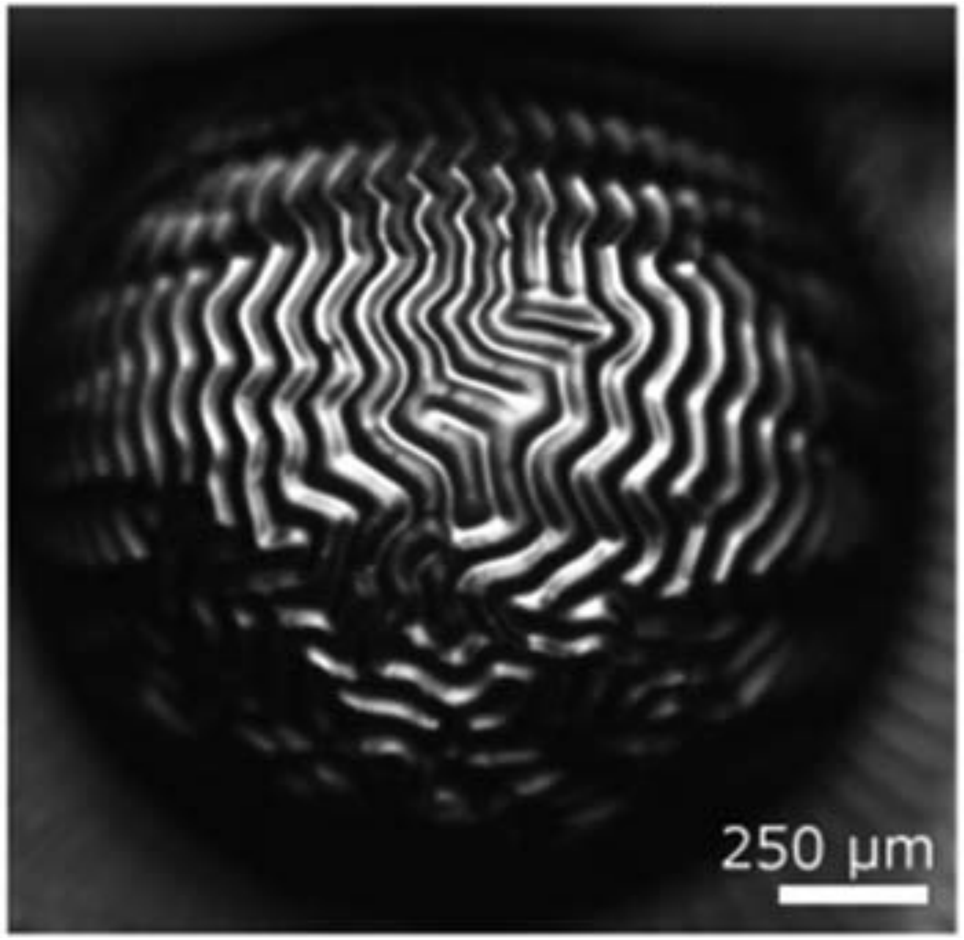
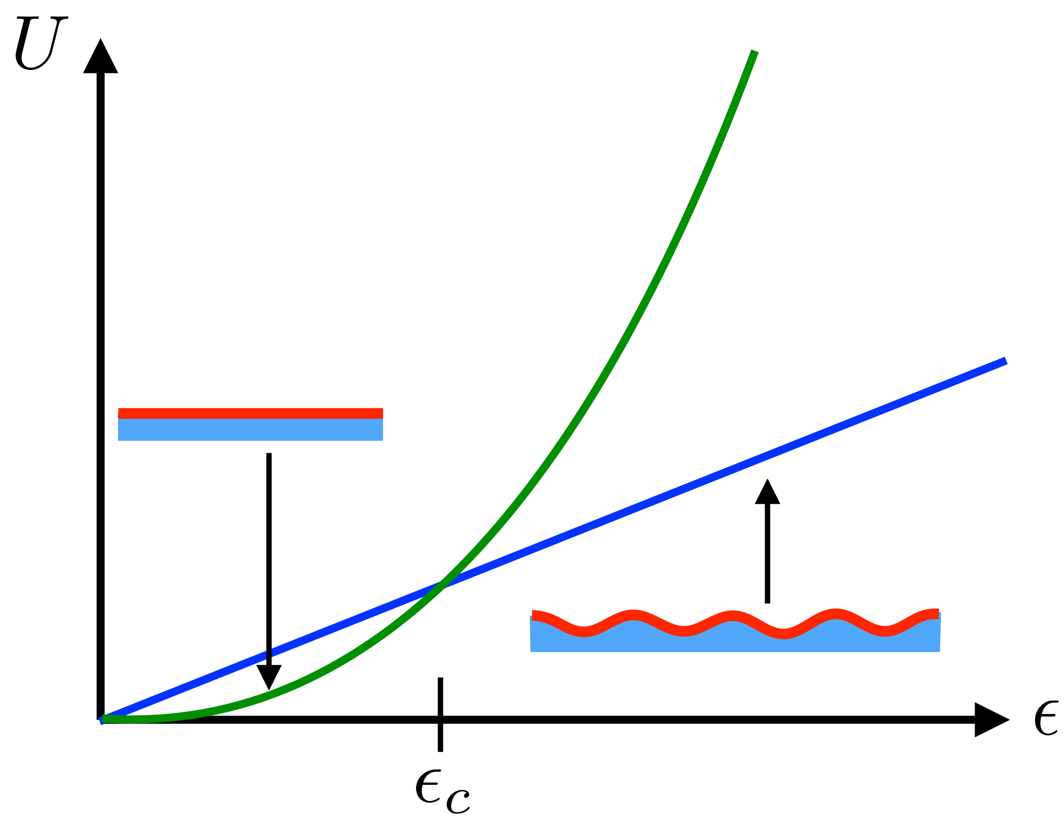
MAE 545: Lecture 6 (2/23)

Wrinkled surfaces



Compression of stiff thin sheets on liquid and soft elastic substrates

liquid substrate



wrinkles are stable above the critical strain

$$\epsilon > \epsilon_c \sim \sqrt{\frac{\rho g d}{E_m}}$$

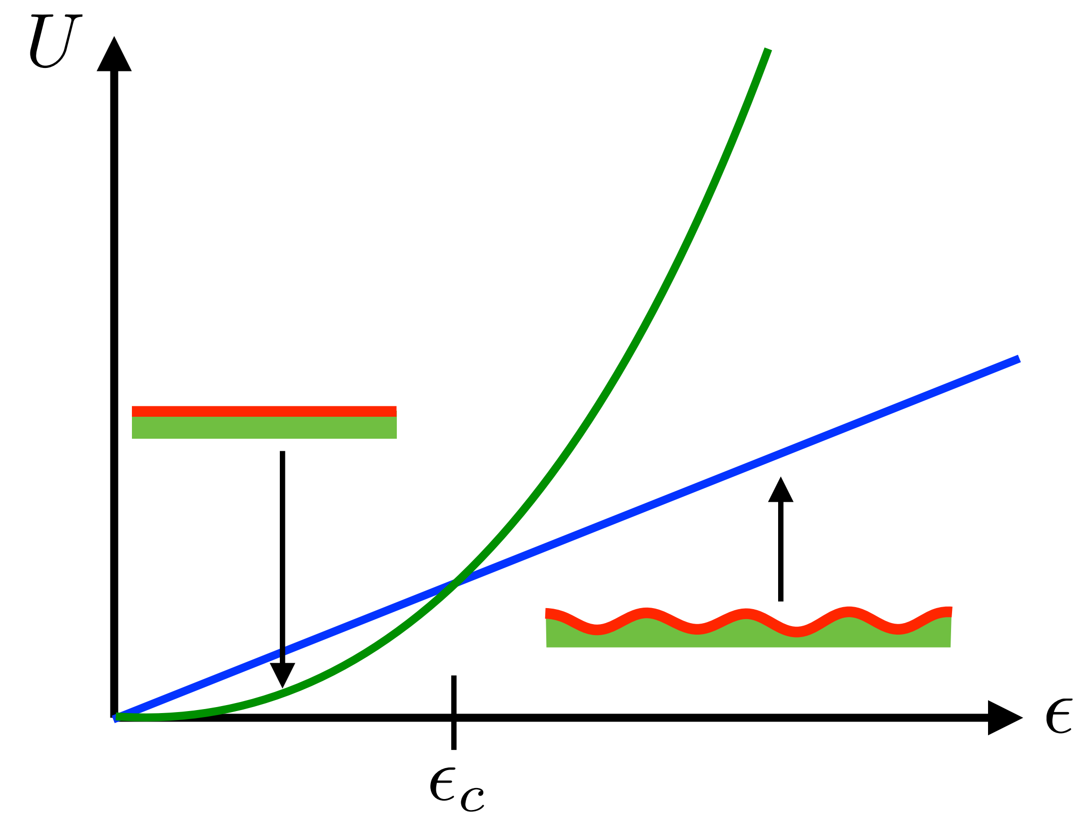
wavelength of wrinkles

$$\lambda \sim \left(\frac{E_m d^3}{\rho g}\right)^{1/4}$$

amplitude of wrinkles

$$h_0 \sim \lambda\sqrt{\epsilon}$$

soft elastic substrate $E_s \ll E_m$



wrinkles are stable for large strains

$$\epsilon > \epsilon_c \sim \left(\frac{E_s}{E_m}\right)^{2/3}$$

wavelength of wrinkles

$$\lambda \sim d\left(\frac{E_m}{E_s}\right)^{1/3}$$

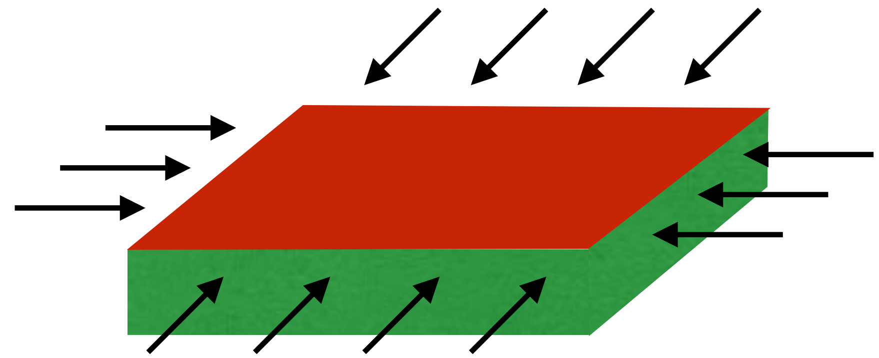
amplitude of wrinkles

$$h_0 \sim \lambda\sqrt{\epsilon}$$

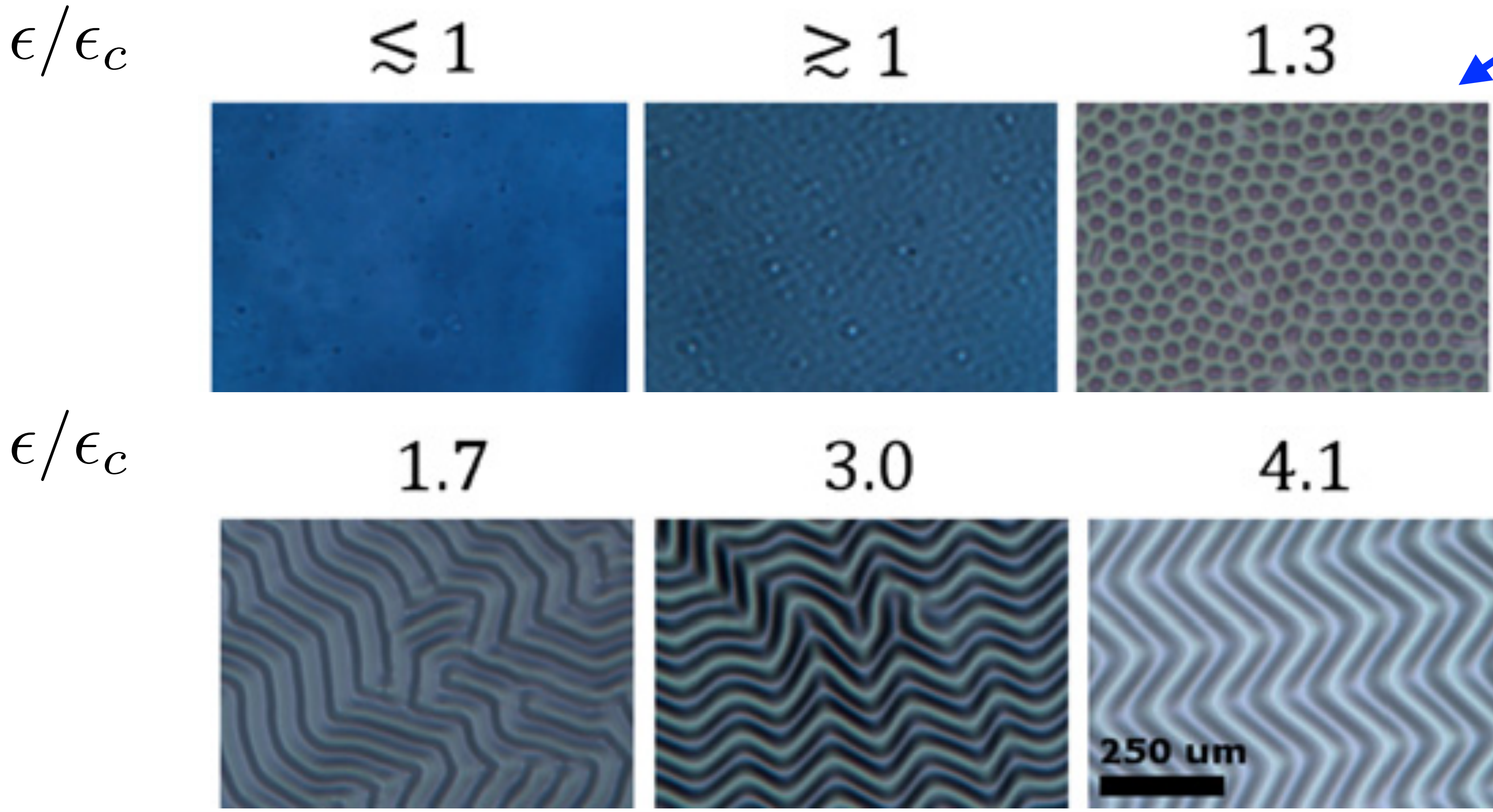
Uniform compression of stiff thin membranes on soft elastic substrates

critical strain

$$\epsilon_c \sim \left(\frac{E_s}{E_m}\right)^{2/3}$$



hexagonal crystal of bumps



characteristic wavelength

$$\lambda \sim d \left(\frac{E_m}{E_s}\right)^{1/3}$$

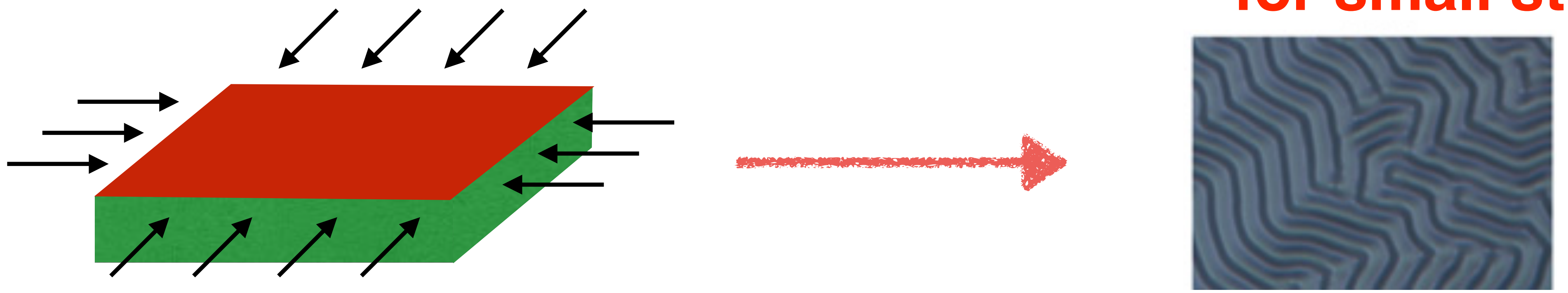
herringbone pattern

S. Cai et al., <u>J. Mech. Phys. Solids</u> **59**, 1094 (2011)

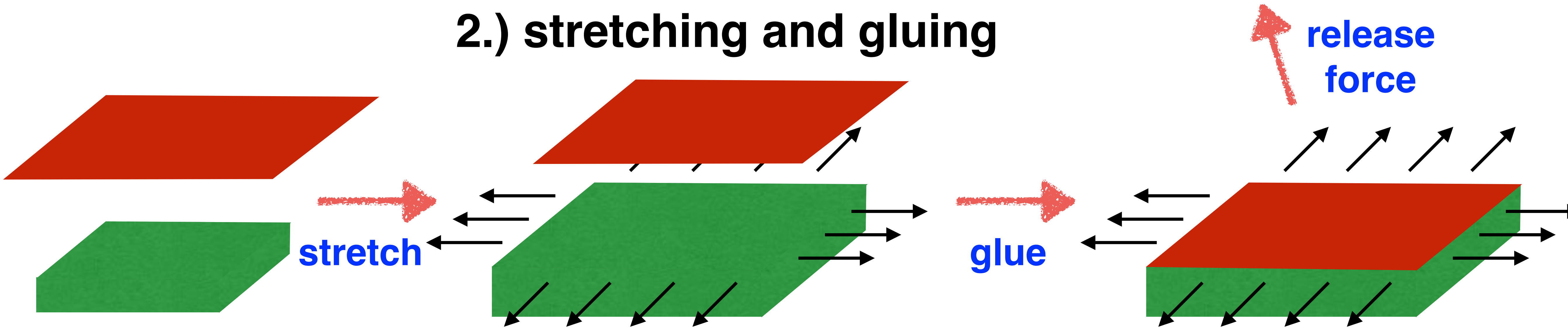
Experimental protocols

All protocols produce equivalent results for small strains!

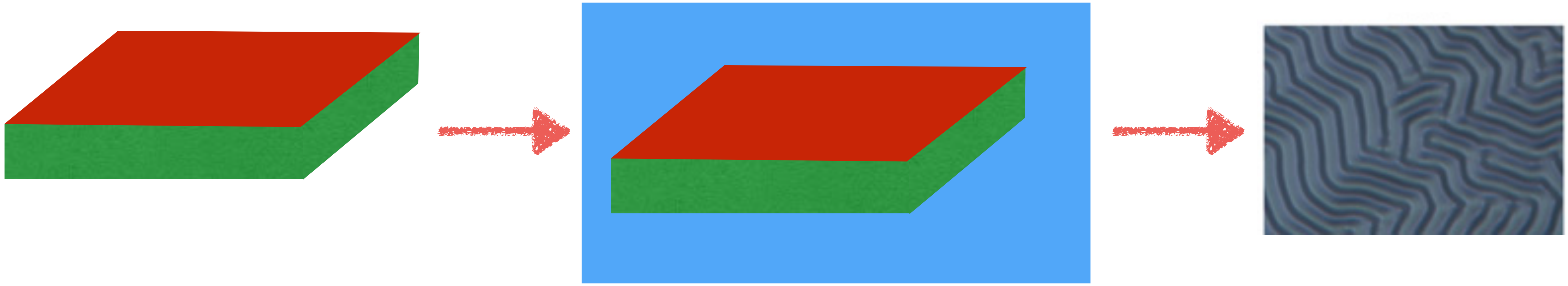
1.) compression



2.) stretching and gluing



3.) differential swelling of gels

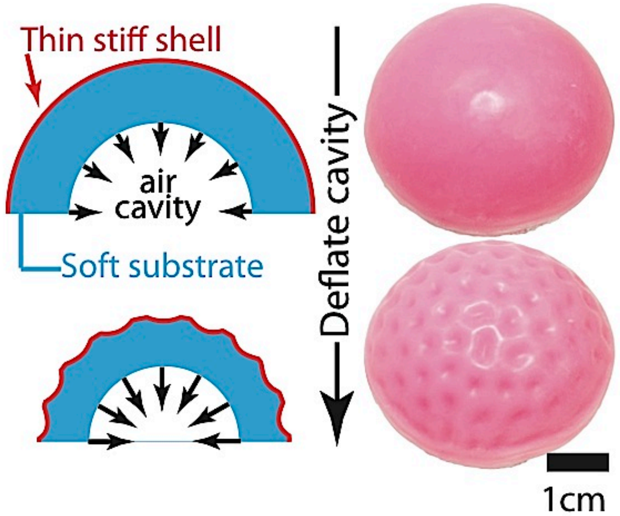


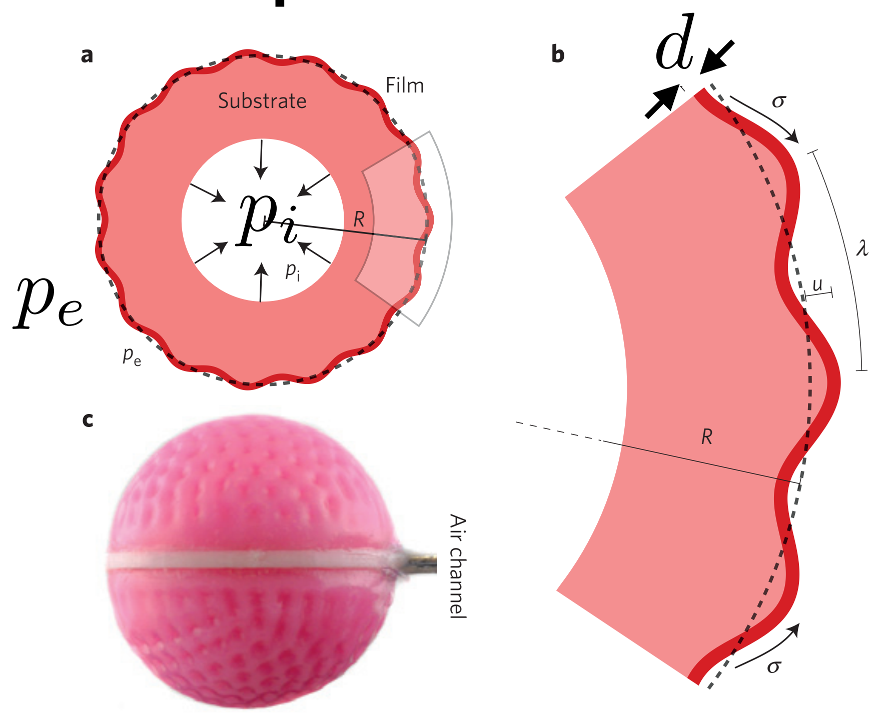
red gel swells more than the green gel

4.) differential growth in biology
5.) differential expansion due to temperature, electric field, etc.

