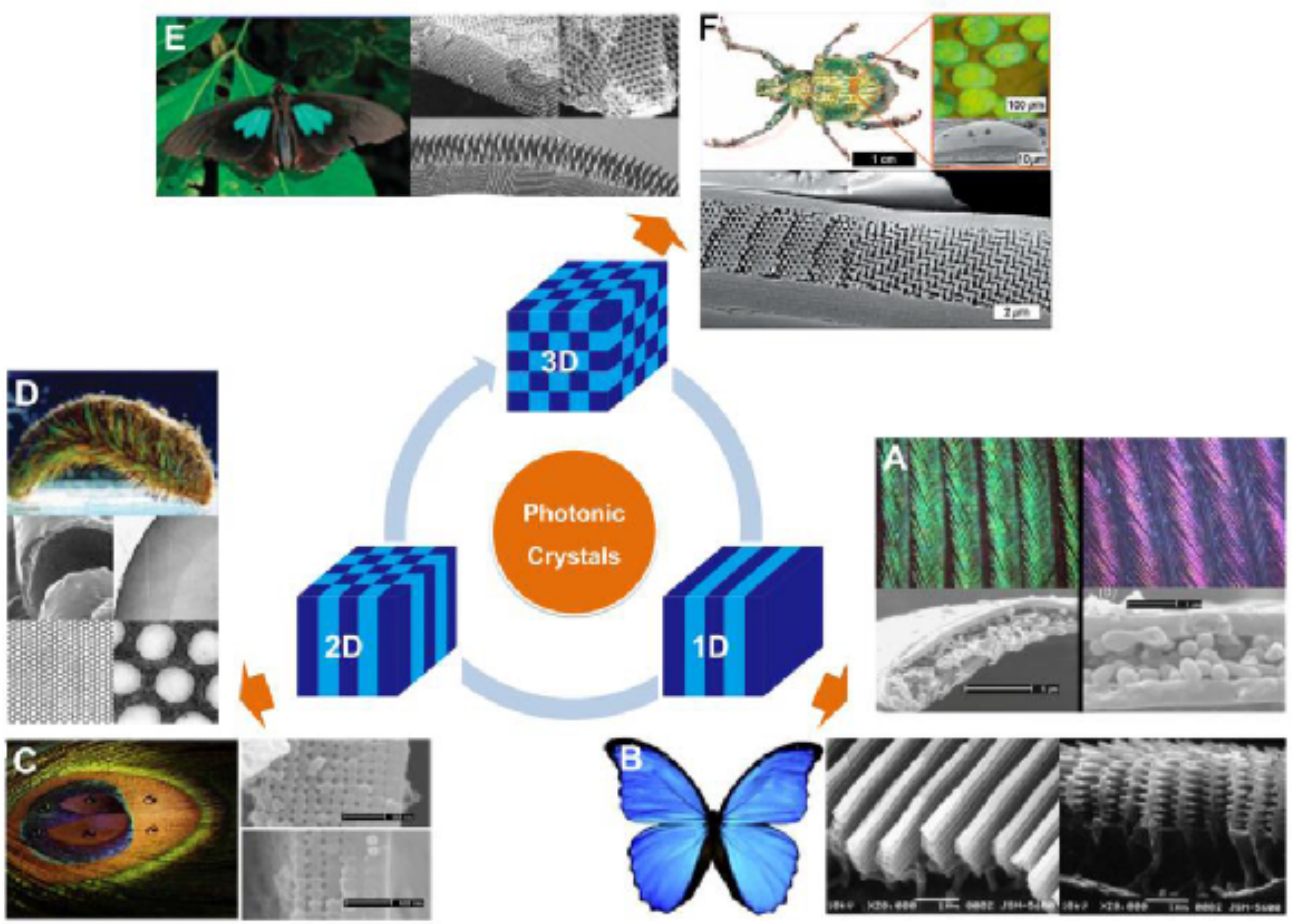
Rainbow color waves are produced by the beating of cilia, which change the orientation of periodic structure from which the ambient light is reflected.



