$\text{cture } 5 (2/21)$ $t_{\rm{max}}$ substrate, refer to the lm and substrate, respectively. **Wrinkled surfaces MAE 545: Lecture 5 (2/21)**

2

tronics
20 Original province in the metropology in the Science 320–912 \mathcal{M} aforementioned natural systems can be idealized natural systems can be idealized natural systems can be idealized in L. Pocivavsek et al., Science **320**, 912 (2008) F. Brau et al., Soft Matter **9**, 8177 (2013)

F. Brau et al., Soft Matter 9, 8177 (2013) the system size. However, the surface deformation energy of the

Buckling vs wrinkling

Compressed thin sheets buckle

S^1 \sim Ω \sim Ω **Compressed thin sheets on liquid and soft elastic substrates wrinkle** S^1 \sim Ω \sim Ω

Fig. 1 Qualitative comparison between the evolution with respect to confinement of the morphology of compressed sheets resting on a liquid⁷⁸ (left panels, adion mandri on the hydra on ook cradito capolita Fig. 1 Qualitative comparison between the evolution with respect to confinement of the morphology of compressed sheets resting on a liquid⁷⁸ (left panels, **Exploration on and operation on our chaptic foundation (right parallel " In compressed thin sheets on liquid and soft elastic substrates global buckling is suppressed, because it would result in very large energy cost associated with deformation of the liquid or soft elastic substrate!**

Note: upon compression the liquid surface also raises, but we will measure the potential energy relative to this new height!

Compression of stiff thin membranes on liquid substrates from the construction of the full expression for the full expression for the full expression for the full expression for the following construction for the full expression for the full expression for the full expression fo

scaling analysis $ln(\lambda)$

exact result

 β F. Brau et al., **Soft Matter 9**, 8177 (2013) σ and partially coross-linked to extend the performed to extend the substrate have been performed to extend the substrate σ

How to go beyond the simple scaling analysis to determine the nonlinear post-buckling behavior?

Find shape profile h(s) that minimizes total energy

$$
U_b + U_p = W \int_0^L \frac{ds}{2} \left[\frac{\kappa h''^2}{(1 + h'^2)^3} + \rho g h^2 \sqrt{1 - h'^2} \right]
$$

subject to constraint

$$
L - \Delta = \int_0^L ds \sqrt{1 - h'^2}
$$

9 F. Brau et al., Soft Má

9 F. Brau et al., <u>Soft Matter</u> 9, 8177 (2013) \mathbf{B}, \mathbf{O} ulk \mathbf{C} Uly

Compression of stiff thin membranes on liquid substrates increase in amplitude that gives rise to an increase in energy for the system. <u>the motor and sta</u> the writing transition. physical data are slightly shifted to the right as sion of St on of an and and physical data attests that the essential physical photography is captured in the photography. simulation. Both experiments show that a as the sum of linear and nonlinear terms, we ratae ðK=2Þ∫ L $\overline{}$

Comparison between theory (infinite membrane) and experiment studied a thin polyester film on water and nu-'ison between theol to the exergence of the energy function \mathbf{r} beyond which the surface geometry becomes v (infinite membrang $\sum_{i=1}^{n}$ and exneriment **Comparison between theory (infinite membrane) and experiment** Fig. 4 (a) Definitions of the amplitudes A⁰ and A1. (b) Comparison between the

10

L. Pocivavsek et al., **Science 320**, 912 (2008) infinite sheet. Inset: representative membrane profiles for various values of D/l0. and Chai (Circles). Experimental data including taken for several membershoep. In the N = 3.5, including when $\frac{1}{2}$ 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, and 8.0. Dark solid lines show numerical results for a sheet sheets \mathcal{S} and the evolution predicted by the exact solution (21) obtained for an and \mathcal{S} L. Pocivavsek et al., <u>Science</u> 320, 912 (

F. Brau et al., Soft Matter **9**, 8177 (2013) Γ D Ω and staled Ω of the latter Ω of 77 (0.010) ϵ_{10} F. Brau et al., <u>Soft Matter</u> 9, 8177 (2013) $\frac{1}{\sqrt{1}}$

Compression of stiff thin membranes on soft elastic substrates *d* **initial undeformed configuration**

Consider the energy cost for two different scenarios:

L

1.) thin membrane is compressed (no bending)

11

Note: soft elastic substrate is also compressed, but we will measure the substrate elastic energy relative to this base value!

deformation of the soft substrate decays exponentially away from the surface

 $h(s, y) \approx h_0 \cos(2\pi s/\lambda)e^{-2\pi y/\lambda}$

F. Brau et al., Nat. Phys. **7**, 56 (2010)

$$
2h_0 \downarrow \qquad \qquad \frac{d\downarrow}{L}
$$

amplitude of wrinkles

$$
h_0 = \frac{\lambda}{\pi} \sqrt{\frac{\Delta}{L}} = \frac{\lambda \sqrt{\epsilon}}{\pi}
$$

deformation of the soft substrate decays exponentially away from the surface

y

 $1/3$

$$
h(s, y) \approx h_0 \cos(2\pi s/\lambda) e^{-2\pi y/\lambda}
$$

 $U_b, U_s \sim A d\epsilon \left(E_s^2 E_m \right)$

bending energy of \mathbf{B} stiff membrane

deformation energy $U_s \sim V \times E_s \times \epsilon_s^2$
of soft substrate

$$
U_b \sim A \times \kappa \times \frac{1}{R^2} \sim A \times E_m d^3 \times \frac{h_0^2}{\lambda^4} \sim \frac{A E_m d^3 \epsilon}{\lambda^2}
$$

$$
U_s \sim V \times E_s \times \epsilon_s^2 \sim A \lambda \times E_s \times \frac{h_0^2}{\lambda^2} \sim A E_s \lambda \epsilon
$$

minimize total energy (*U***b+***Us***) with respect to**

14

 $\sqrt{\frac{1}{3}}$

 E_m

 E_s

 $\lambda \sim d$

16 F. Brau et al., **Soft Matter 9**, 8177 (2013) $\frac{10}{2}$

Compression of stiff thin membranes on soft elastic substrates Software Review of the Software Review of the Software Review of the Software Review of the Software Review of

 $\frac{1}{\sqrt{2}}$ ϵ and sheet to the sheet of the sheet, being proportional to the square of the the full nonlinear deformation **Tempt 2008** the system size. However, the surface deformation energy of the **of the soft substrate!** Fig. 2 \sim 1 \sim 1 \sim 2 \sim 1 \sim 1 \sim 1 \sim 1 \sim 0 \sim 1 \sim **11.6 cm in an elastic foundation (right panels confidence**ly particular particular foundation particular in par **In order to explain period doubling (quadrupling, …) one has to take into account** Published on 25
Princeton University on 25/11/2015 15:27.
Princeton University on 25/11/2015 15:19:27.

Tranglet al., Soft Matter 9, 8177 (2013) PDMS and E ¼ 3.2 GPa, s ¼ 0.35, h ¼ 218 nm for polystyrene (PS). Data:⁵⁷ E ¼ 130 $_{17}$ F. Brau et al., <u>Soft Matter</u> 9, 8177 (2013)

S. Cai et al., J. Mech. Phys. Solids 59, 1094 (2011)

Fig. 4. A sequence of pictures of the hexagonal mode to a ''sequence to a '' (disorganized herringbone) pattern with the herringbone of the herringbone of the herringbone) pattern with the herringbone of the herringbone (d uincreasing overstress.
The contract of the contract o
Support of the contract A.) differential growth in biology and the uvorstreated Poles as described in the text. The UVO-text. The text. The text. The UVO-text. The text. The UVO-text. The text. The text. The UVO-text. The UVO-text. The UVO-text. treatment times from left to right are 10, 15, 20, 30, 45, and 60 min. **5.) differential expansion due to temperature, electric field, etc.**

red gel swells more than the green gel

19

Compression of stiff thin membranes BS on a spherical soft substrates Increasing e!ective radius *R*/*h u*min/*h u*/*h u*max/*h* based on numerical steady-state solutions of equation (1). Colour red (blue) signals inward (outward) wrinkles. Simulation parameters: (**a**) ⁰ =0.029, *^a*=0.00162, *^c*=0.0025; (**b**) ⁰ ⁼0.04, *^a*=1.26⇥106, *^c*=0.002; (**c**) ⁰ ⁼0.02, *^a*=1.49⇥104, *^c*=0.0025 (see Table 1). **^d**–**f**, Experimentally negligible geometric non linearities, even if the strong material properties remain linear. Furthermore, the strong material properties remain linear and the strong material properties remain linear and the strong material rooting of the far-from-threshold (*a.k.a.i.a.i.a.)* regions on general and universal and univer $u_{\text{min}}/h = u_{\text{min}}/h = u_{\text{max}}/h$ functional mechanisms based on instabilities can be instantiated over a wide range of scales. The *second motivation* regards the **engineering significance** and the **broader impact for technology** of