Compression of stiff thin membranes on a spherical soft substrates

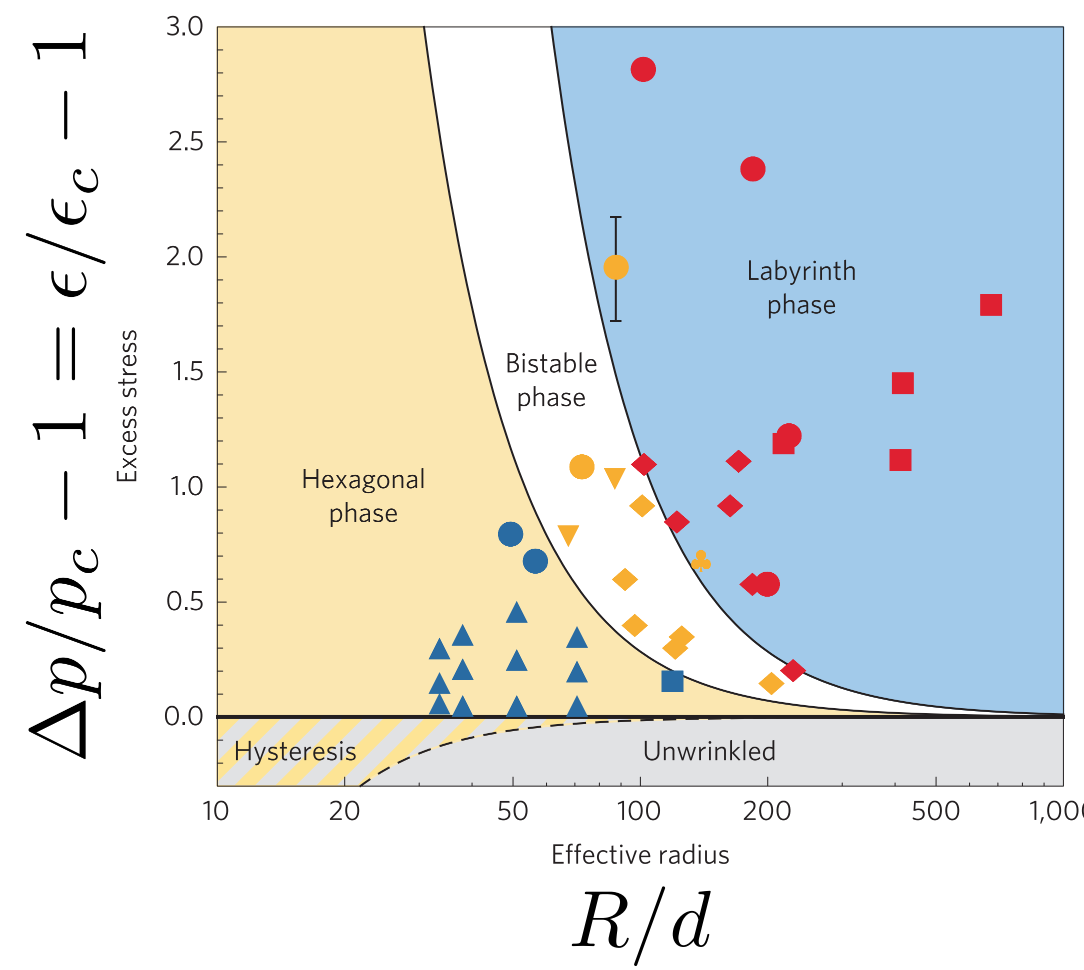
Spherical shells are compressed by reducing internal pressure



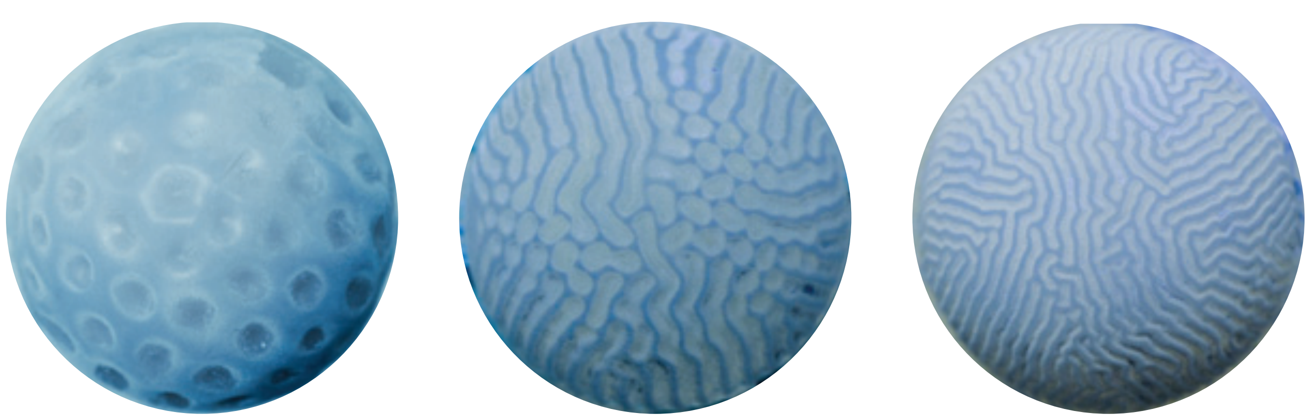


$R = 20\mathrm{mm}$

Phase diagram of dimples/wrinkles



hexagonal phase **bistable phase** **labyrinth phase**



R/d

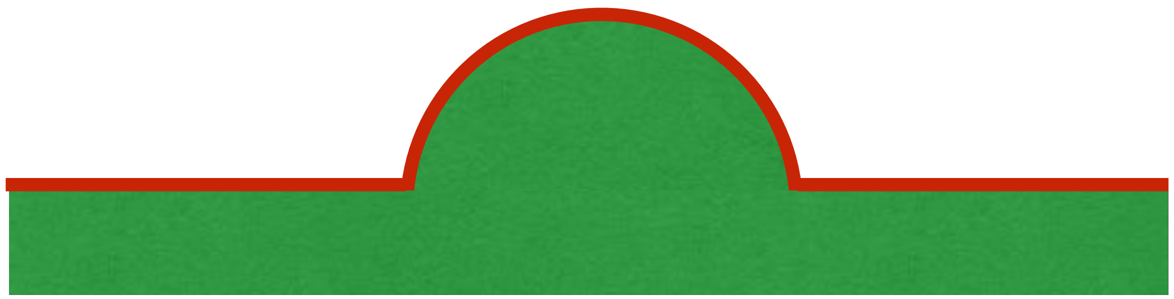
characteristic wavelength is almost independent of radius R

$$\lambda \sim d \left(\frac{E_m}{E_s} \right)^{1/3}$$

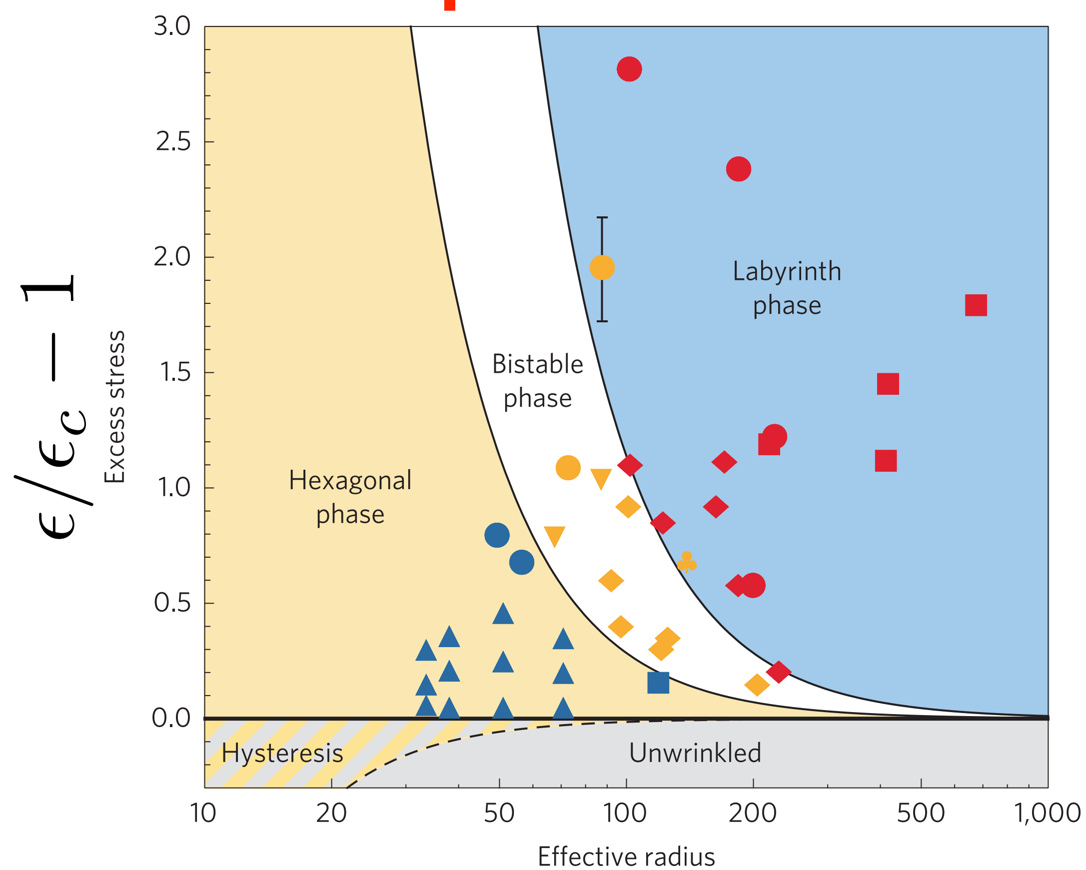
N. Stoop et al., Nat. Materials **14**, 337 (2015)

Compression of stiff thin membranes on a spherical soft substrates

Swelling of gels (red gel swells more than the green one)



Phase diagram of dimples/wrinkles

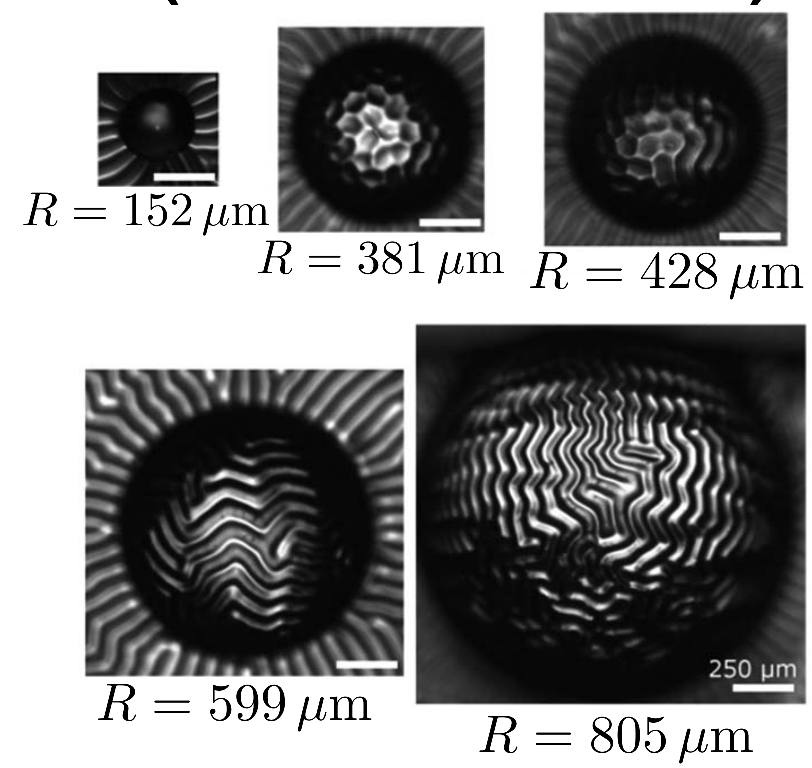


R/d

D. Breid and A.J. Crosby,

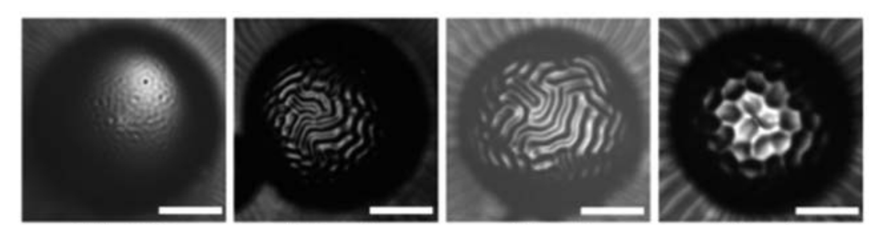
Soft Matter **9**, 3624 (2013)

Modifying radius *R* (fixed thickness *d*)



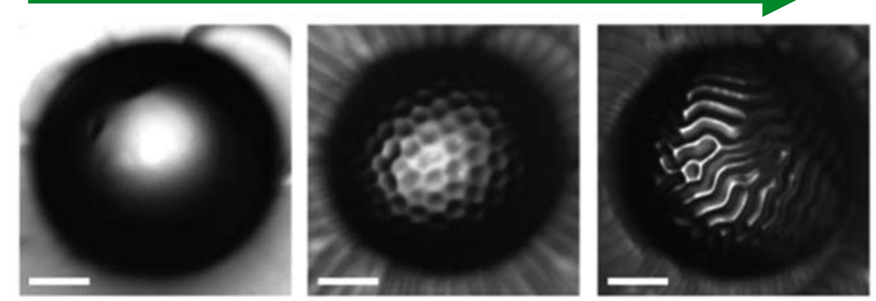
Modifying membrane thickness *d*

$R = 381\mu\mathrm{m}$

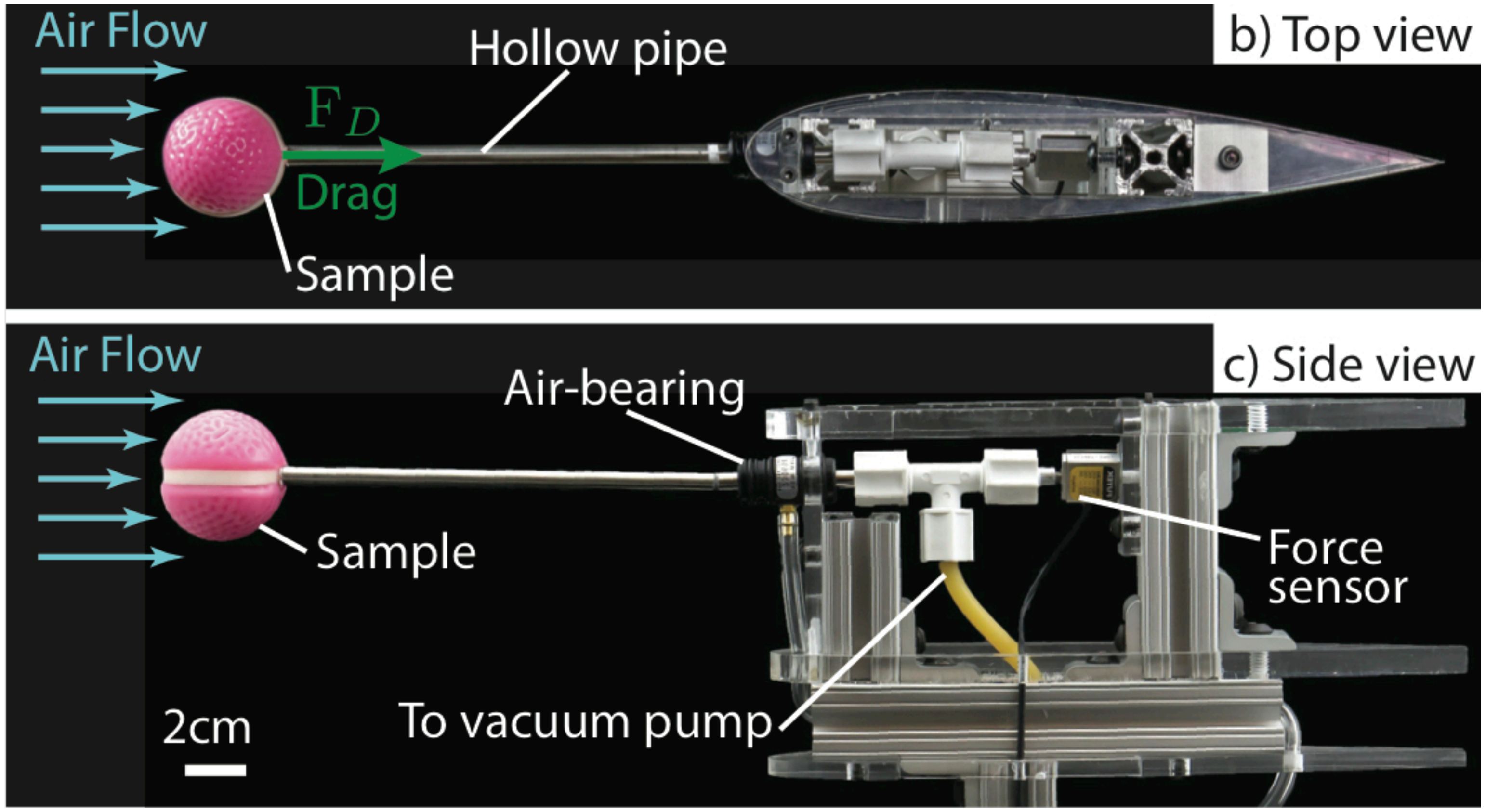


Modifying swelling strain ϵ

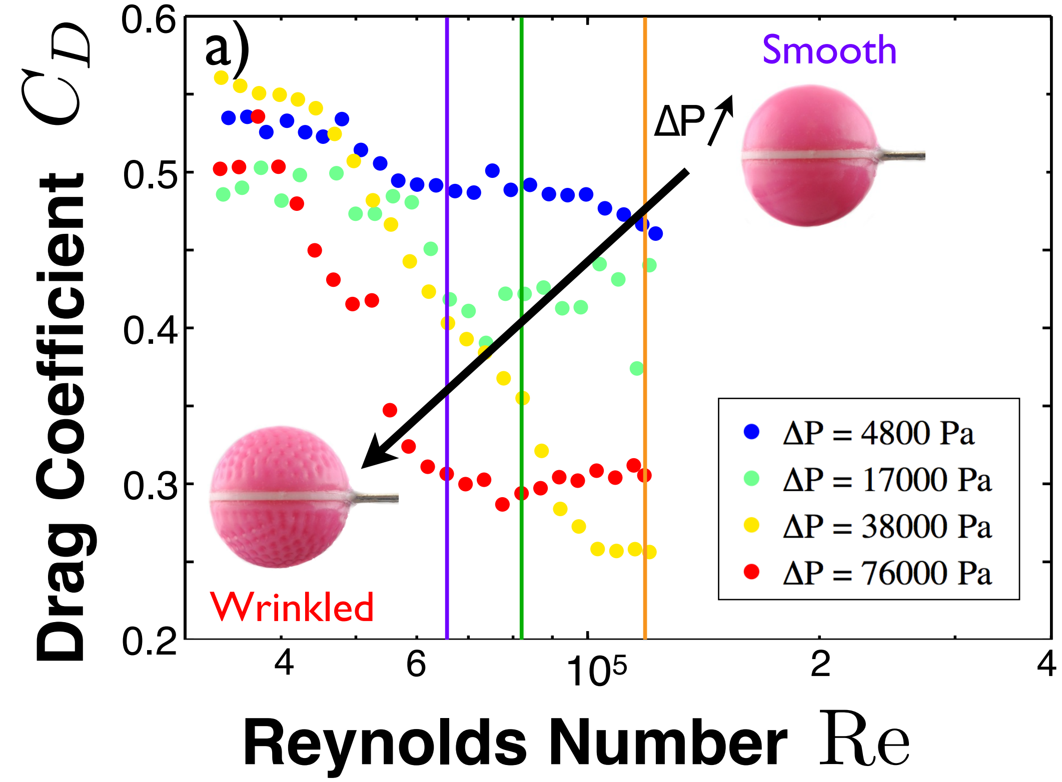
$R = 522\mu\mathrm{m}$



Tuning drag coefficient via wrinkling



Wrinkling reduces drag coefficient



Drag Force

$$F_d = \frac{1}{2} C_D \rho u^2 A$$

ρ **air density**

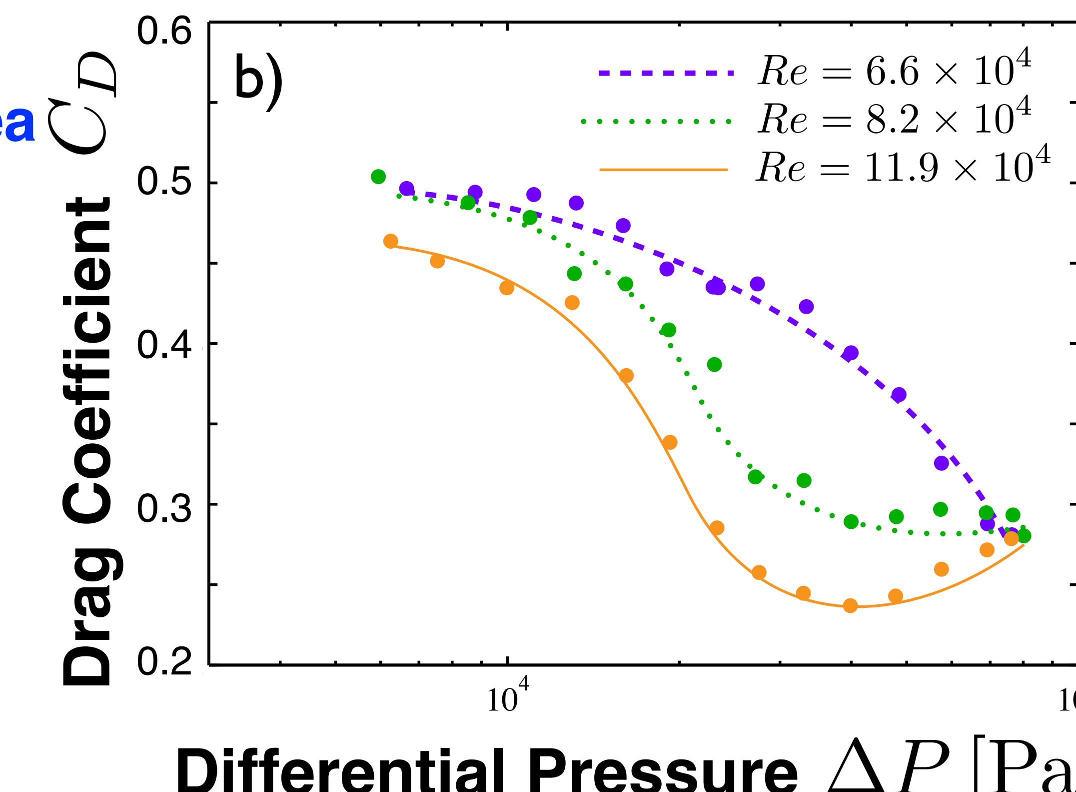
u **air flow speed**

R **sphere radius**

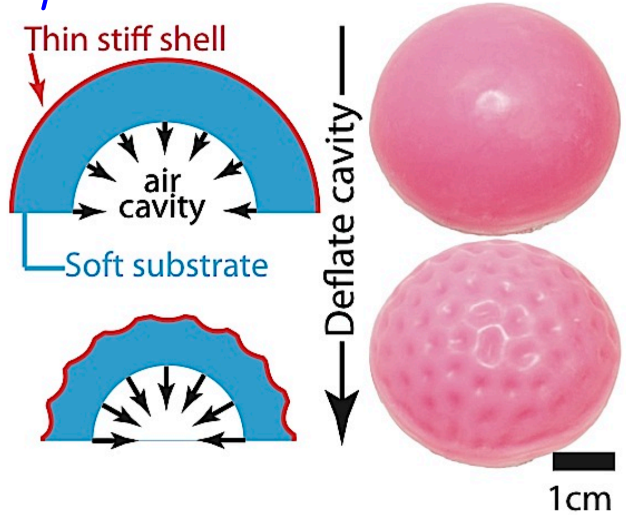
$A = \pi R^2$ **sphere cross-section area**

μ **air viscosity**

$$\mathrm{Re} = \frac{\rho u (2R)}{\mu} \gg 1$$ **Reynolds Number**



Depth of wrinkling is controlled via the reduction of internal pressure ΔP.