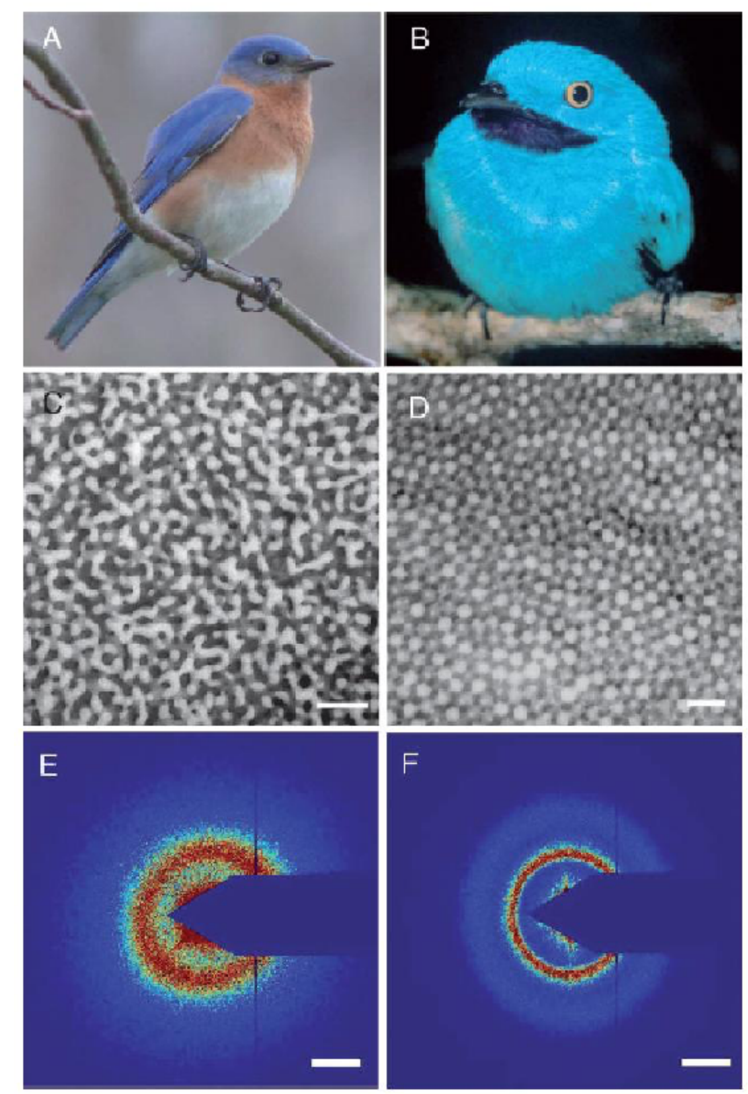
$1\mu m$

Structural colors

Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.