Compression of stiff thin membranes on a spherical soft substrates

2(1+⌫)

 \overline{a}

Soft Matter **9**, 3624 (2013)

<u>SOIL Matter</u> **9**, 3024 (2013)

(fixed thickness *d*) **Modifying radius** *R* \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L} are so \mathcal{L}

6(1+⌫)⌘2*/*³ **Modifying membrane thickness** *d* between radial displacement of the shell and the stretching \mathbf{b} radial displacement of the shell and the stretching \mathbf{b} energy in the shell like $\mathbf s$ changes this dependence from a quadratic to a linear relation-

250 µm

energy in the shell limit of curvature in the shell. The introduction of curvature in the introduction of curvature

surface with radius of curvature R. The denition of the curved

Modifying membrane thickness d	
$R = 381 \mu m$	$R = 805 \mu m$
$R = 381 \mu m$	$R = 805 \mu m$
$R = 381 \mu m$	$R = 805 \mu m$
$R = 599 \mu m$	$R = 805 \mu m$
$R = 522 \mu m$	$R = 805 \mu m$

Modifying swelling strain ϵ critical stress s^R approach of Hutchinson²⁹ for buckling of hollow spherical shells. In brief, similar to the case of understanding instabilities i in a at system, critical deformation modes of the following α

Tuning drag coefficient via wrinkling Fig. 1b) will involve advanced manufacturing techniques (*e.g.* digital fabrication and rapid prototyping)

Self-cleaning property of lotus leaves

Lotus leaves repel water (hydrophobicity) due to the rough periodic microstructure

M. N. Costa et al., Nanotechnology **25**, 094006 (2014)

V 1ey

Tuning wetting angle via wrinkling

Water droplet on a flat surface

front view

 $\overline{\mathbf{N}}$ dropletWater droplet on a wrinkled surface **(wrinkling increases contact angle)**

but

 $\mathbf i$

a

front view t
I $\overline{\mathbf{o}}$ n
I \overline{v} front view
view

front view $\frac{1}{24}$ J. Y. Chung et al., **Soft Matter 3**, 1163 (2007) \mathcal{C} compression, the equilibrium contact angles (wH and wI) \mathcal{C} 24 b. r. Ondrig et an, <u>bont iviation</u> **b**, rroplet showing evidence of pinning of an elongated showing 24 Fig. 2 (a) Top view optical micrograph of the smooth surface. (b)–(d) Optical micrographs of a water dropped on the smooth surface. (b)–(d) Optical micrographs of a water dropped on the smooth surface. (b)–(d) Optical micr Dettre is $\frac{1}{2}$

diew

side view

found

 ϕ k

side view

side view

Tuning adhesion via wrinkling

25

Flat complaint surface has enhanced adhesion (larger contact area)

Wrinkling reduces adhesion (smaller contact area)

mmm

Wrinkled structures can be used for flexible electronics

Figure 1. Images and design features of a simple, multifunctional device with skin-like physical characteristics and capabilities in both sensing and B. Xu et al., Adv. Mater. **28**, 4462 (2016)

How are villi formed in guts?

Villi increase internal surface area of intestine for faster absorption of digested nutrients.