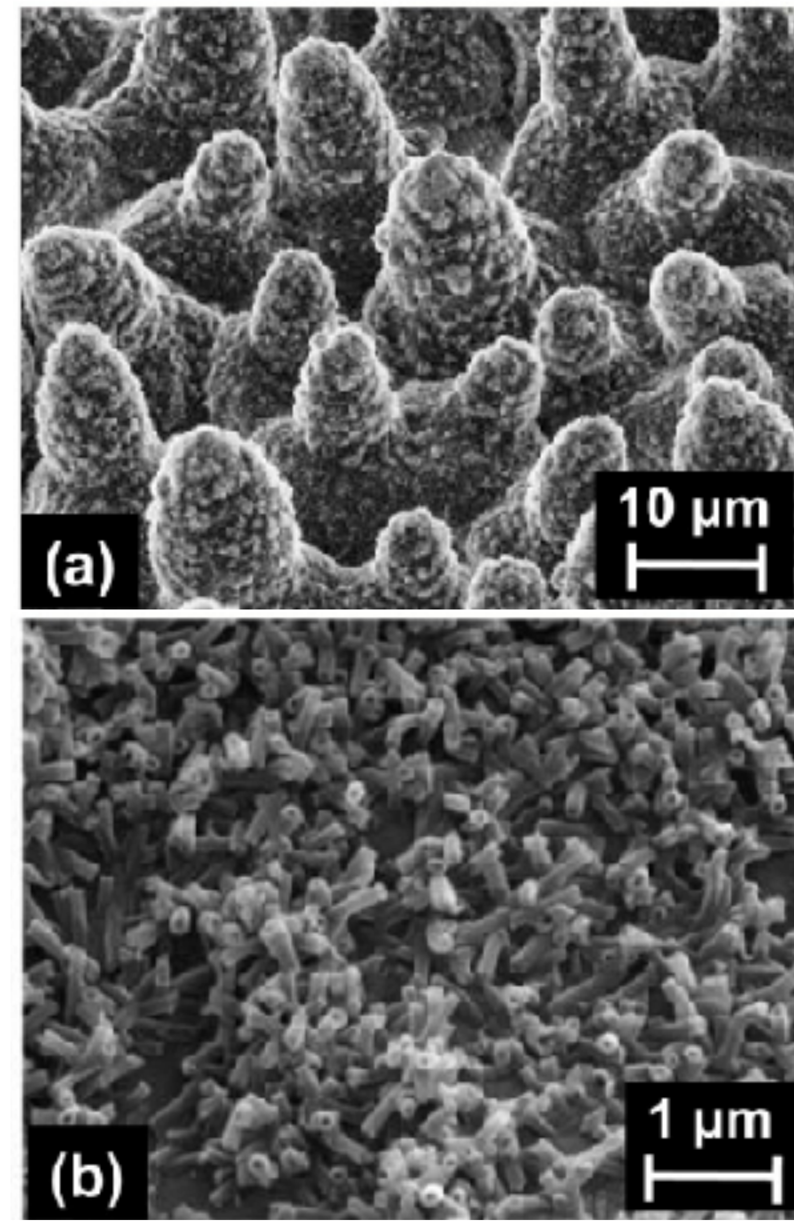


7

D. Terwagne et al., Adv. Mater. **26**, 6608 (2014)

Self-cleaning property of lotus leaves

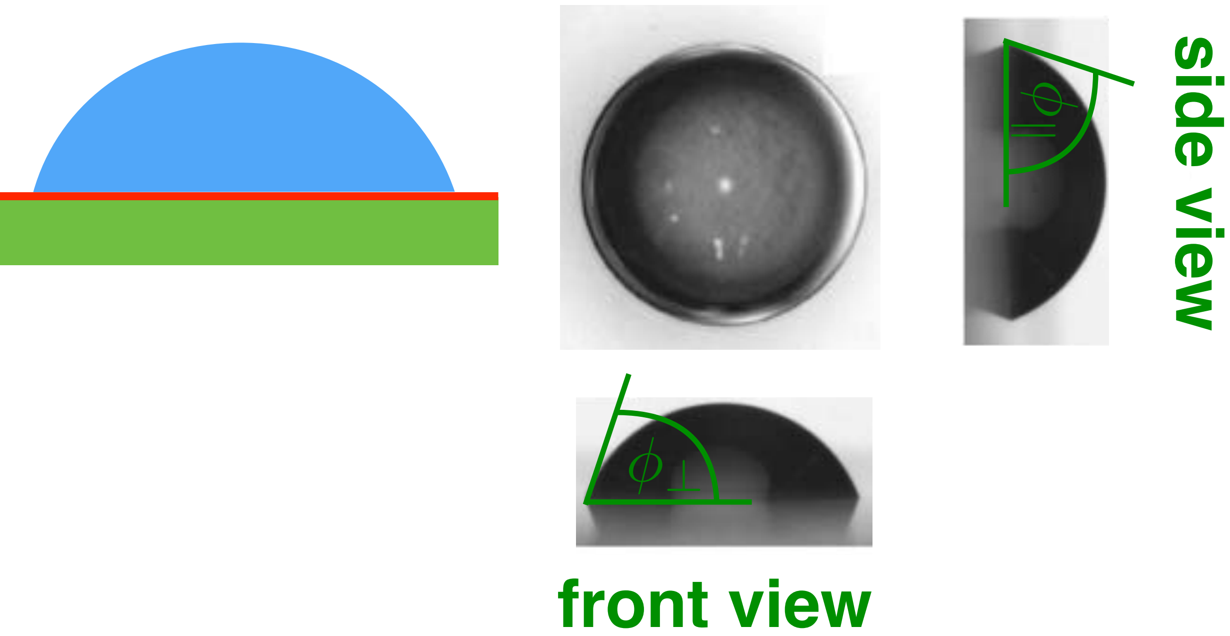
**Lotus leaves repel water (hydrophobicity)
due to the rough periodic microstructure**



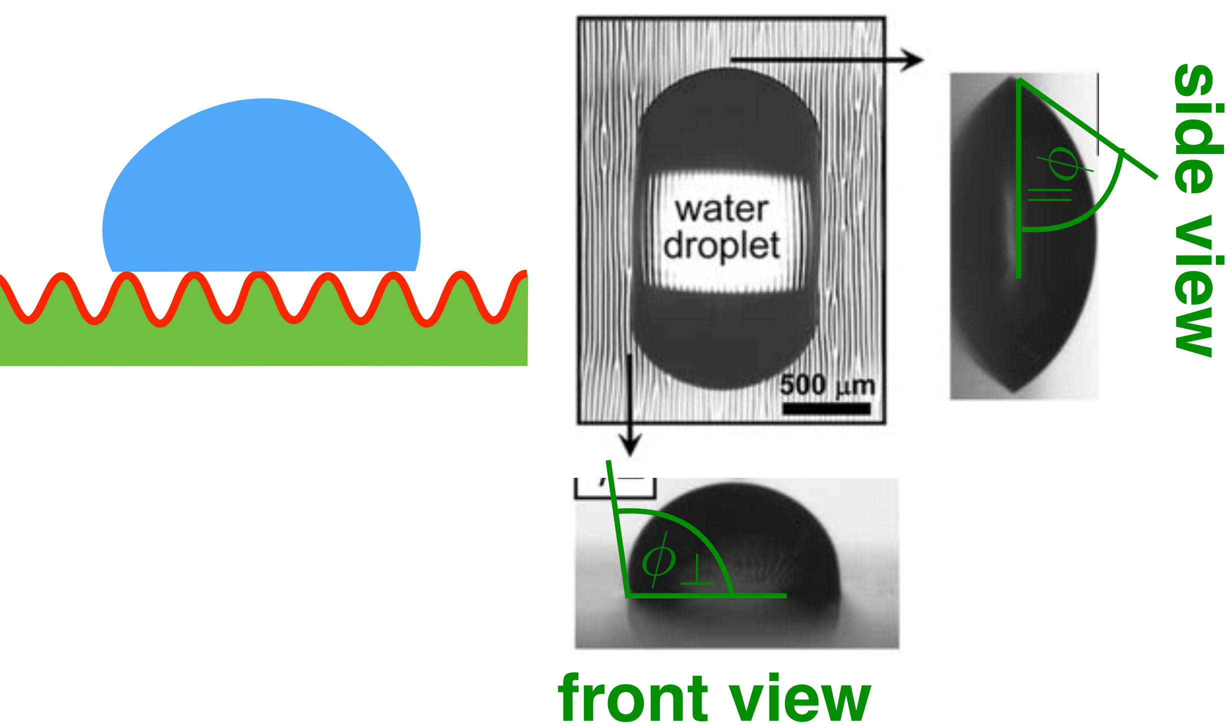
M. N. Costa et al.,
Nanotechnology **25**, 094006 (2014)

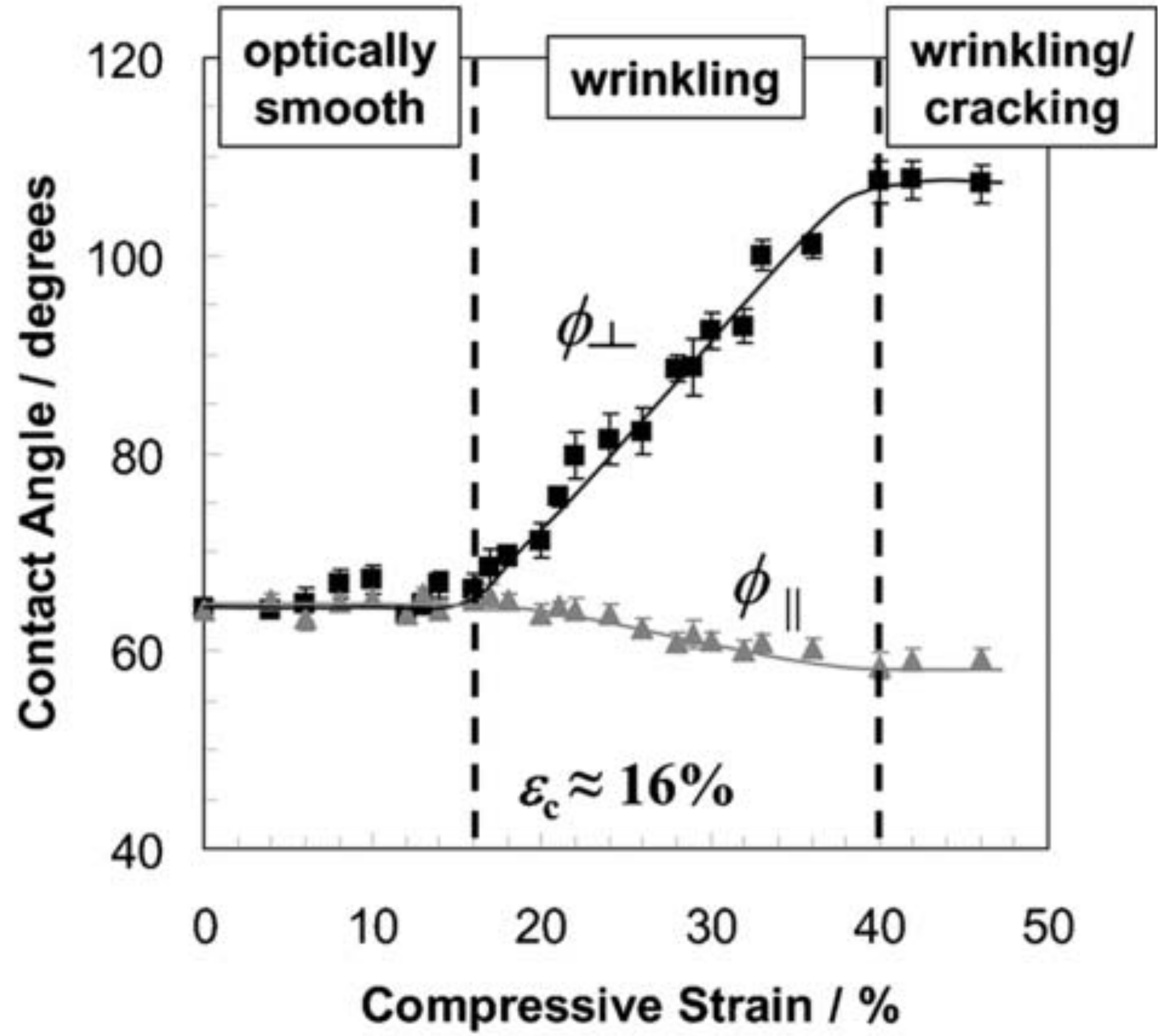
Tuning wetting angle via wrinkling

Water droplet on a flat surface



Water droplet on a wrinkled surface (wrinkling increases contact angle)

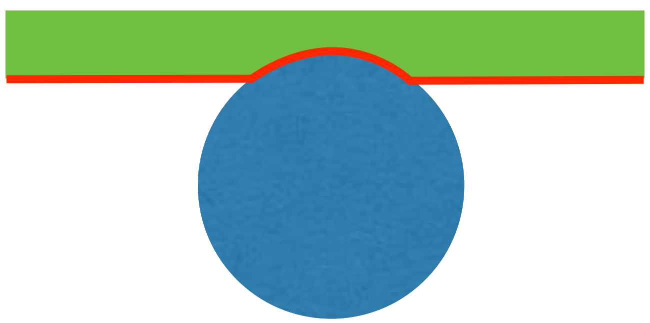




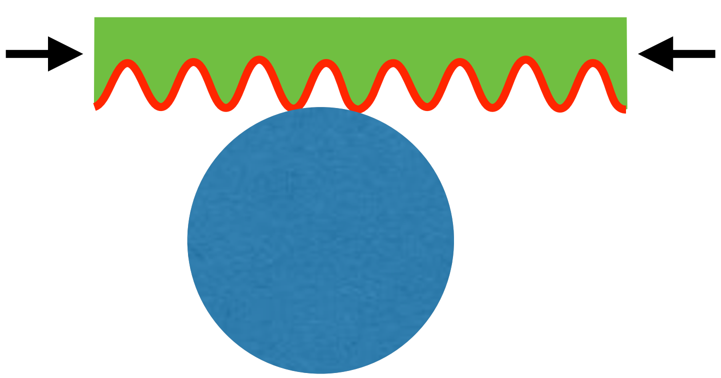
J. Y. Chung et al., Soft Matter **3**, 1163 (2007)

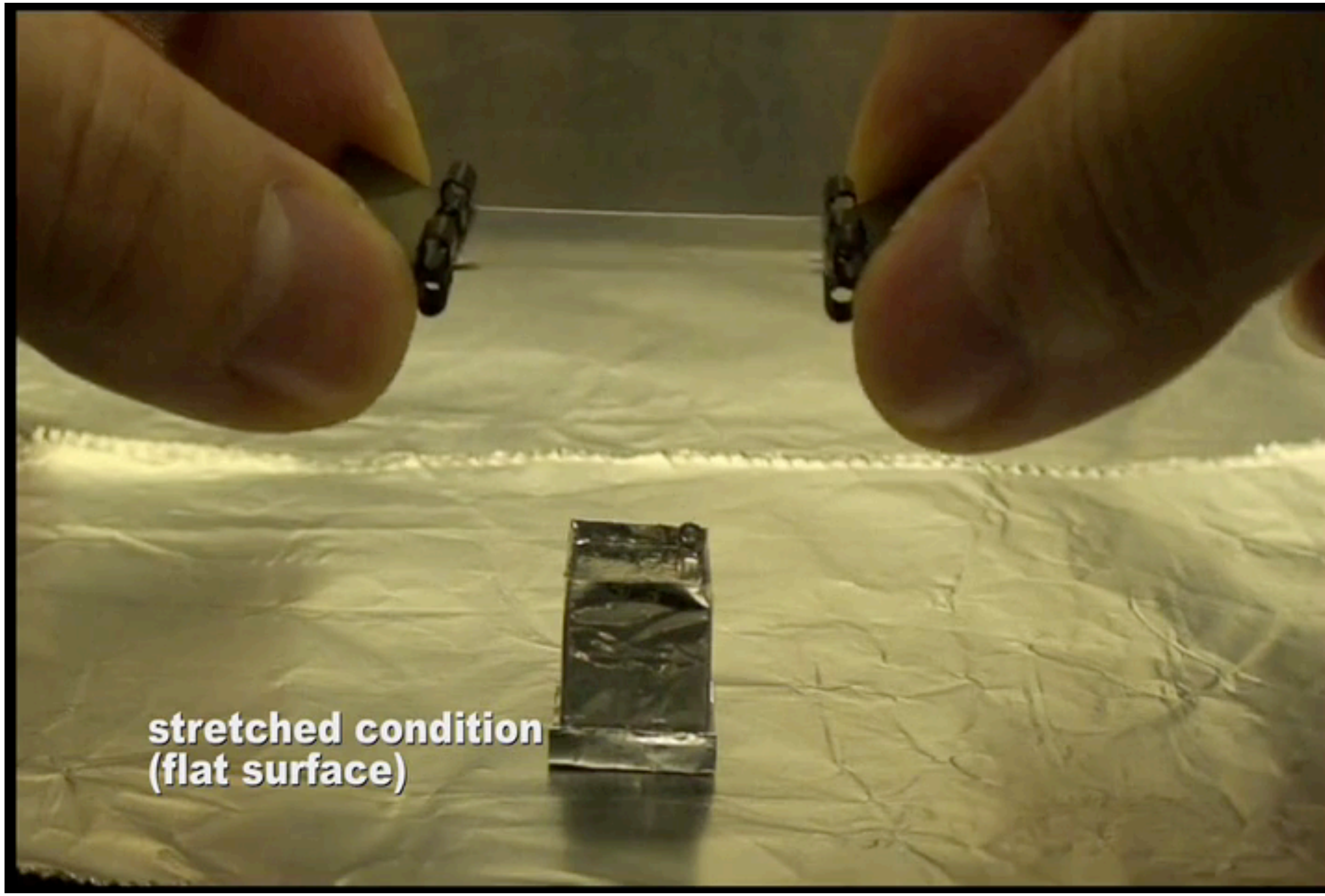
Tuning adhesion via wrinkling

Flat complaint surface has enhanced adhesion (larger contact area)



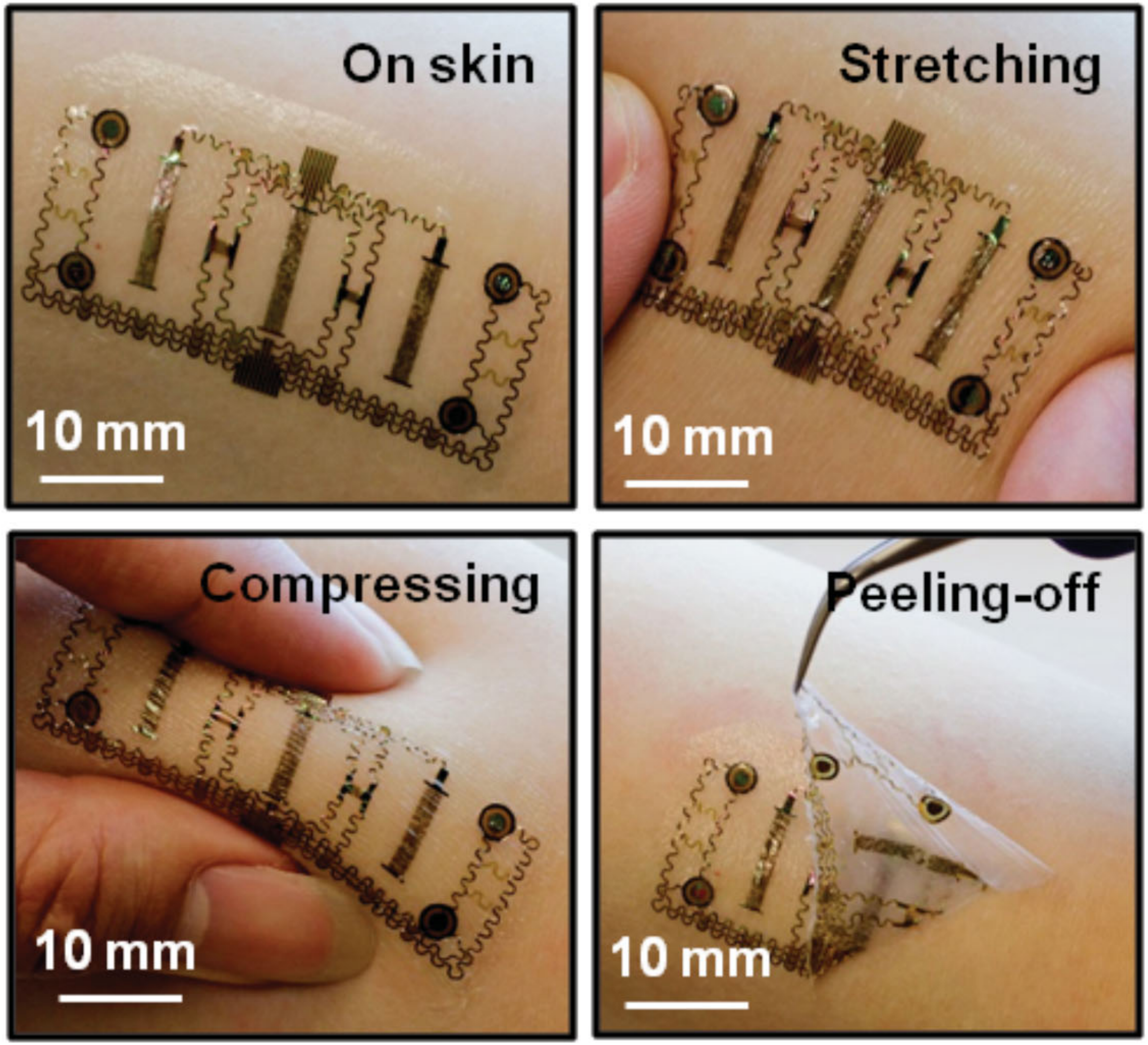
Wrinkling reduces adhesion (smaller contact area)