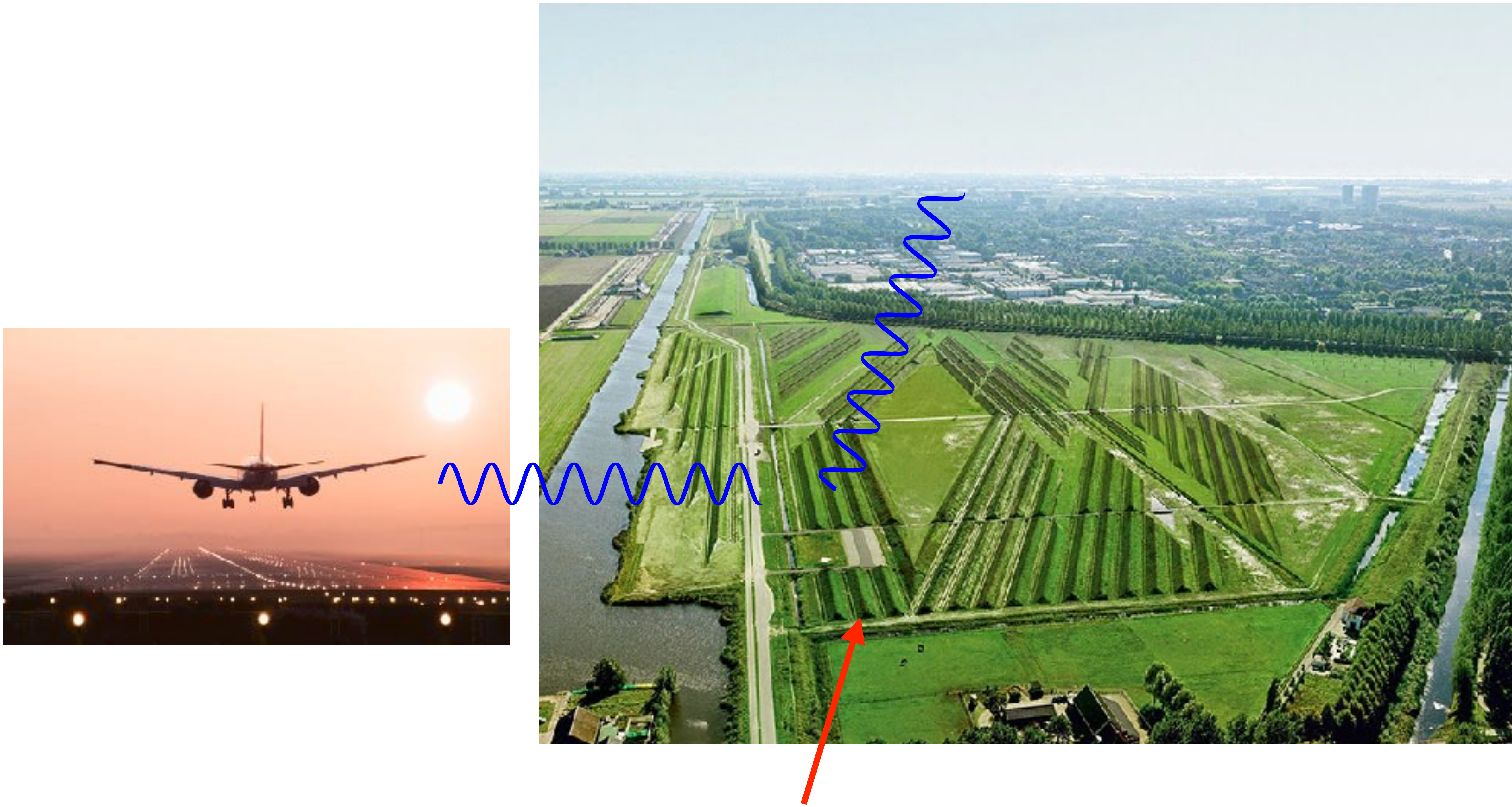


H. Wang and K-Q. Zhang,
Sensors 13, 4192 (2013)



V. Saranathan et al.,
J. R. Soc. Interface 9, 2563 (2012)

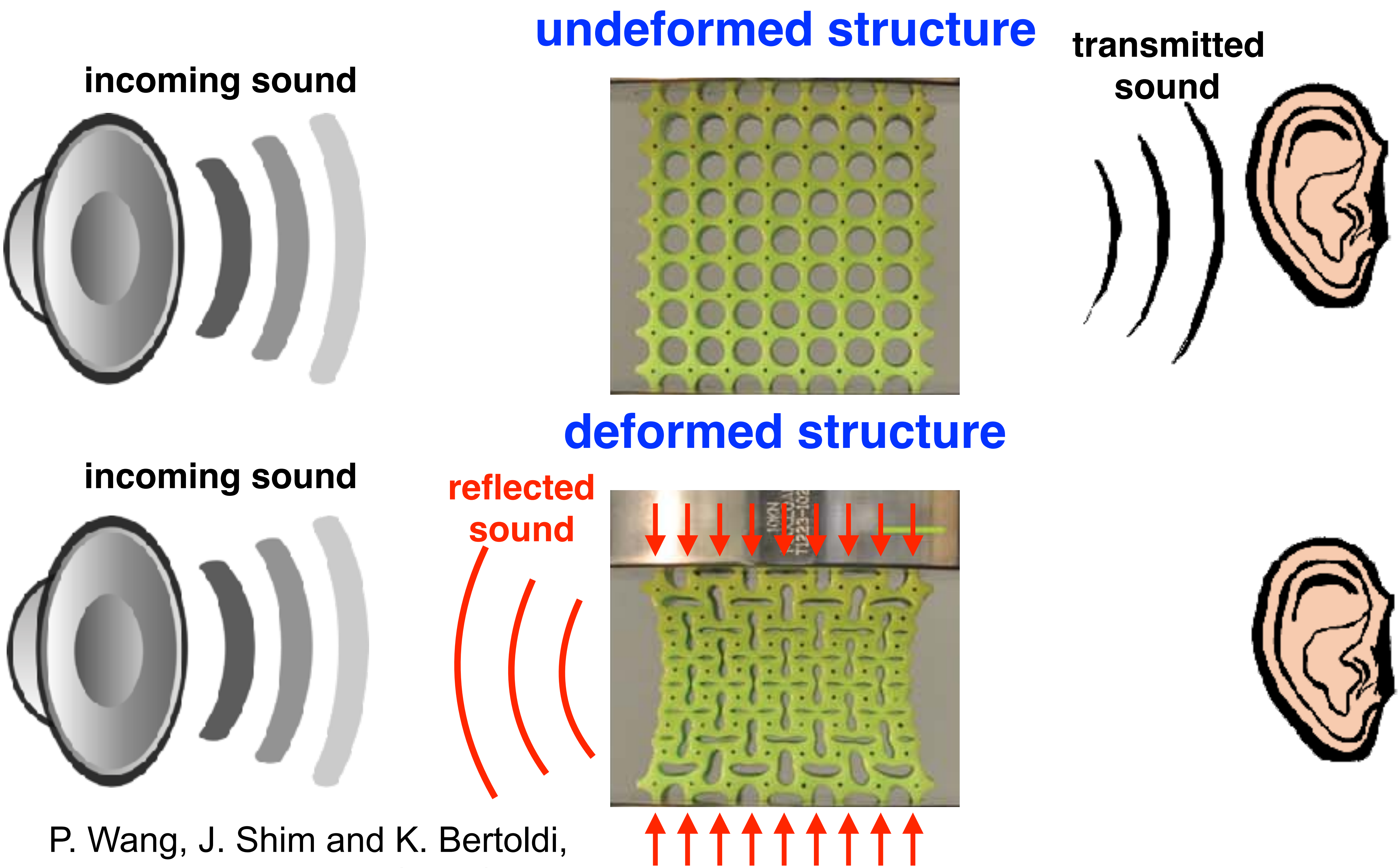
Noise barriers around the Amsterdam airport