Lumen patterns in chick embryo and an inner, luminal endo stearna in ahialz alichis fillus formation FI-40014Jyväskylä, Finland. ⁴ inhrvo Organismic and Evolutionary Biology, Harvard University, Cambridge, MA 02138, USA. ⁶ array of fingerlike projections termed intestinal villi **unich pattens i** ahial ambryo inner endormal tion of smooth muscle is a substantial of the smooth muscle in the smooth muscle is a substantial of the smooth mends in chick a sion of the gut segments of the grown in vehicle alone developed a layer of circular smooth muscle and formed luminal folds.

face of the guidant of the gut the gu The Convolution DAPI marks cell nuclei Morphogenesis and Differentiation of Constrained Azimuthal Growth of the

as in marked and birds, the angles of and birds, the set of and birds, the set of and birds, the set of an organized and birds, the set of and birds, the set of an organized and birds, the set of an organized and birds, th \mathbb{R} abrus interved interved in the Miles aSMA marks smooth muscle actin Endoderm-Mesenchyme Composite th muscle actin

E…: age of chick embryo in days tissues can lead to epithelial buckling is classical mbryo in days **compared to example.** plain longitudinal ridge formation in healthy and

(Middle) Close-ups of left photos, showing muscle layers. (Right) Whole-mountains \mathbb{R}^n

sity, Cambridge, MA 02138, USA. ⁷ Wyss Institute for Biologically It muscles arow slower us maconoo grom oromor an softer mesen *These authors contributed equally to this work. and endoderm layers Quantifying the constraint provided by the mus-**Stiff muscles grow slower** of the muscle layer in the control samples to the than softer mesenchyme

This individual compression due of the inner circumference of the inner circumferences striped wrinkles to differential growth

to 0.55 across the developmental stages from E8 to E12 (Fig. 2B). However, the separation of the separation **endoderm from the composite of mesenchyme**

28 A. Shyer et al., **Science 342**, 212 (2013) $\frac{1}{\sqrt{2}}$ second longitudinal muscle layer forms, interior to the formation of villiant $\frac{1}{\sqrt{2}}$ Schematic illustration of villiant $\frac{1}{\sqrt{2}}$ compression and buckling. This suggests that suggests that \mathcal{L}

Lumen patterns in chick embryo (1, 2) although a variety of morphologies such as \blacksquare attarne in ahiak allerns in Ghick *These authors contributed equally to this work. \mathbf{m} hrv \mathbf{n} \mathbf{r} indiyo \mathbf{v} the Chick Midgut **LUNTURII PALLEI** University, Cambridge, MA 02138, USA. is in chick em †Corresponding author. E-mail: lm@seas.harvard.edu (L.M.); face of the gut transforms from a smooth surface **Lanch pattens in Giller** \blacksquare USA. ⁸ Kavli Institute for Nanobio Science and Technology, Harvard University, Cambridge, MA 02138, USA.

ferentiation of smooth muscle layers. (Left photos) Transverse sections of $\mathcal{M}(\mathcal{M})$ close-ups of left photos, showing muscle layers. (Right) Whole-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount-mount

29 exterior to the circular layer of α , with the formation of α a second al., Shyer et al., Science **342**, 212 (2013)

Lumen patterns in chick embryo Until embryonic day 7 (E7), the gut tube, with its inner endodermally derived epithelium and outer <u>tabina de l</u> axial compression that mimics the role of the arne in chick a za aristmenters (fundameters (fundameters (fundameters ϵ derm (supplementary materials, fig. S9, and movie mhryn initially, the endoderm and mesenchyme are the endoderm and mesenchyme are the endoderm and mesenchyme a assumed to have \mathbf{y}

Villi start forming at E16 because **Though and Lease and Lease and Lease Canada** of the faster growth in valleys across the mesenchyme and endoderm before villi **Propertion in Addition in Addition** VIIII start forming \sim f the fer Although additional compression from the inner

Zigzag Zigzag Twisting Bulges

images of corresponding gut lumen pattern; longitudinal axis runs top to bottom. Scale bars indicate 100 mm; time is in days past fertilization (e.g., E6). (A) Lumen is smooth before muscle layers form. A, anterior; P, posterior. (B) Longitudinal ridges for $\mathbf{v}_\mathbf{I}$ exterior to the circular layer (arrow coincident with the formation of ϵ is maintained but with increasing and compactness over ϵ **The same mechanism for** Previous work in mouse has shown that, al**villi formation also works** in other organisms! arise, as villi form, proliferating cells are found **in other organisms!**

30 \overline{a} and \overline{b} these changes in problems in pr 30 A. Shyer et al., **Science 342**, 212 (2013)

Why are guts shaped like that?