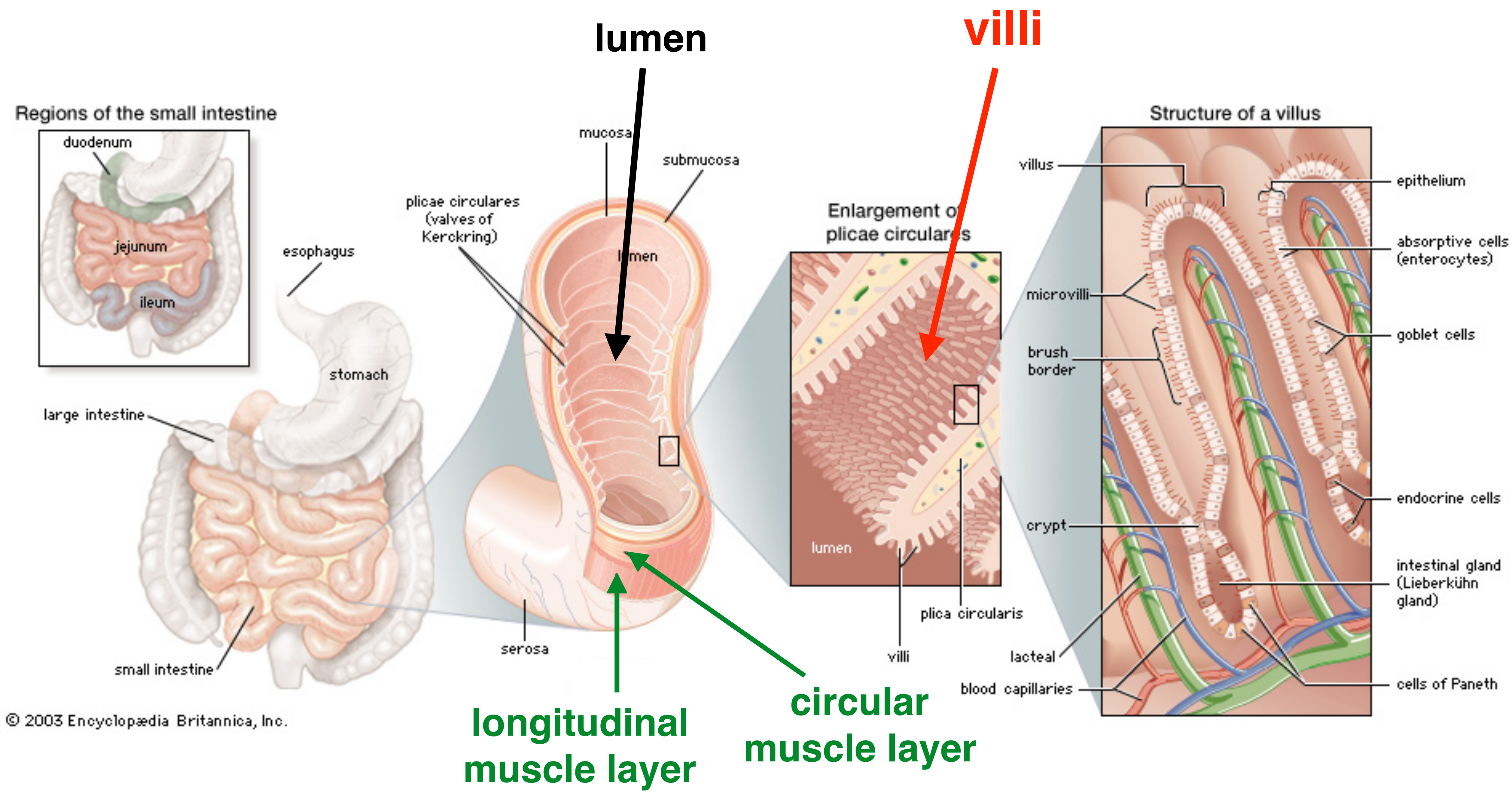
P.-C. Lin et al., Soft Matter **4**, 1830 (2008)

Wrinkled structures can be used for flexible electronics



B. Xu et al., Adv. Mater. **28**, 4462 (2016)

How are villi formed in guts?



Villi increase internal surface area of intestine for faster absorption of digested nutrients.

Lumen patterns in chick embryo

DAPI marks cell nuclei

aSMA marks smooth muscle actin

E…: age of chick embryo in days

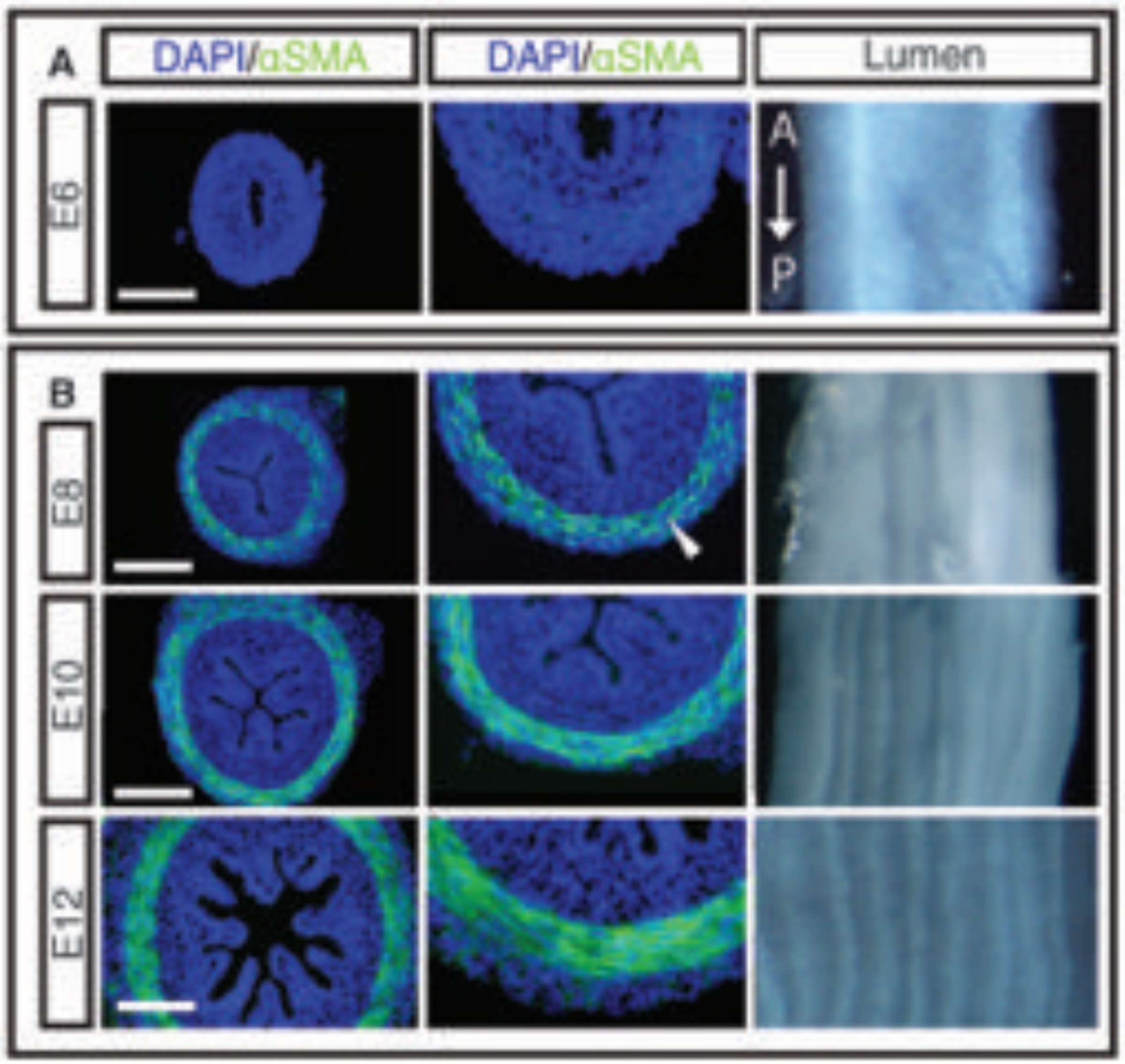
Stiff muscles grow slower than softer mesenchyme and endoderm layers

$E6$

$E8$

$E10$

$E12$

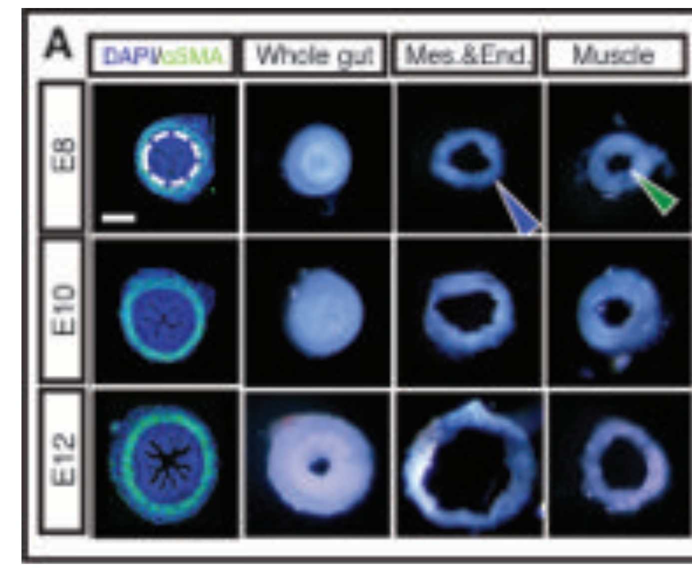


$100\mu m$

$E8$

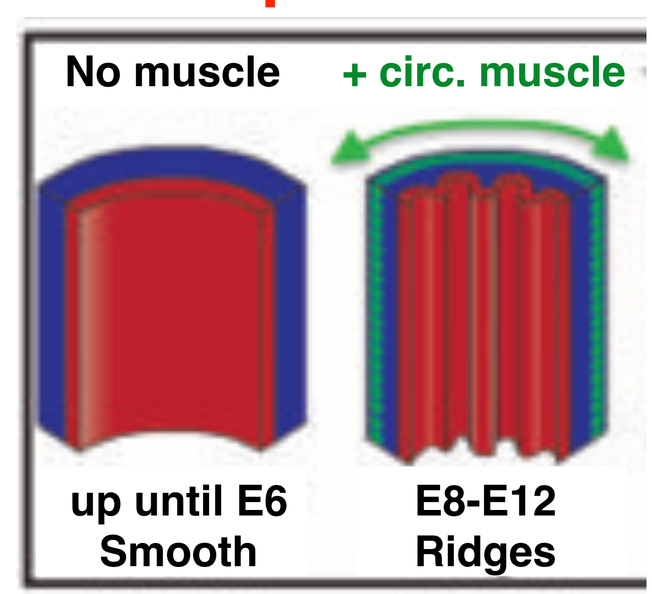
$E10$

$E12$



radial compression due to differential growth produces striped wrinkles

endoderm
mesenchyme
muscle



A. Shyer et al., Science **342**, 212 (2013)

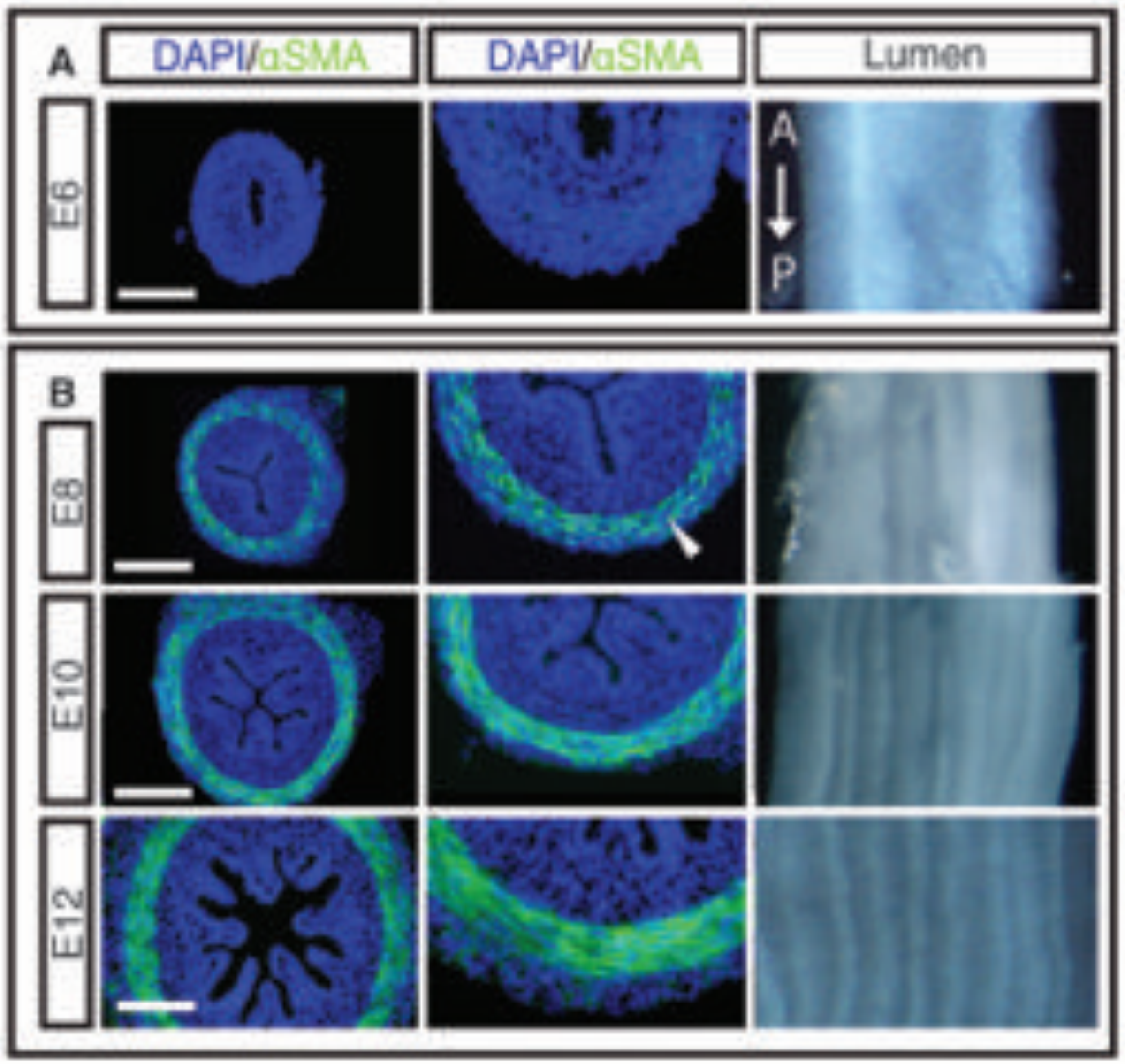
Lumen patterns in chick embryo

$E6$

$E8$

$E10$

$E12$



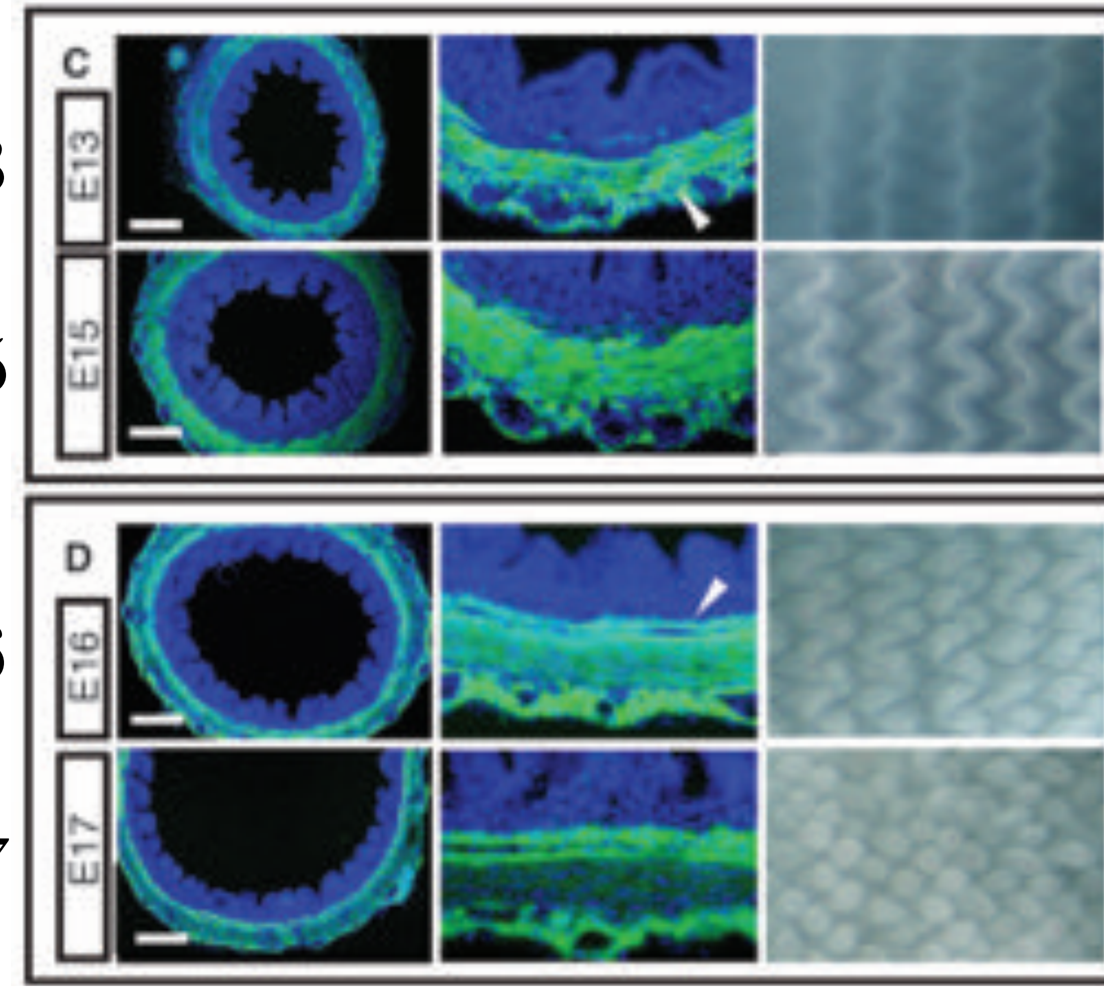
Formation of longitudinal muscles at E13 produces longitudinal compression