Sound from airplanes that are landing and taking off is reflected from artificial barriers into the atmosphere.

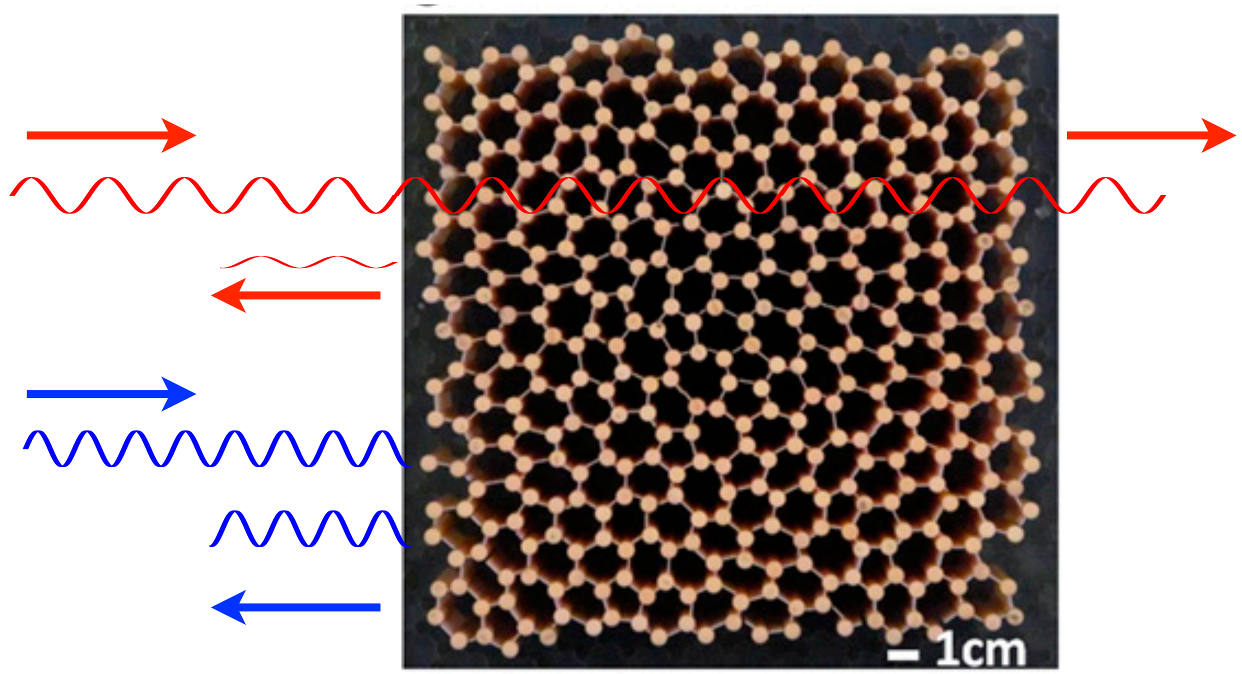
Controllable sound filters

In periodic structures sound waves of certain frequencies (within a “band gap”) cannot propagate. The range of “band gap” frequencies depends on material properties, the geometry of structure and the external load.