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RESEARCH ARTICLE **Guts in chick embryo** RESEARCH ARTICLE finite radius of the loop. To deform the gut into a loop of radius R, the in chick embryo exerted by the membrane with strain energy and a length α width with strain energy and a length R

above the surgically removed guts from chick embryo surgically removed guts from chick embryo $\mathbf{f}_{\mathbf{M}}$ \mathbf{y} uture to a thin, double-epithelial sheet with no observable left–

separation from **mesentery** Cells per mm2 \overline{a} 12 **Tube straightens after** attached, there is a differential strain,e, that compresses the tube axially

we faetor than Tube grows faster than were under the dorsal to cut the dorsal SMA in order to cut the document of the document of the during guide a mesentery sheet! \mathbf{A} comparison of the results of our predictions with \mathbf{A} and \mathbf{A} and \mathbf{A} and \mathbf{A} igntens after the measurement of the grows faster than

32

T. Savin et al., **Nature 476**, 57 (2011) **tio**, si (2011) $\frac{1}{2}$ $\frac{1}{2}$ T Covin at al b, Prodition in the End of Andrew States (blue) and the End method is and mediant of the End method is and mes vin e

Synthetic analog of guts tion whereas the mesentery relaxes to an almost flat configuration implies that the tissues behave elastically, a fact that will allow us to **Synthetic a**

Rubber model of guts

Chick guts at E12

stretched uniformly along its length and then stitched to a straight, unstretched rubber tube (gut) along its boundary; the differential strain minics of the differential strain minics the differential strain minics the differential strain minics the differential strain minics of the differential strain differential growth of the two times. The system was then allowed to relax, free to relax, free to relax, free
, the system was the $F = \frac{F}{2}$ constructed wavelength of the rubber model of the rubber sheet (mesentery) was sheet (mesentery) was sheet (me
The rubber sheet (mesentery) was sheet (mesentery) was sheet (mesentery) was sheet (mesentery) was sheet (mese stretched uniformly along the composite rubber model to a straight, uniformly along the composite rubber model \mathbf{r}_i rubber tube (gut) along its boundary; the distribution of the $\frac{d}{dt}$ differential growth of the two times. The system was then allowed to relax, free allowed to relax, free allowed to relax, $\frac{d}{dt}$ **What is the wavelength of**

Compression of stiff tube on soft elastic mesentery sheet

$$
\begin{array}{c}\n 2h_0 \uparrow \\
\xrightarrow{L}\n \end{array}
$$

$$
\text{assumed profile} \quad h(s) = h_0 \cos(2\pi s/\lambda)
$$

amplitude of wrinkles

$$
h_0 = \frac{\lambda}{\pi} \sqrt{\frac{\Delta}{L}} = \frac{\lambda \sqrt{\epsilon}}{\pi}
$$

deformation of the soft mesentery decays exponentially away from the surface

w

d

 $2r_0$ $2r_i$

y

 $h(s, y) \approx h_0 \cos(2\pi s/\lambda)e^{-2\pi y/\lambda}$

bending energy of stiff tube

$$
U_b \sim L \times \kappa_t \times \frac{1}{R^2} \sim L \times E_t I_t \times \frac{h_0^2}{\lambda^4} \sim \frac{LE_t I_t \epsilon}{\lambda^2}
$$

deformation energy of soft mesentery

$$
U_m \sim A \times E_m d \times \epsilon_m^2 \sim L\lambda \times E_m d \times \frac{h_0^2}{\lambda^2} \sim L E_m d\lambda \epsilon
$$

minimize total energy (U_b+U_m) **with respect to**

$$
\lambda \sim \left(\frac{E_t I_t}{E_m d}\right)^{1/3}
$$

bending stiffness of tube $\kappa_t = E_t I_t$ $\kappa_t \propto E_t(r_0^4 - r_i^4)$

Wavelength of oscillations in guts \mathbf{M}_{e} similar set of measurements the course of gut development in mouse with α birds, the geometrical and biophysical properties of the developing might be regulated by mechanical feedback. Discussion $T_{\rm tot}$ investigate the physical origins of the physical origins of the physical origins of the physical origins $T_{\rm tot}$ developed a simple simulacrum of the gut–mesentery composite \mathbf{u} silicone rubber tube (mimicking the guide the guide sheet) and a thin latex sheet shee

35 Savin et al., Nature $476, 57$ (2011) T. Savin et al., <u>Nature</u> **476**, 57 (2011)