$E13$

$E15$

$E16$

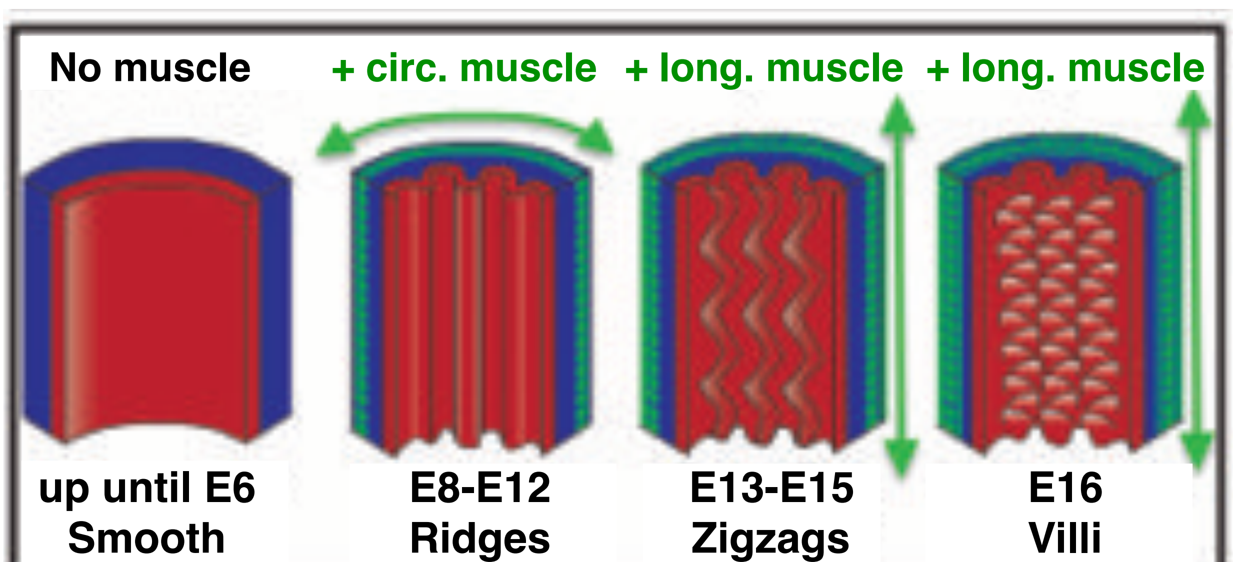
$E17$



endoderm

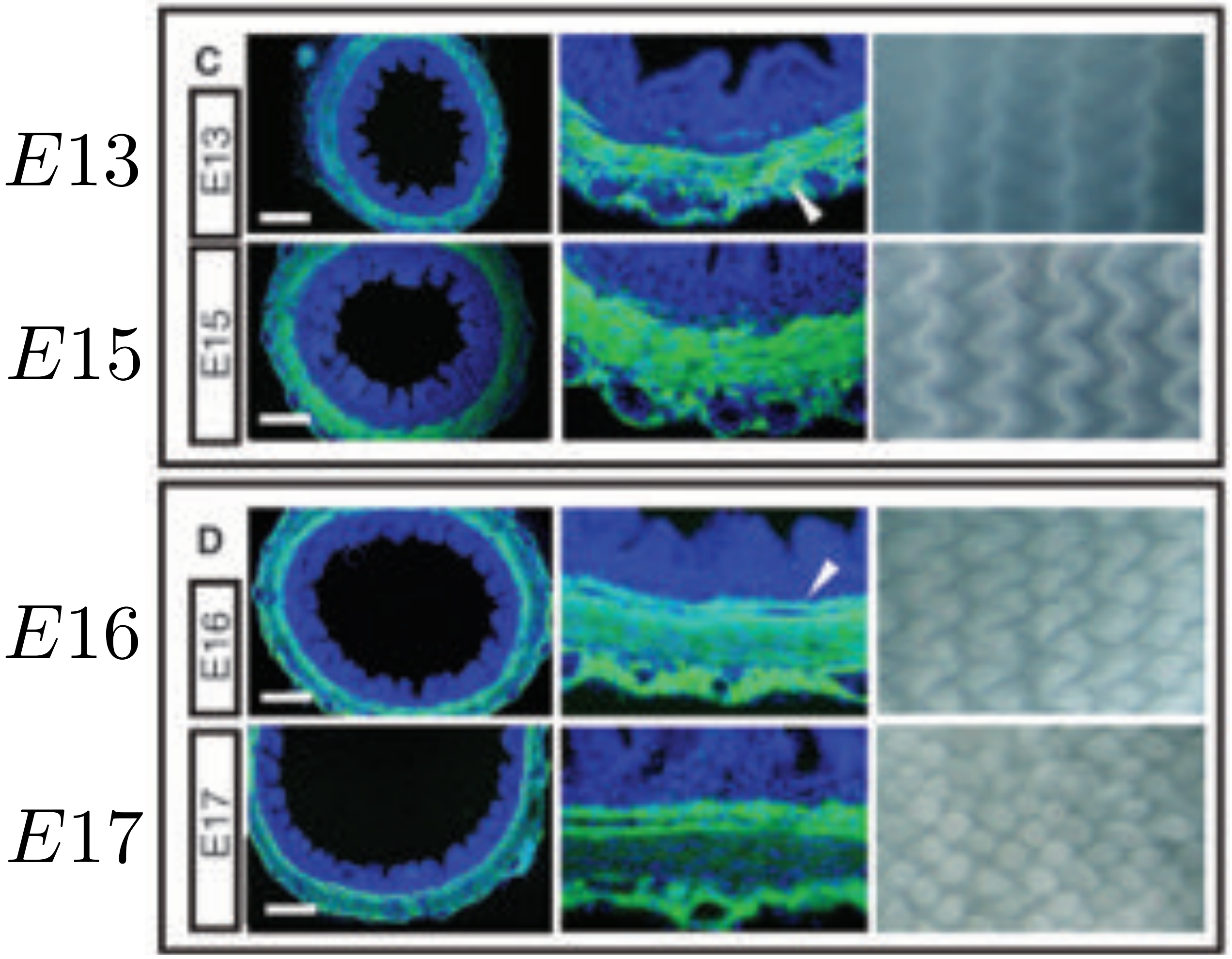
mesenchyme

muscle

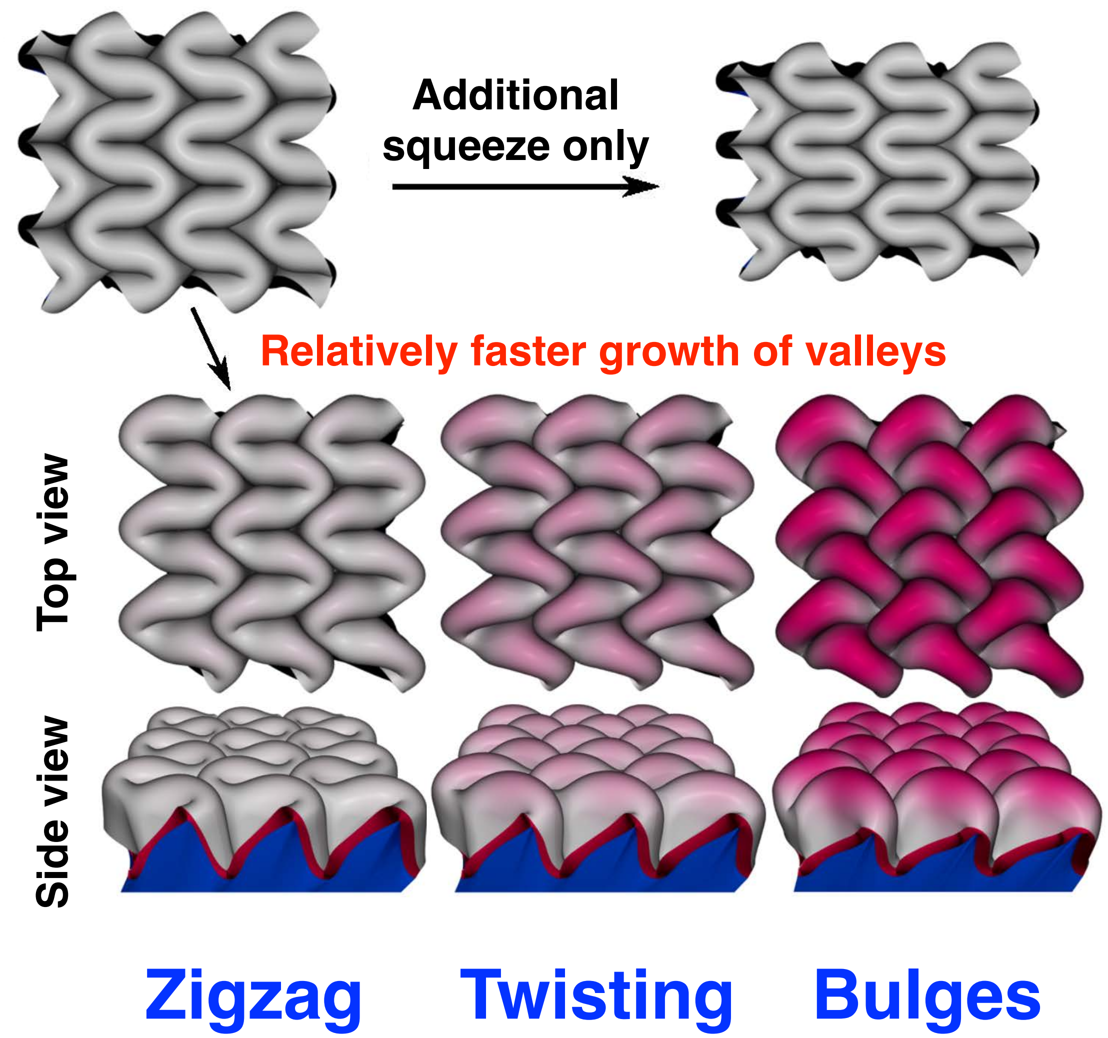


A. Shyer et al., Science **342**, 212 (2013)

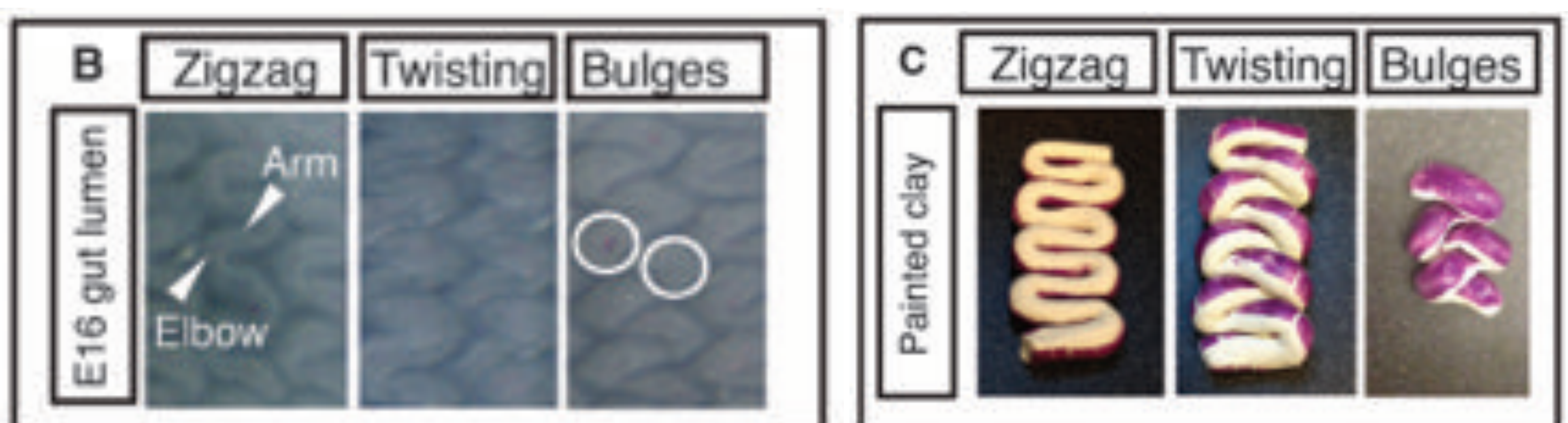
Lumen patterns in chick embryo



Villi start forming at E16 because of the faster growth in valleys

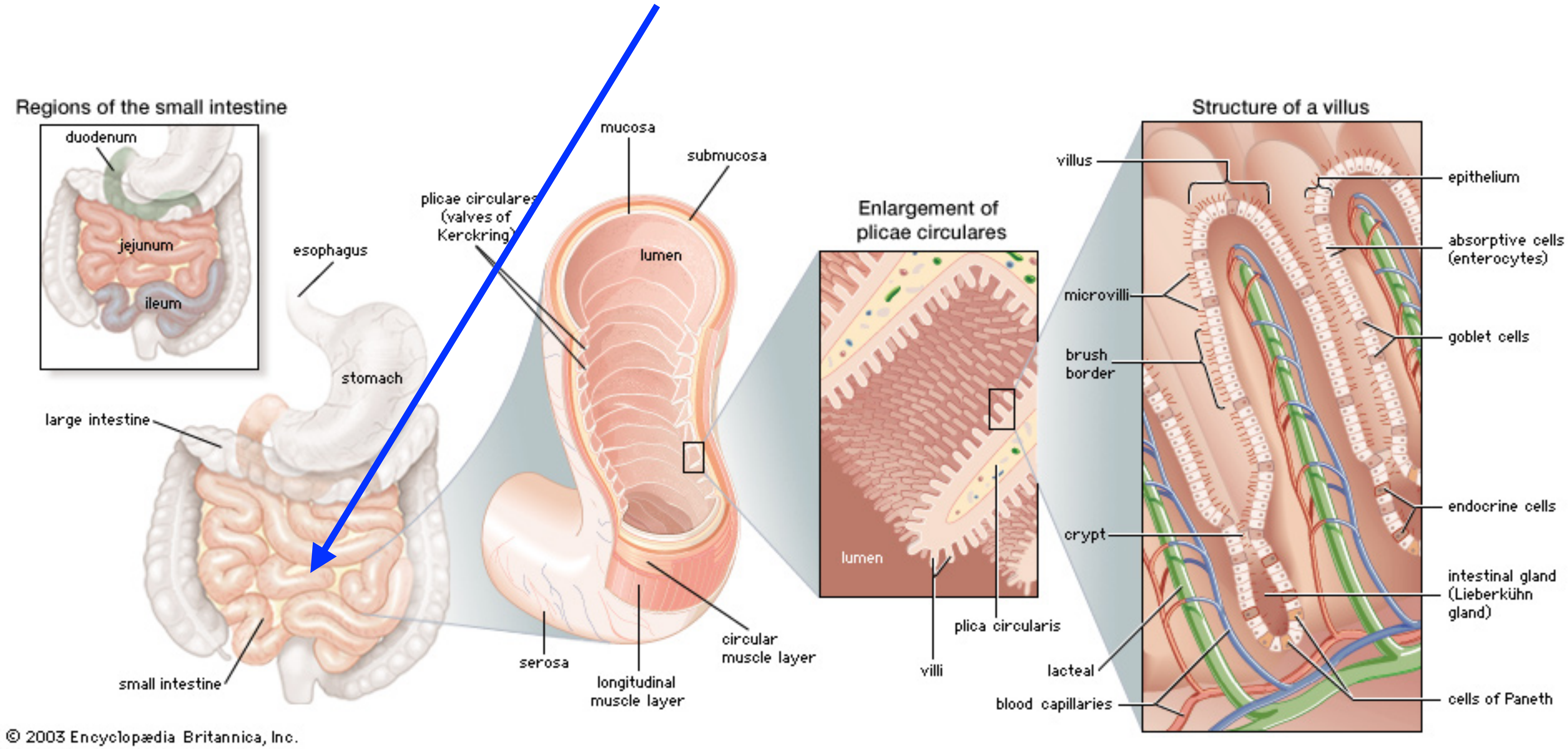


The same mechanism for villi formation also works in other organisms!



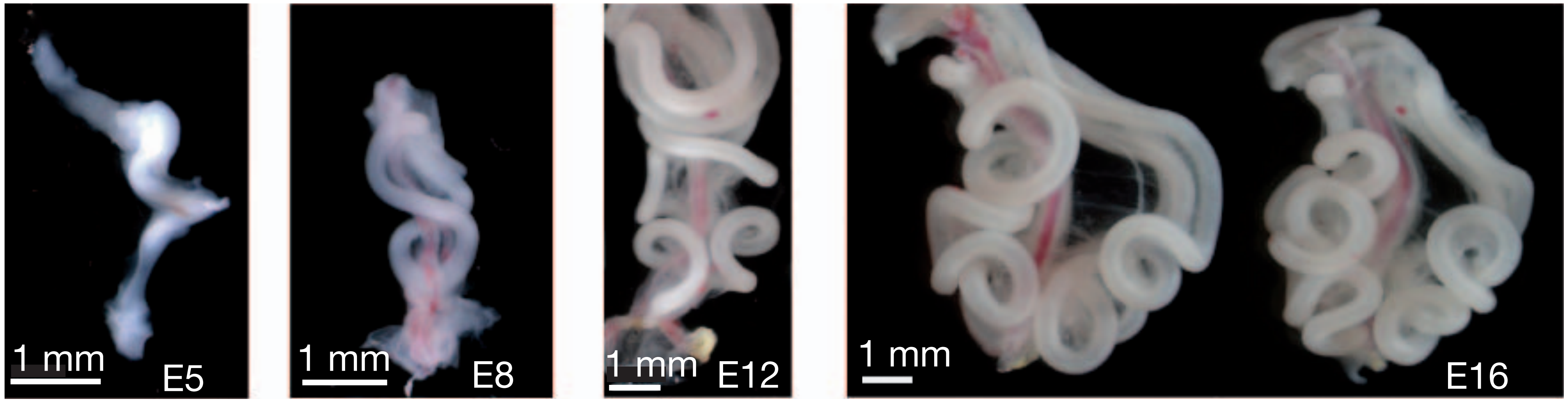
A. Shyer et al., Science **342**, 212 (2013)

Why are guts shaped like that?



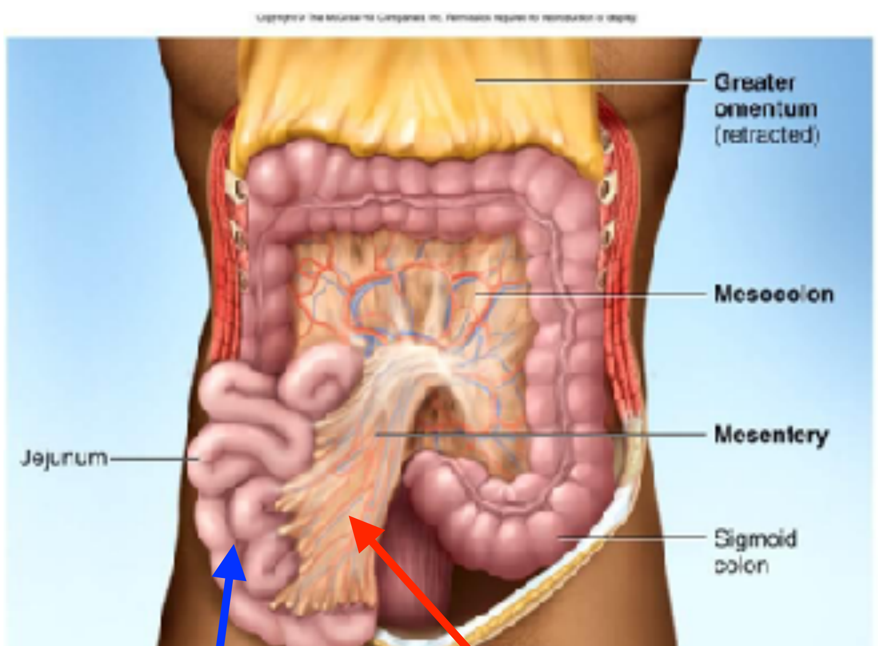
Guts in chick embryo

Surgically removed guts from chick embryo

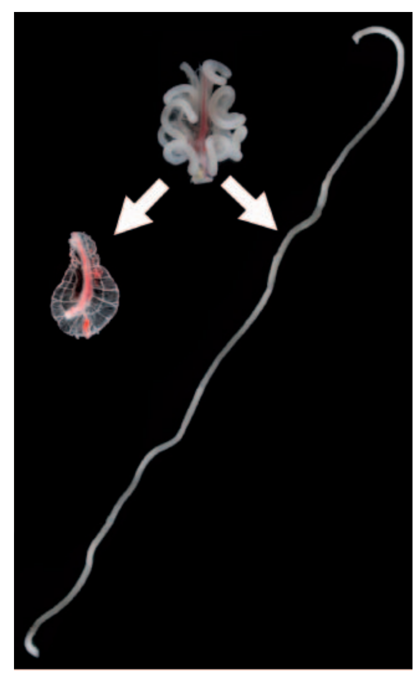


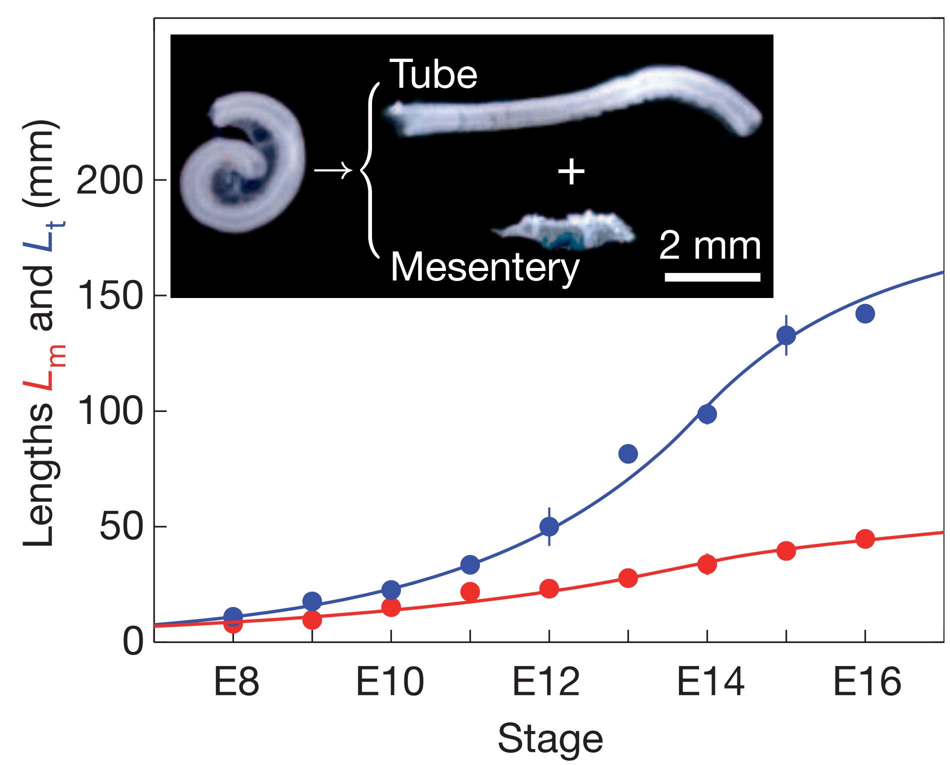
Tube straightens after separation from mesentery

Tube grows faster than mesentery sheet!