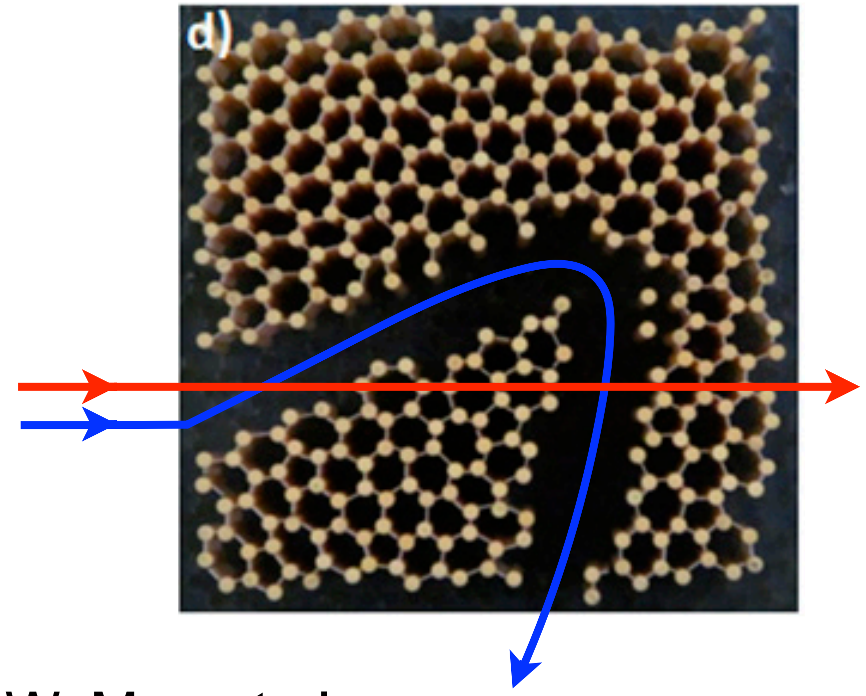
P. Wang, J. Shim and K. Bertoldi, PRB **88**, 014304 (2013)

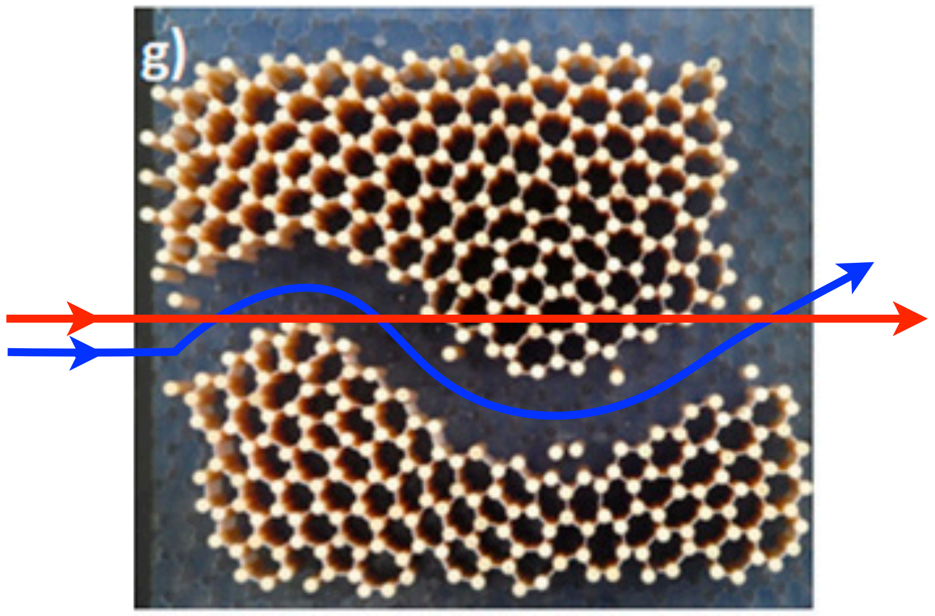
Waveguides in disordered structures



Channels inside structures can be used as guides for waves with wavelengths that are totally reflected from a complete structure!

Note: channels can have arbitrary bends!