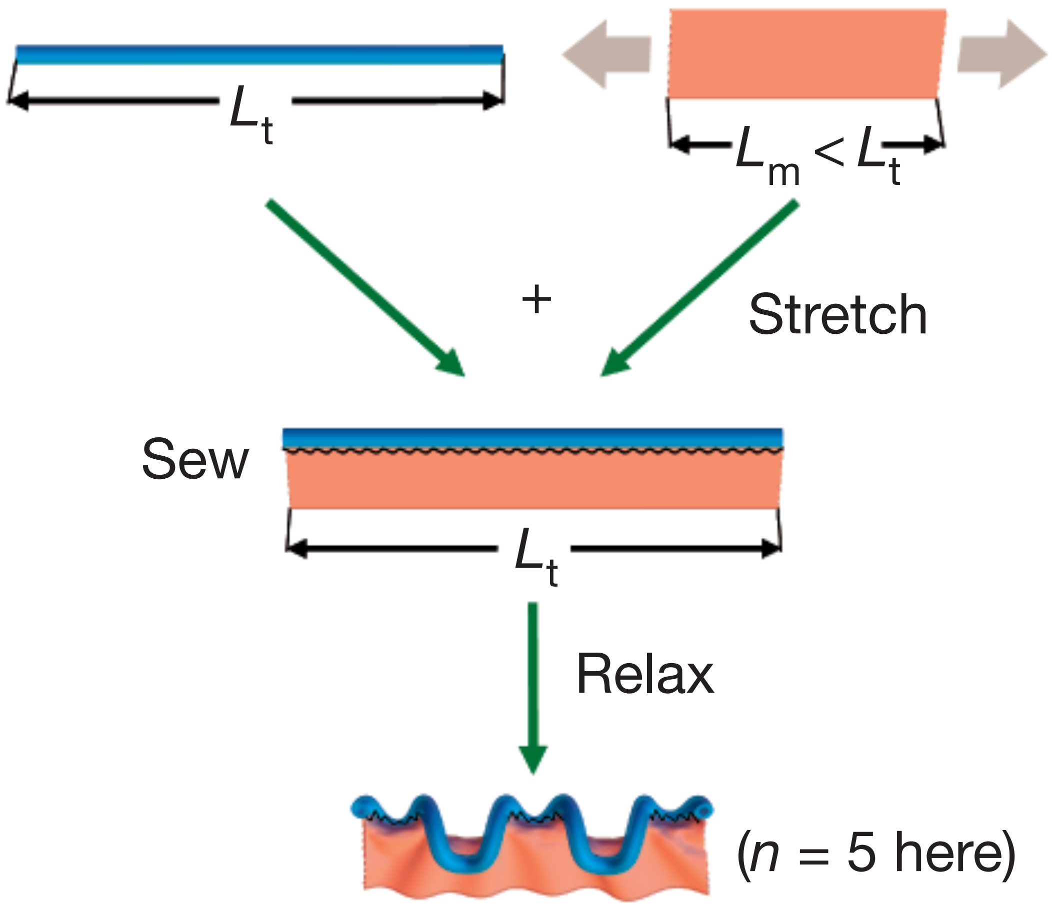
tube **mesentery**



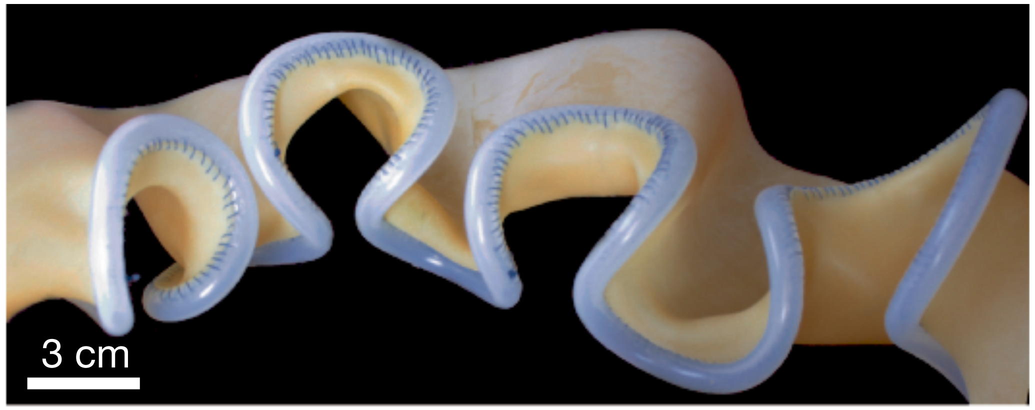


T. Savin et al., Nature **476**, 57 (2011)

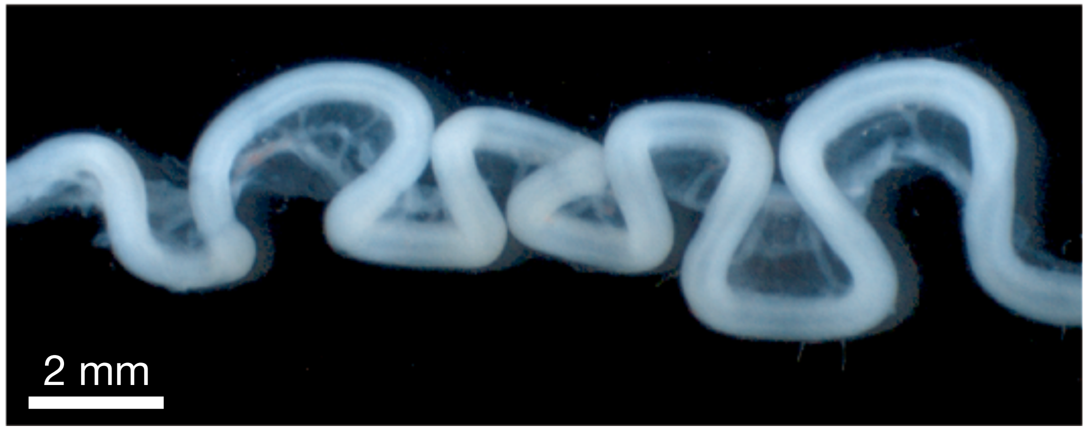
Synthetic analog of guts



Rubber model of guts



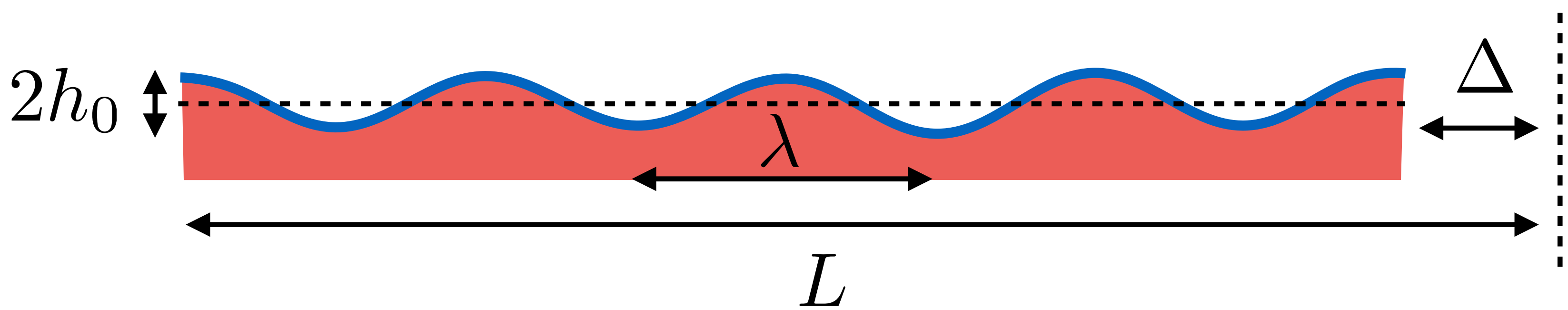
Chick guts at E12

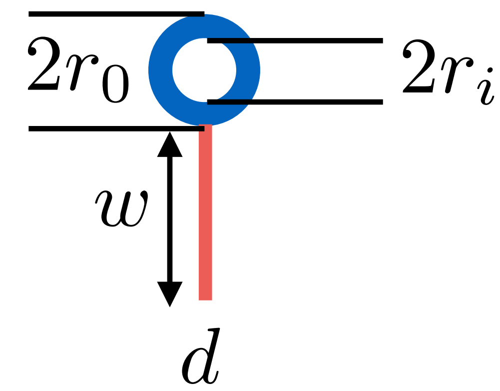


What is the wavelength of this oscillations?

T. Savin et al., Nature **476**, 57 (2011)

Compression of stiff tube on soft elastic mesentery sheet





assumed profile $h(s) = h_0 \cos(2\pi s/\lambda)$

amplitude of wrinkles

$$h_0 = \frac{\lambda}{\pi}\sqrt{\frac{\Delta}{L}} = \frac{\lambda\sqrt{\epsilon}}{\pi}$$

deformation of the soft mesentery decays exponentially away from the surface

$$h(s, y) \approx h_0 \cos(2\pi s/\lambda) e^{-2\pi y/\lambda}$$

bending energy of stiff tube

$$U_b \sim L \times \kappa_t \times \frac{1}{R^2} \sim L \times E_t I_t \times \frac{h_0^2}{\lambda^4} \sim \frac{L E_t I_t \epsilon}{\lambda^2}$$

deformation energy of soft mesentery

$$U_m \sim A \times E_m d \times \epsilon_m^2 \sim L\lambda \times E_m d \times \frac{h_0^2}{\lambda^2} \sim L E_m d \lambda \epsilon$$

minimize total energy (U_b+U_m) with respect to λ →

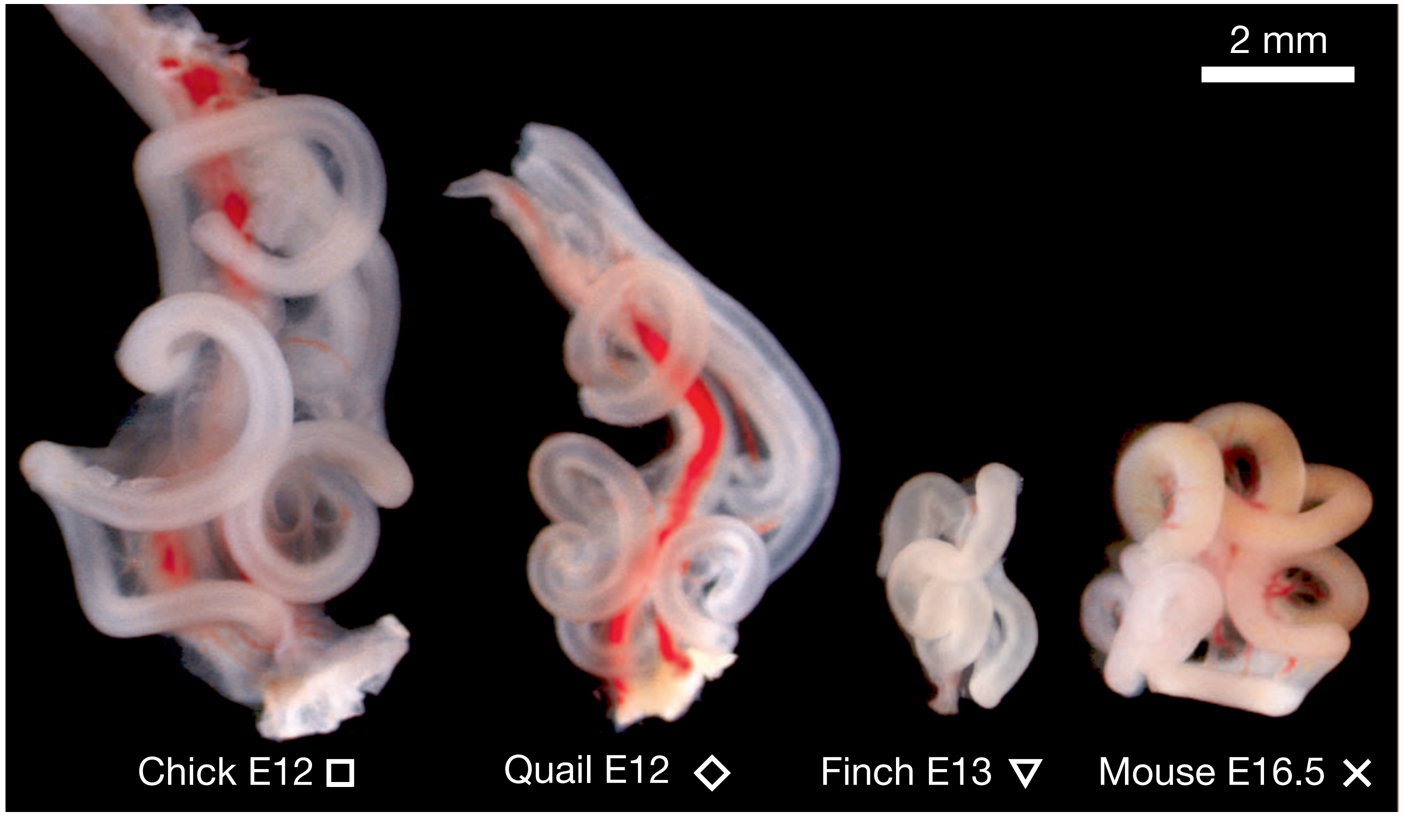
$$\lambda \sim \left(\frac{E_t I_t}{E_m d}\right)^{1/3}$$

bending stiffness of tube

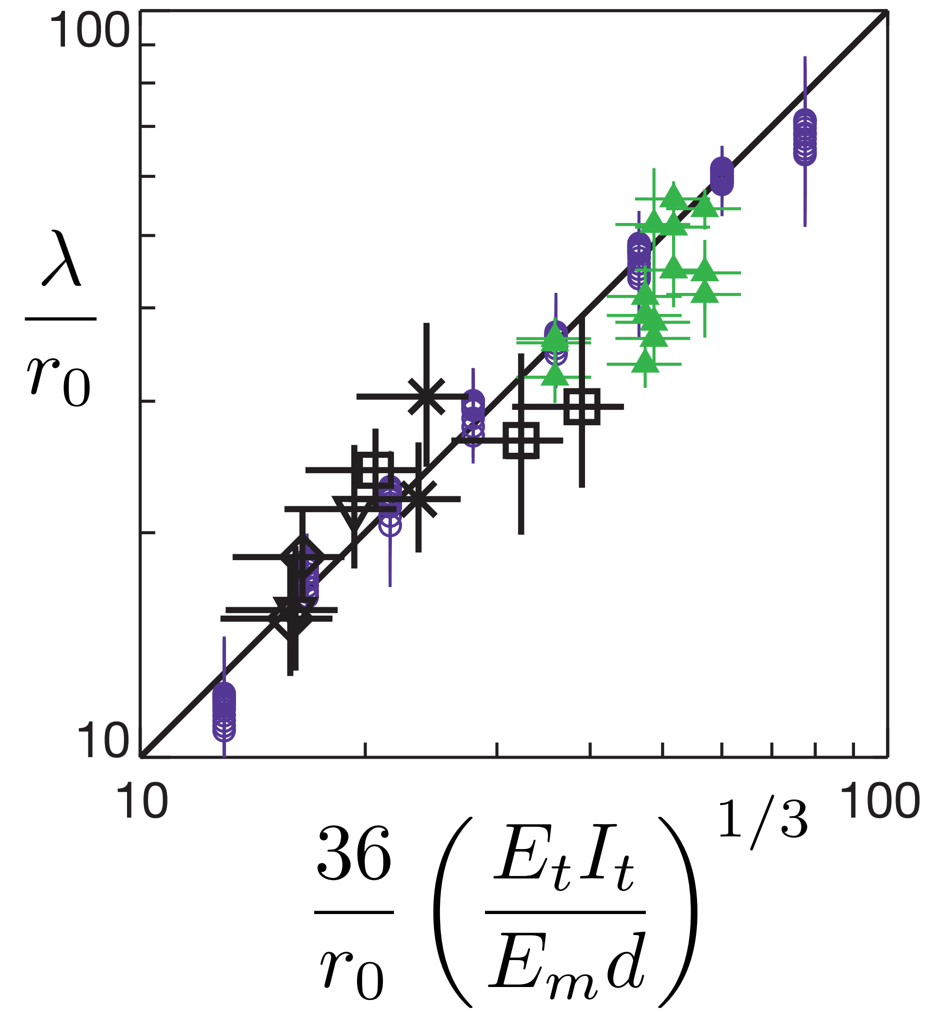
$$\kappa_t = E_t I_t$$

$$\kappa_t \propto E_t (r_0^4 - r_i^4)$$

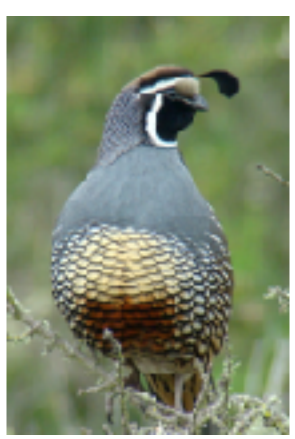
Wavelength of oscillations in guts



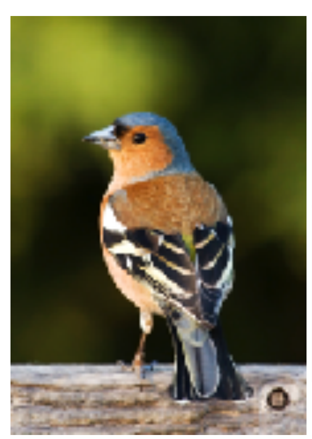
animal data, rubber model, computer simulations



chick

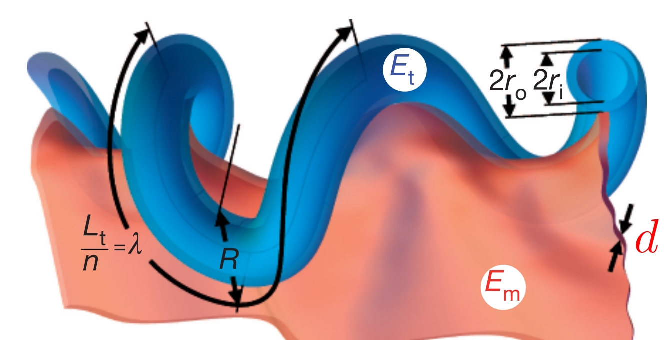


quail



finch

mouse

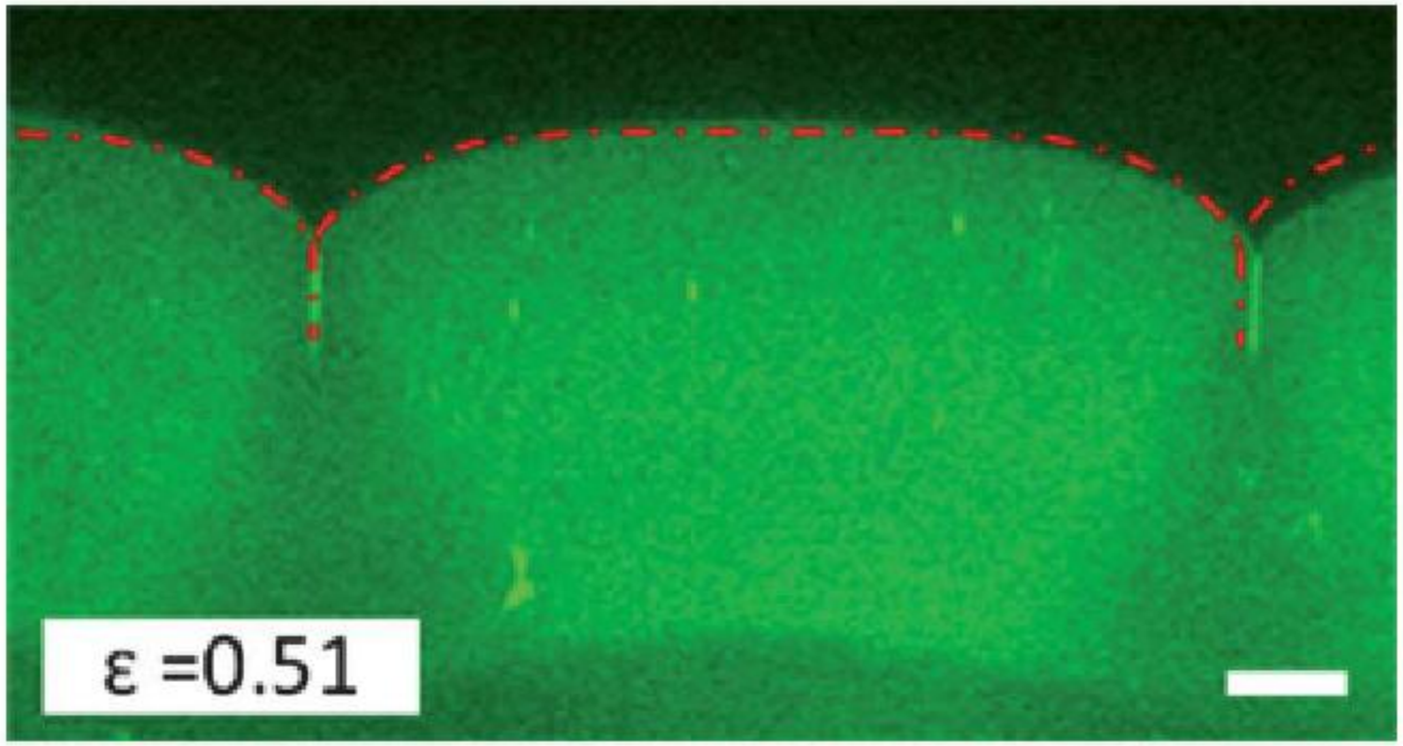


T. Savin et al., Nature **476**, 57 (2011)

Compression of soft elastic material

When soft elastic material is compressed by more than 35% surface forms sharp creases. This is effect of nonlinear elasticity!

swollen gel
on a stiff substrate

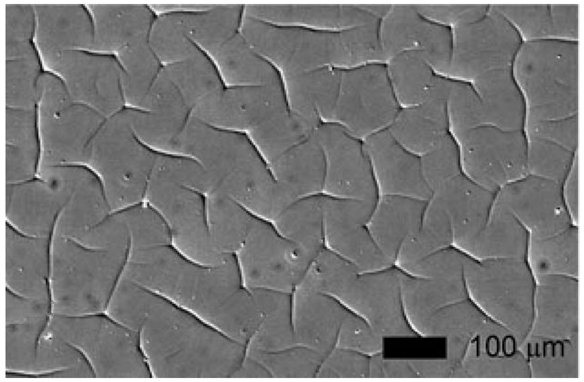


arm of
an infant

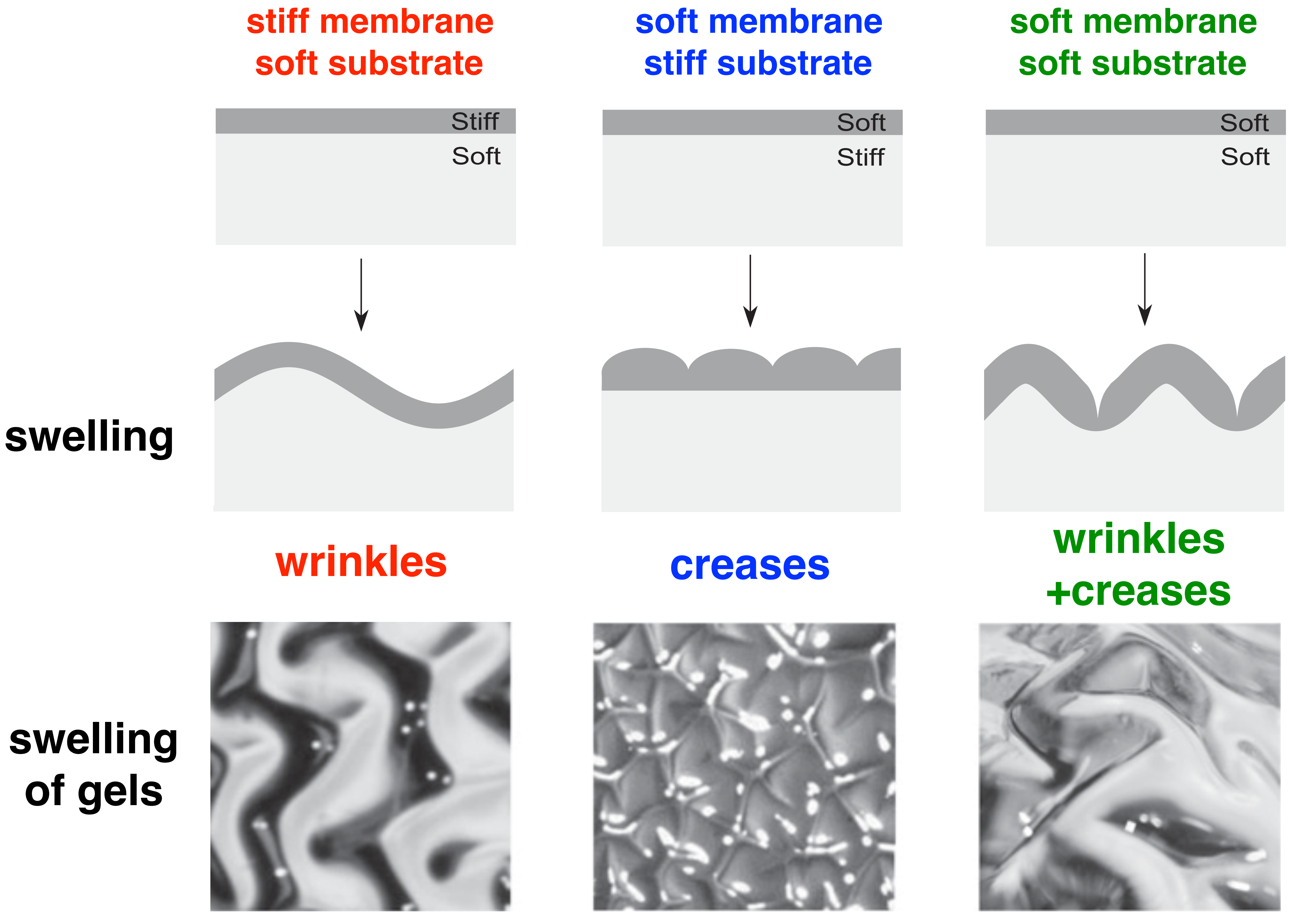
rising
dough

Liangfen
(starch jelly)