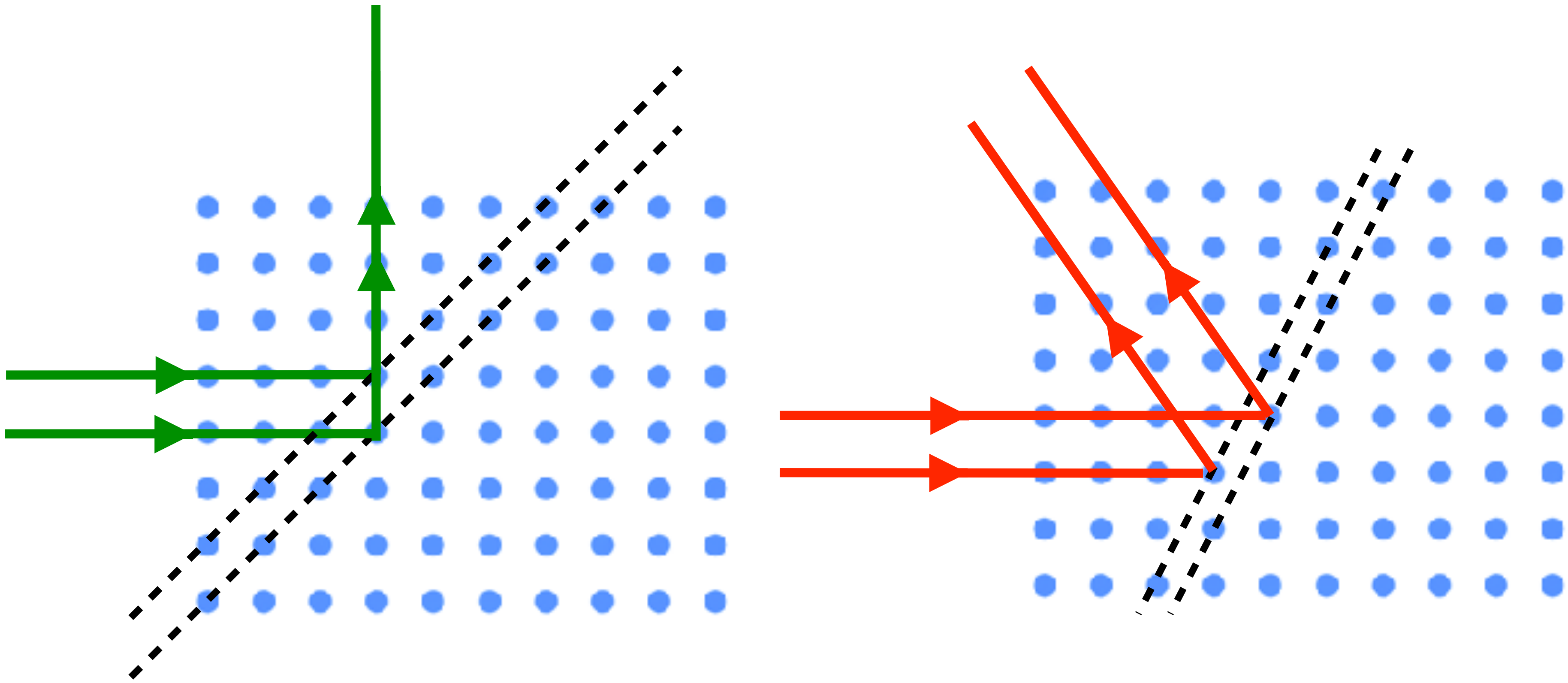




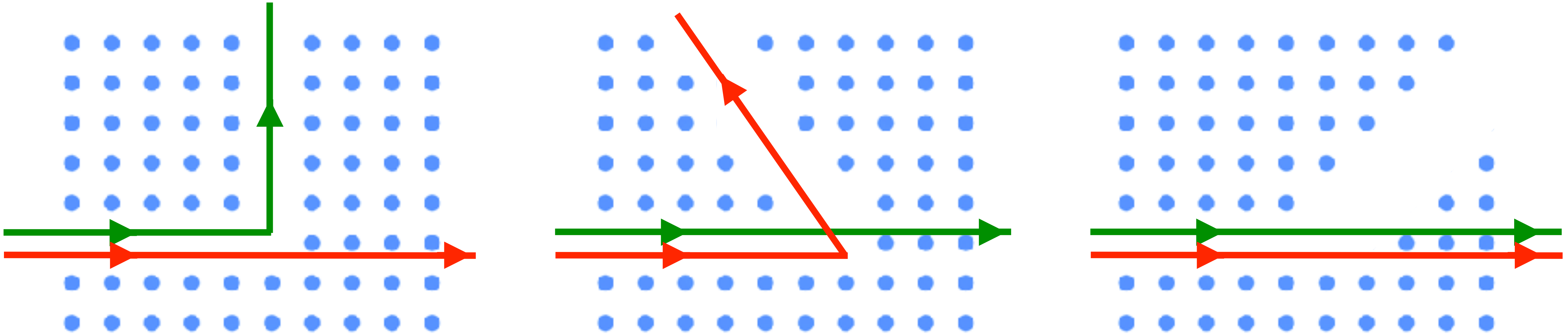
W. Man et al.,
PNAS **110**, 15886 (2013)

Waveguides in periodic structures

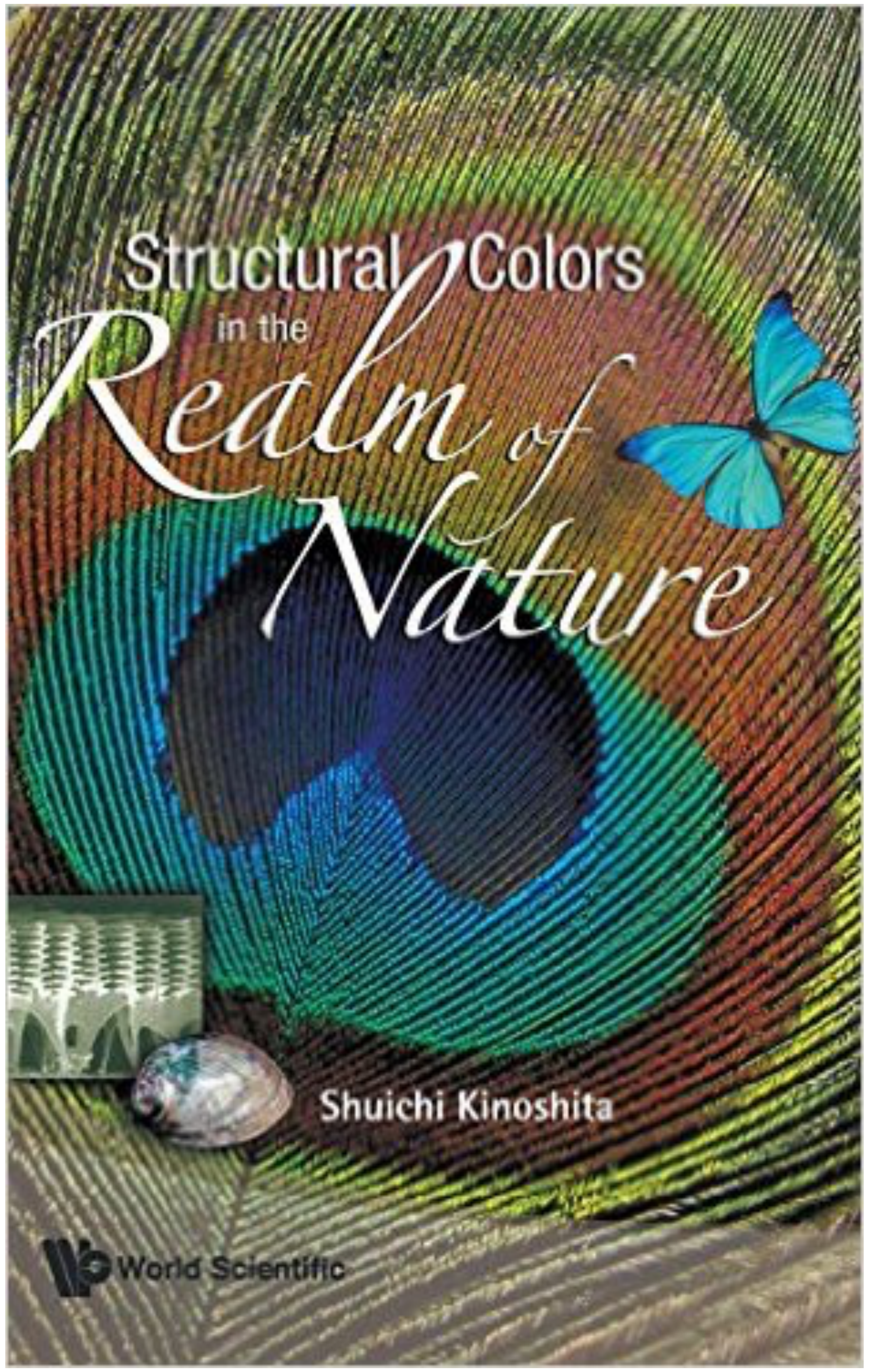
In periodic structures waves are completely reflected only at certain angles.