swollen gel
on a stiff substrate

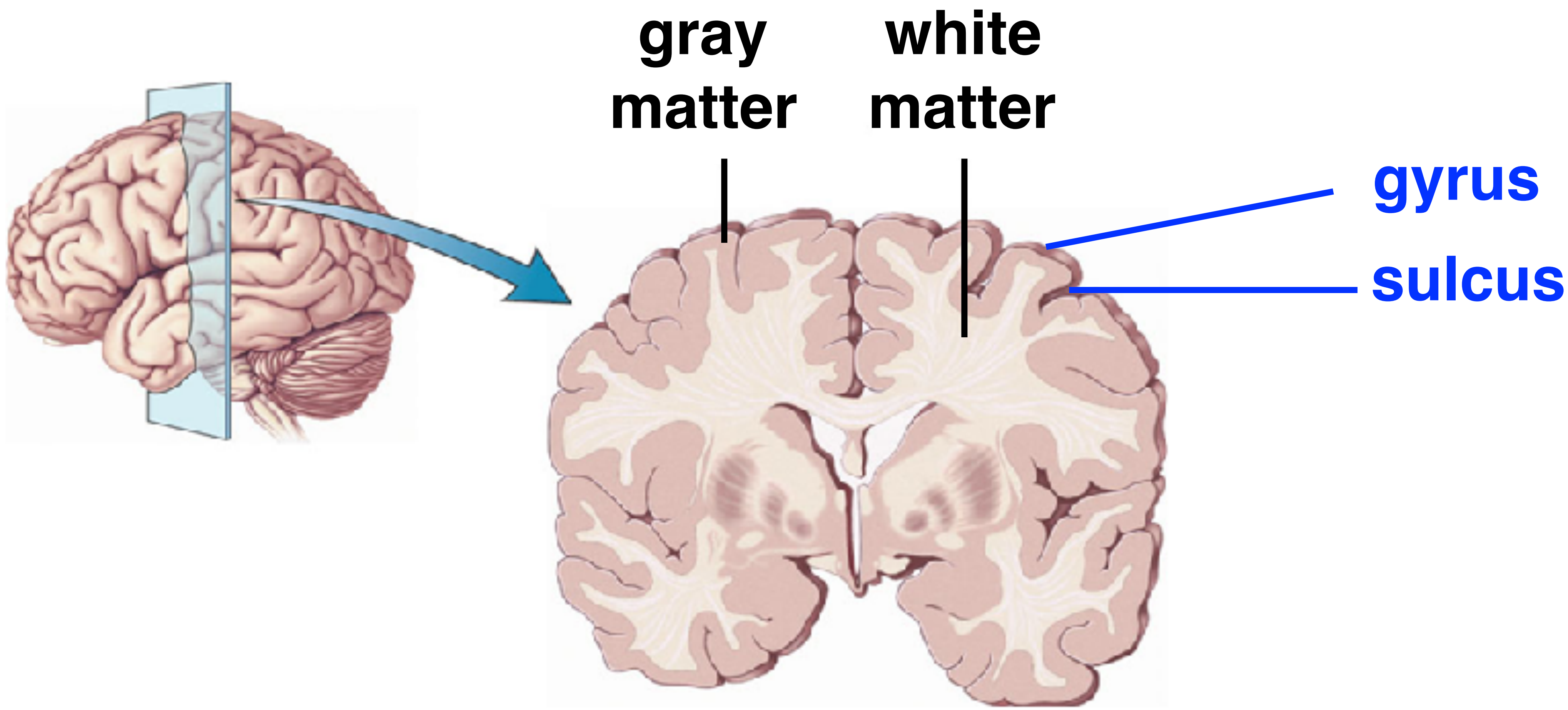


Swelling of thin membranes on elastic substrates

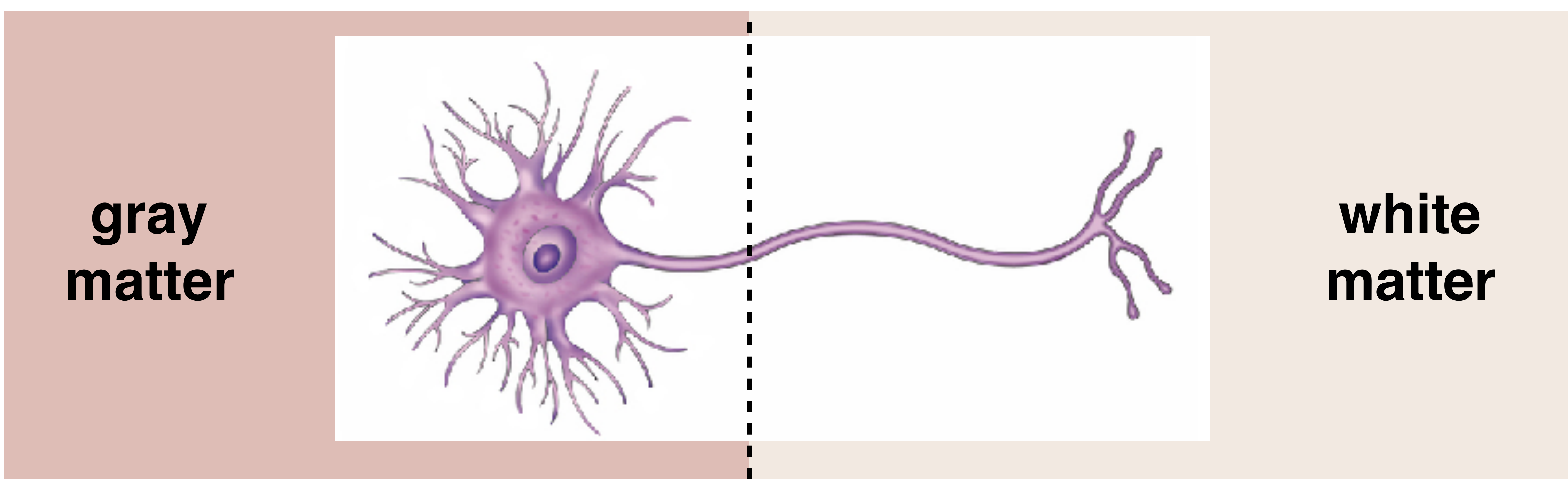


T. Tallinen et al., PNAS **111**, 12667 (2014)

Cortical convolutions in brains

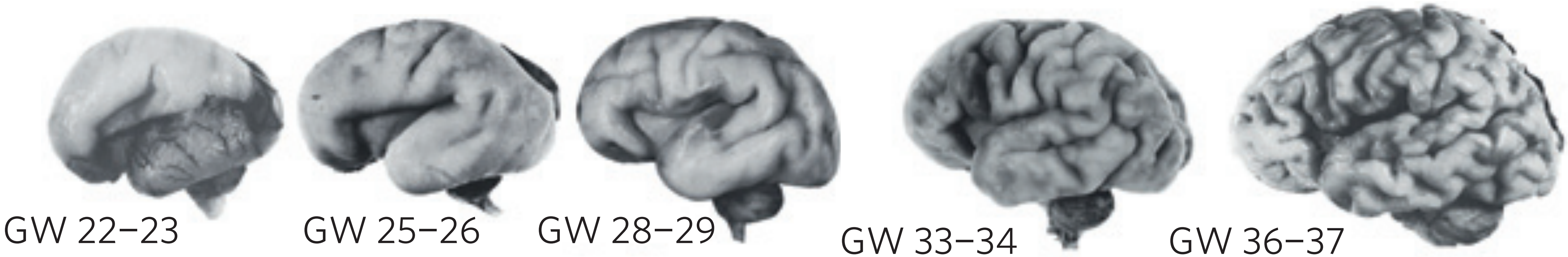


Migration of neurons to the cortex leads to "swelling" of gray matter!



Formation of cortical convolutions in developing brains

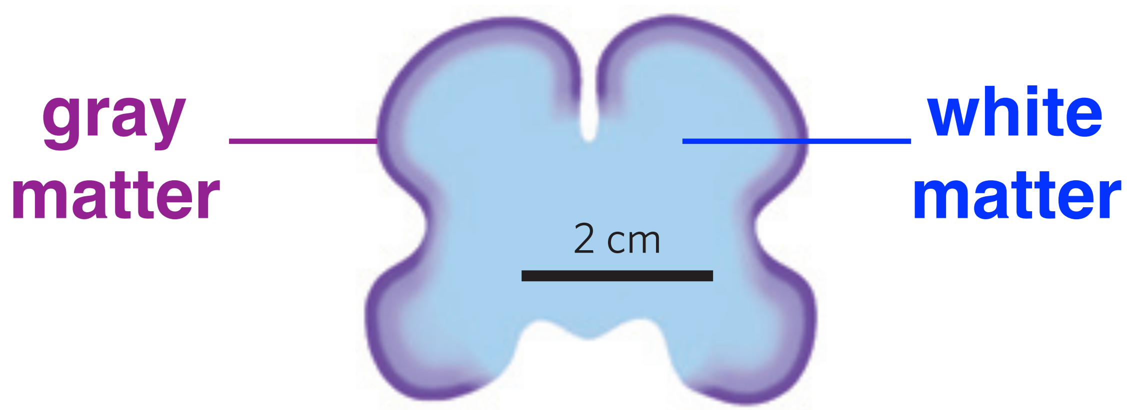
Magnetic resonance images (MRI) of fetal brains



gestational week (GW): age of fetus in weeks

Numerical simulations of developing brain

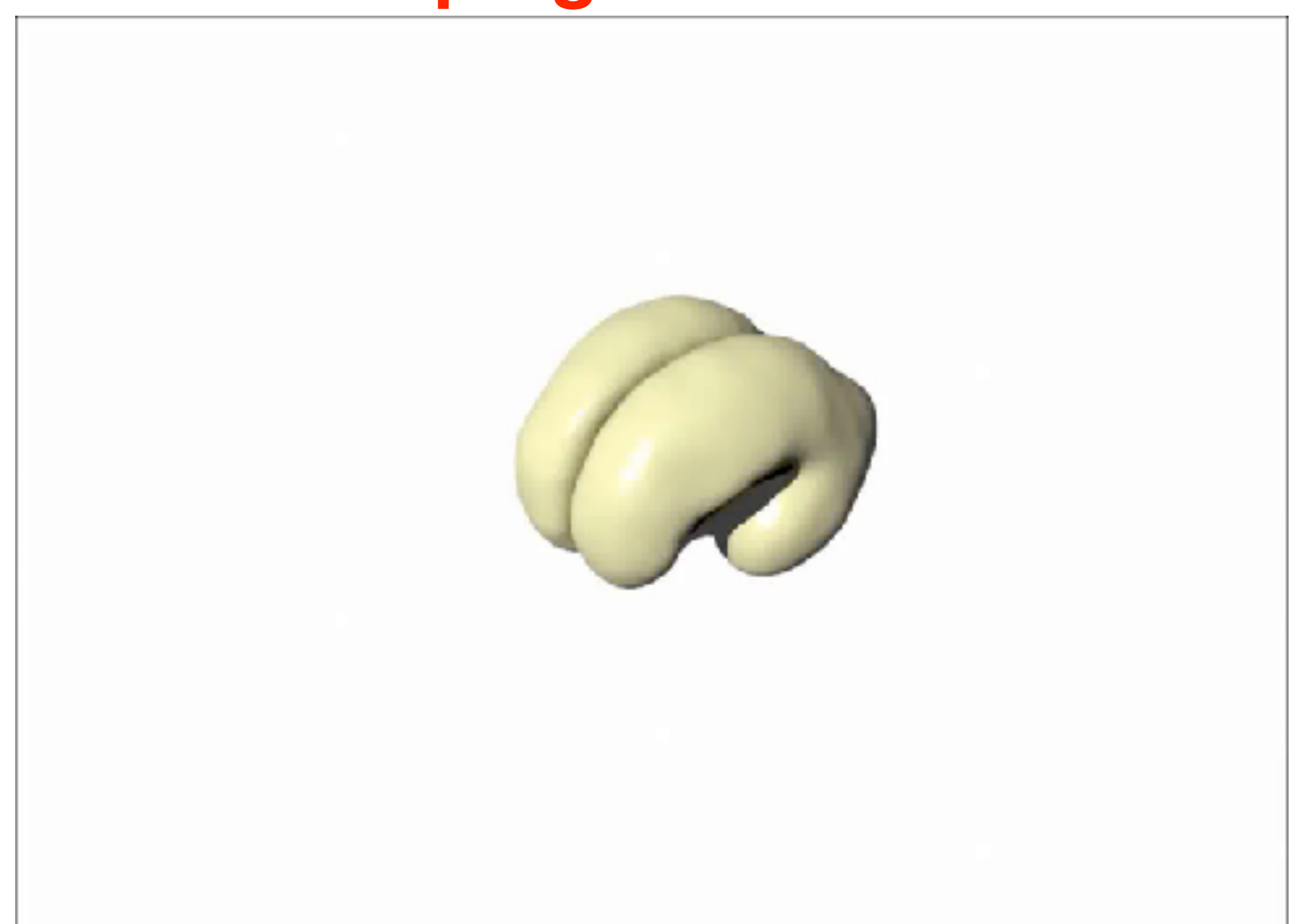
Initial condition: shape from MRI image of fetal brain at GW 22.



Assumptions:

isotropic expansion of white matter

additional tangential expansion of gray matter



T. Tallinen et al., Nature Physics **12**, 588 (2016)

Formation of cortical convolutions in developing brains

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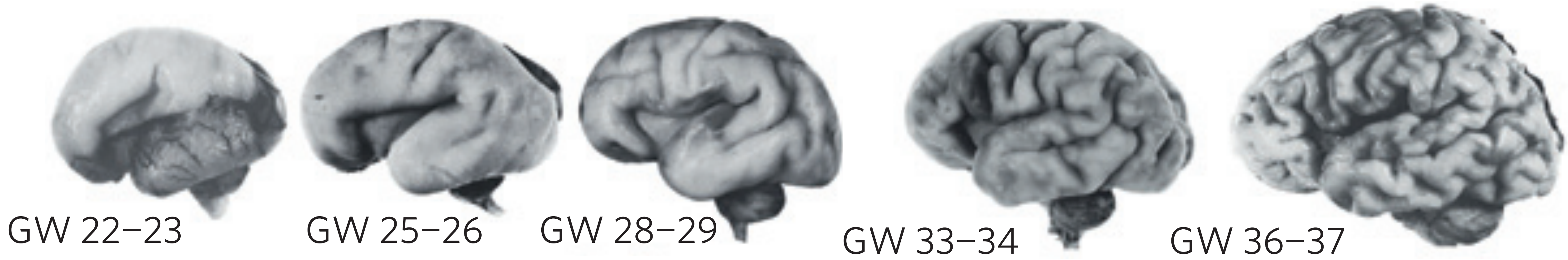
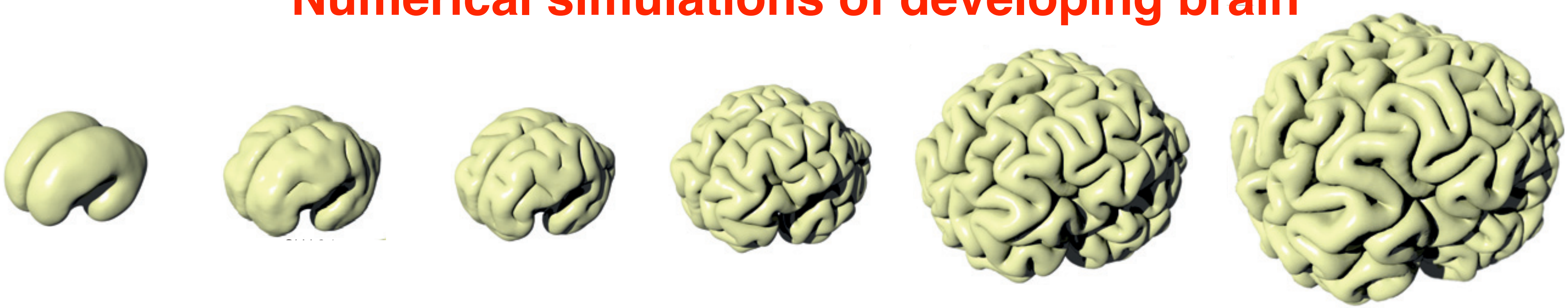


gestational week (GW): age of fetus in weeks

Numerical simulations of developing brain



GW 22 **GW 26** **GW 29** **GW 34** **GW 40** **adult**

From GW 22 to adult stage:

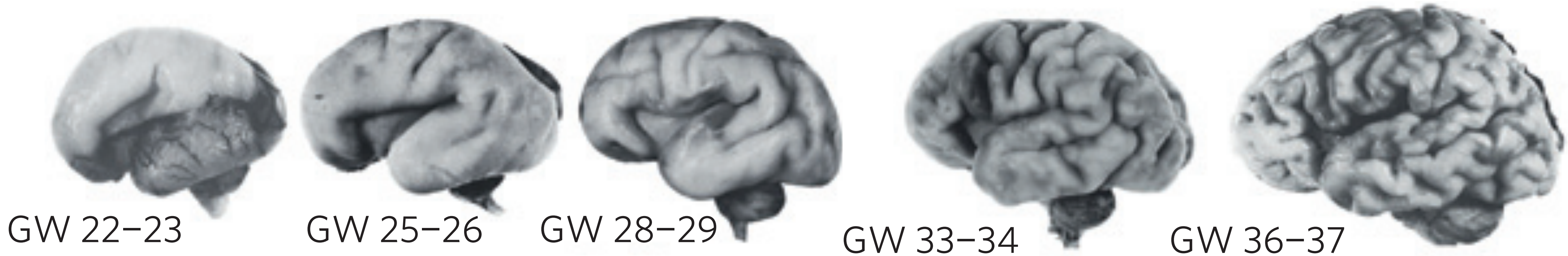
brain volume increases 20-fold from 60 ml to 1,200 ml

cortical area increases 30-fold from 80 cm^2 to 2,400 cm^2

T. Tallinen et al., Nature Physics **12**, 588 (2016)

Formation of cortical convolutions in developing brains

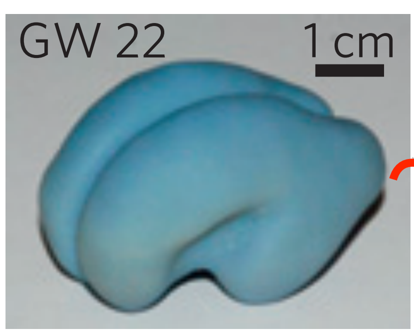
Magnetic resonance images (MRI) of fetal brains



gestational week (GW): age of fetus in weeks

Swelling of gel models of brain

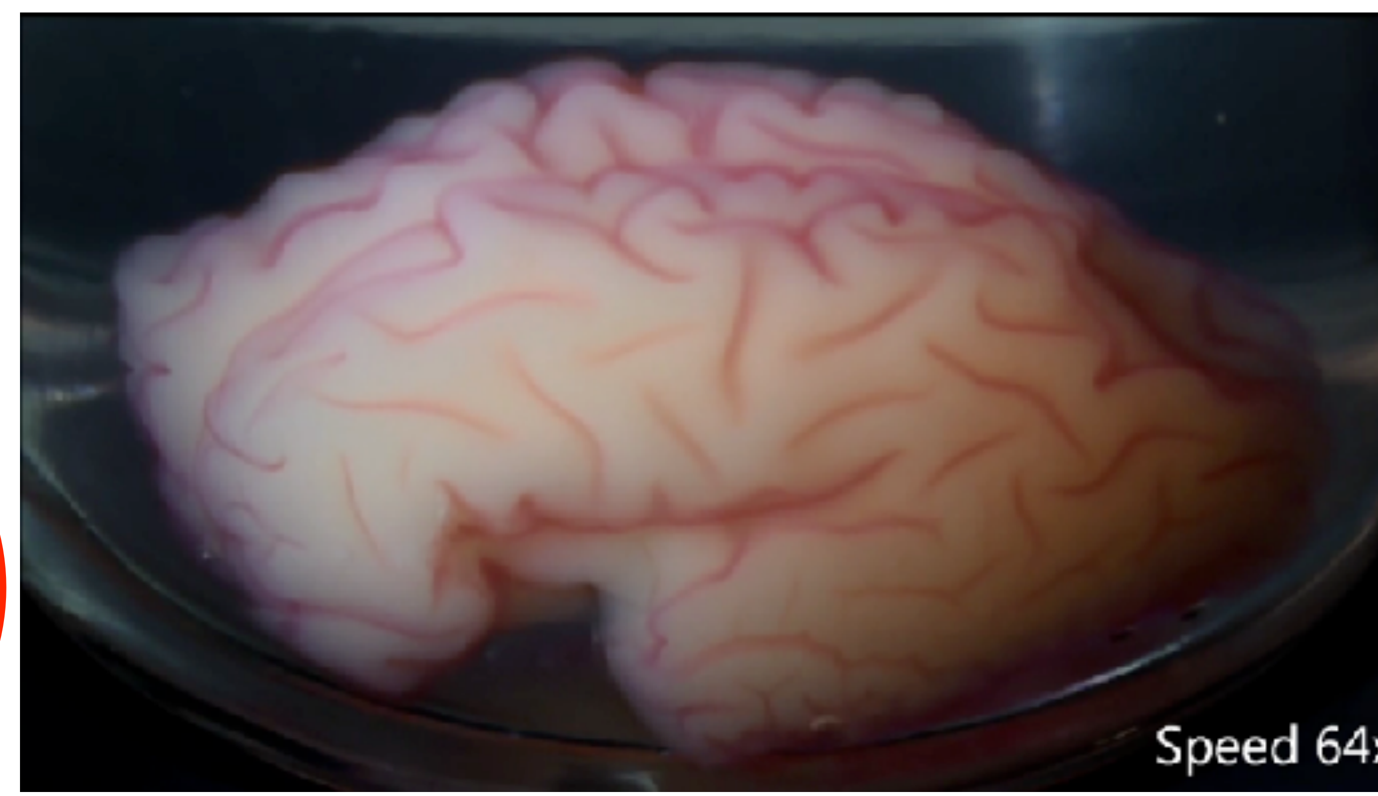
3D-printed brain model



master mould

replicated gel-brain

gel-brain coated with thin layer



In experiments only the thin coated layer swells by absorbing a liquid!