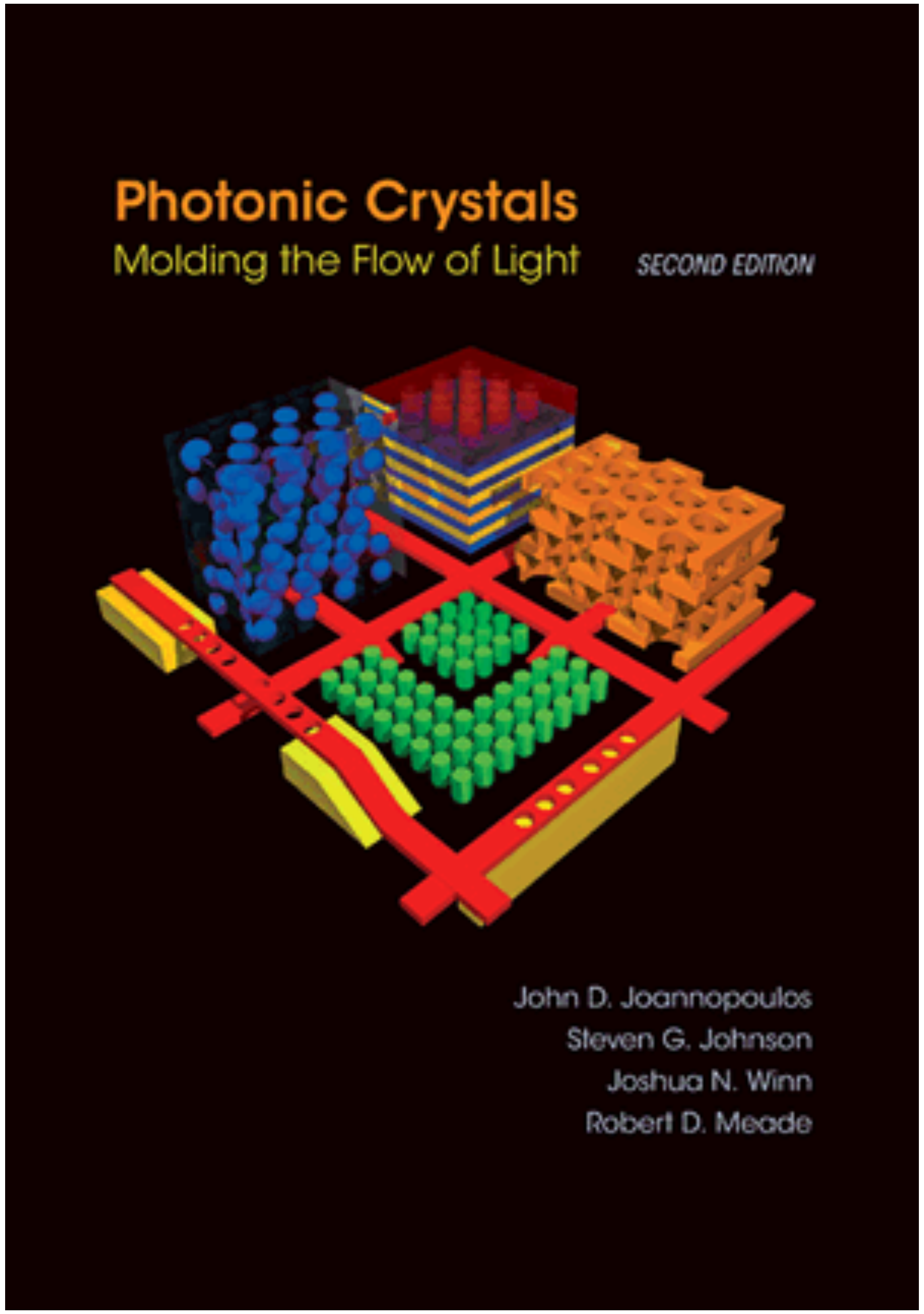


Note: channels with certain bends act as waveguides only for those waves that are completely reflected at these angles!



Further reading





http://ab-initio.mit.edu/book/