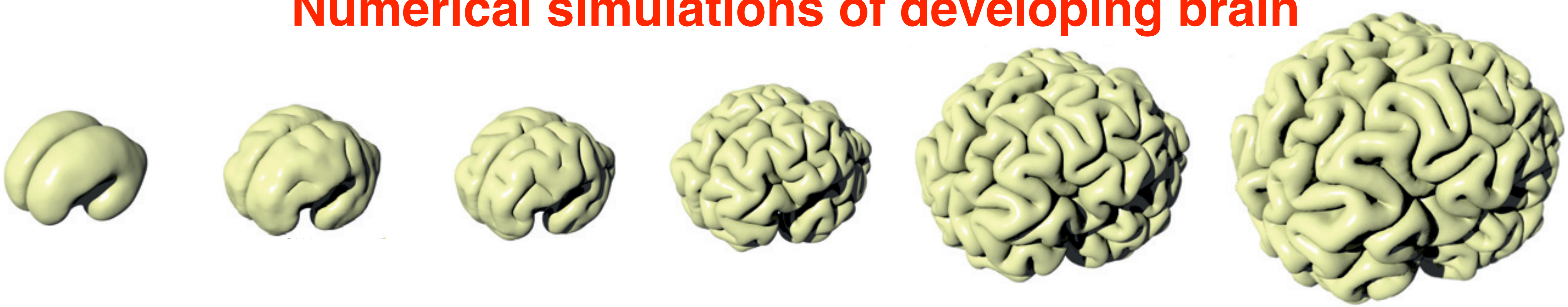
T. Tallinen et al., Nature Physics **12**, 588 (2016)

Formation of cortical convolutions in developing brains

Magnetic resonance images (MRI) of fetal brains

gestational week (GW): age of fetus in weeks

Numerical simulations of developing brain



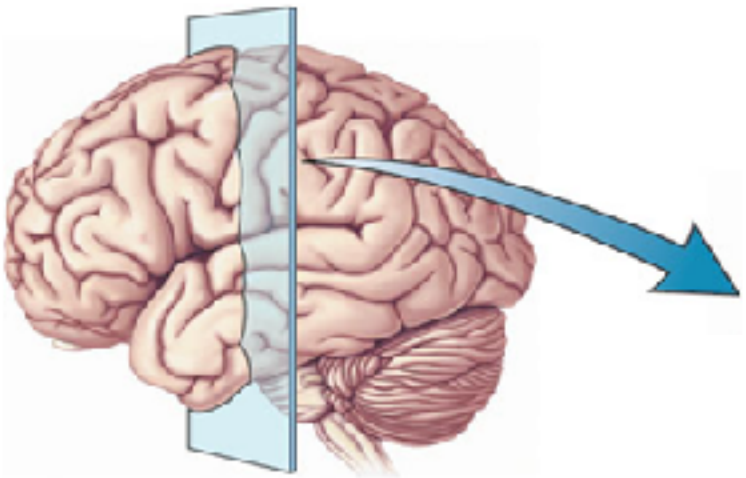
GW 22 **GW 26** **GW 29** **GW 34** **GW 40** **adult**

Swelling of gel models of brain

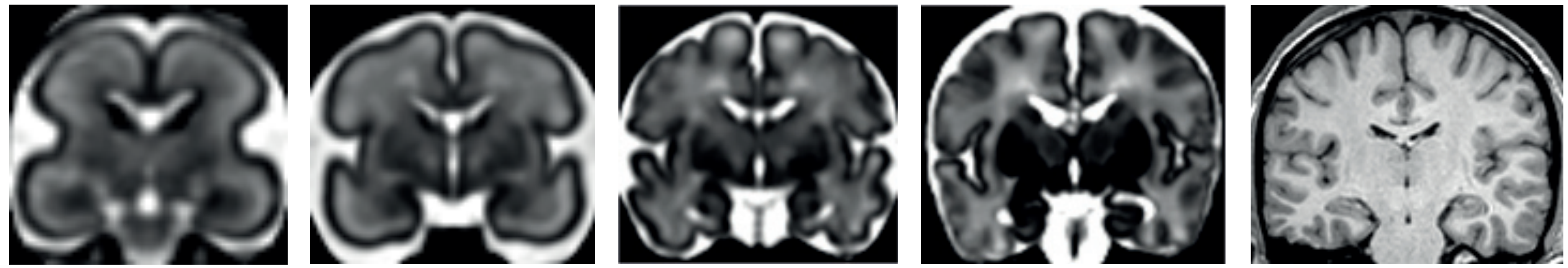
GW 22 (t=0) **GW 26 (t=4 min)** **GW 29 (t=9 min)** **GW 34 (t=16 min)**

T. Tallinen et al., Nature Physics **12**, 588 (2016)

Formation of cortical convolutions in developing brains

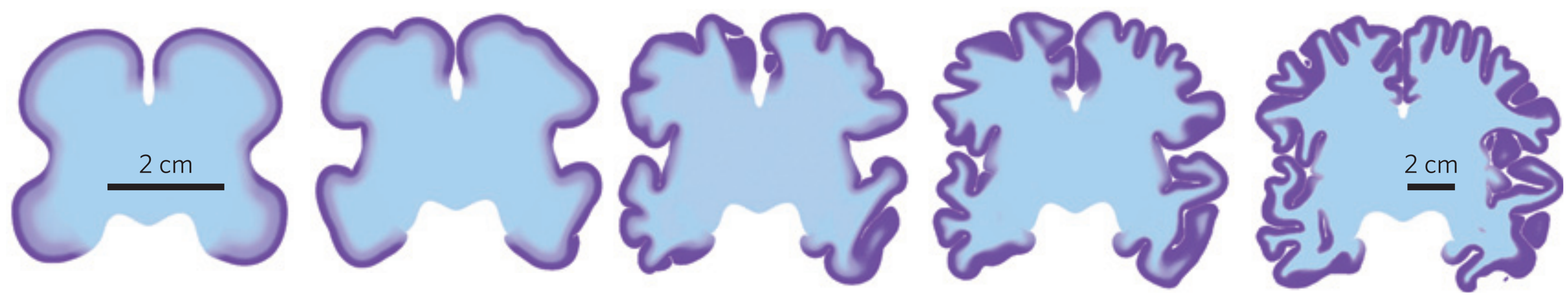


Magnetic resonance images (MRI) of brains



GW 22 | GW 29 | GW 34 | GW 40 | adult

Numerical simulations of developing brain



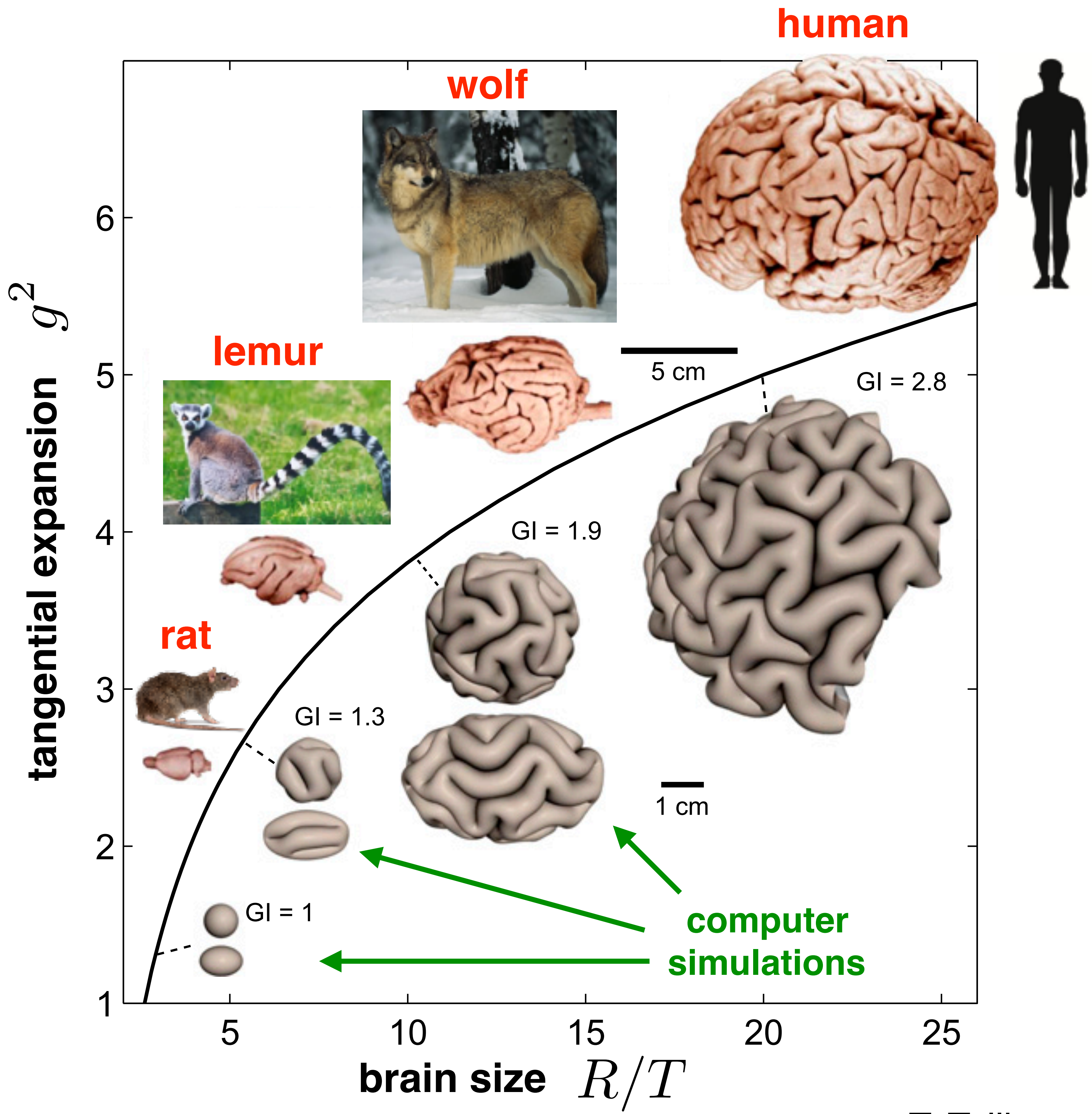
GW 22 | GW 29 | GW 34 | GW 40 | adult

Swelling of gel models of brain

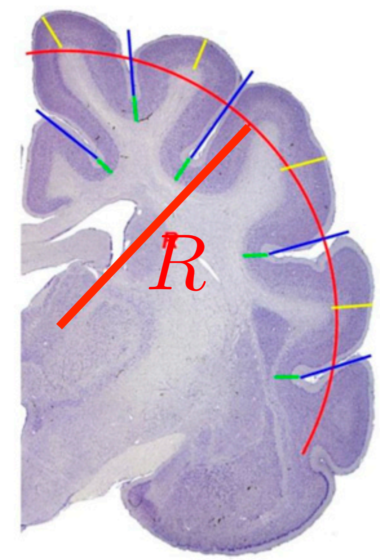
GW 22 (t=0) | GW 29 (t=9 min) | GW 34 (t=16 min)

T. Tallinen et al., Nature Physics **12**, 588 (2016)

Brains for various organisms



measurements of brain parameters



***R*: brain size**

***T*: thickness of gray matter**

tangential expansion

$$g = \frac{\text{contour length of gray matter}}{\text{length of circular section}}$$

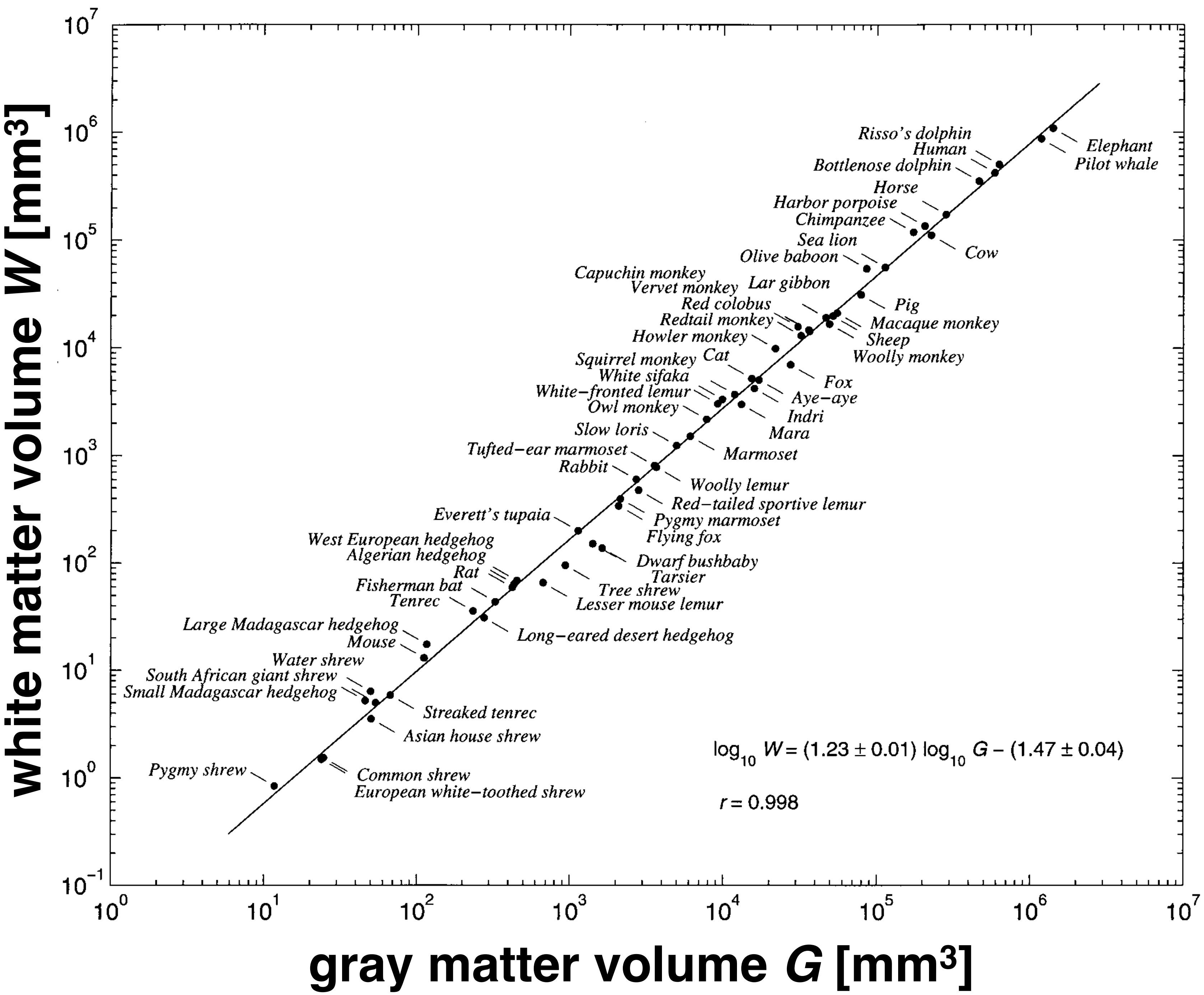
gyrification index

$$\mathrm{GI} = \frac{\text{area of brain surface}}{\text{area of convex hull}}$$

T. Tallinen et al., PNAS **111**, 12667 (2014)

Power law scaling for the brain size of various organisms

$$\text{white matter volume} \propto \left(\text{gray matter volume}\right)^{1.23}$$



another scaling relation

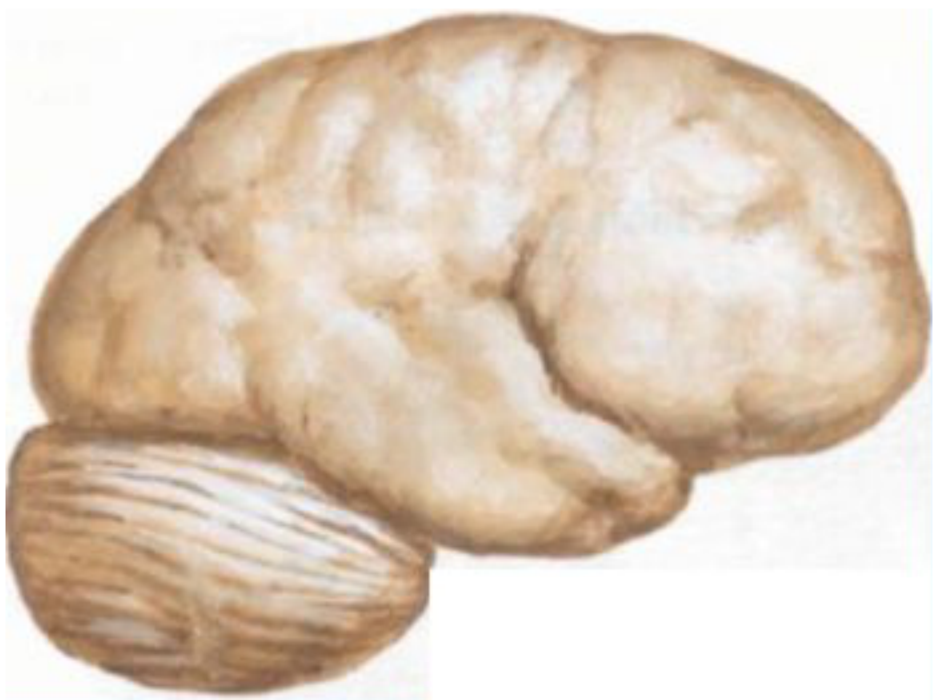
$$\text{gray matter thickness} \propto \left(\text{gray matter volume}\right)^{0.10}$$

Note: power law scalings of various quantities among organisms are very common!

K. Zhang and T.J. Sejnowski, PNAS **97**, 5621 (2000)

Brain malformations

lissencephaly
pachygyria
(small number of larger gyri)

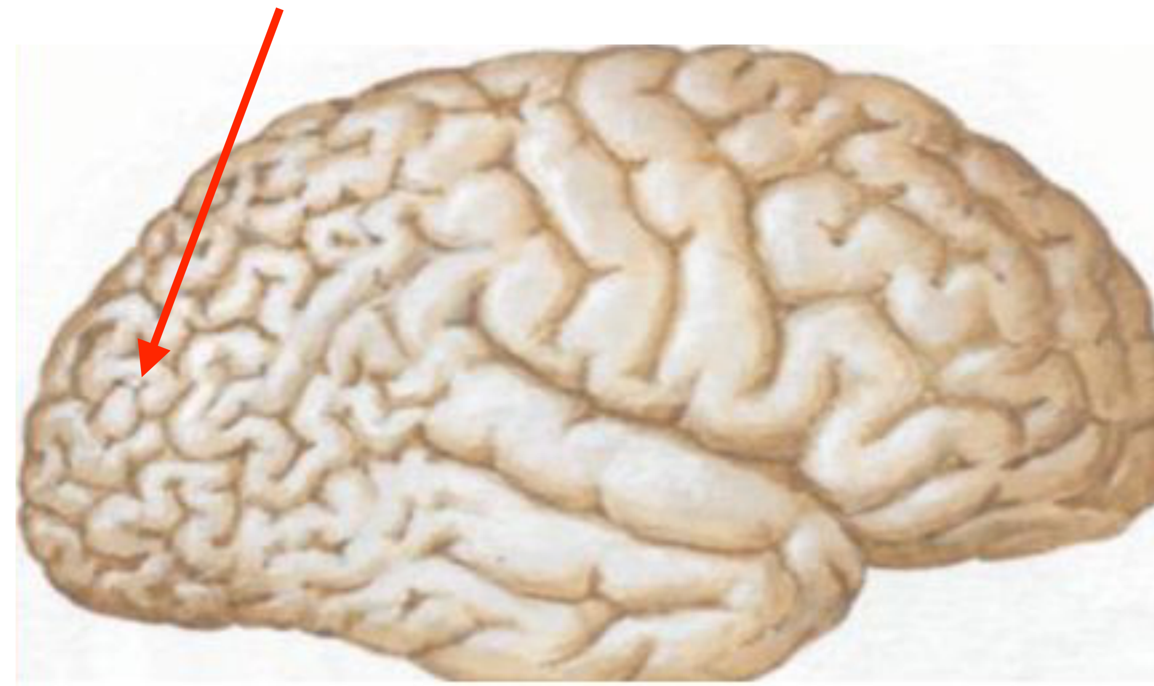


Reduced neuronal migration to cortex

↓

Gray matter is thicker and it swells less!

polymicrogyria
(large number of smaller gyri)

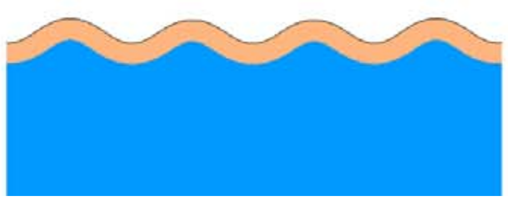


Typically gray matter has only four rather than six layers in the affected areas.

Compression of thin membranes on elastic substrates with finite adhesion

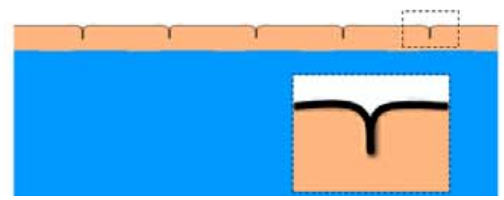
Strong adhesion between membrane and substrate

stiff membrane
soft substrate



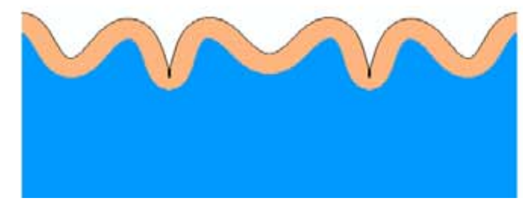
wrinkles

soft membrane
stiff substrate



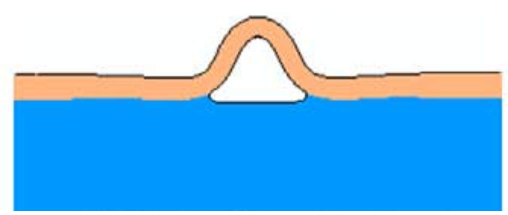
creases

soft membrane
soft substrate



folds

Weak adhesion between membrane and substrate



thin membrane
delaminates/buckles!

The morphology of compressed structures can be obtained by minimizing the total energy

$$U_{\text{total}} = U_{\text{substrate}} + U_{\text{membrane}} + U_{\text{adhesion}}$$

elastic energy of deformed substrate

elastic energy of deformed membrane

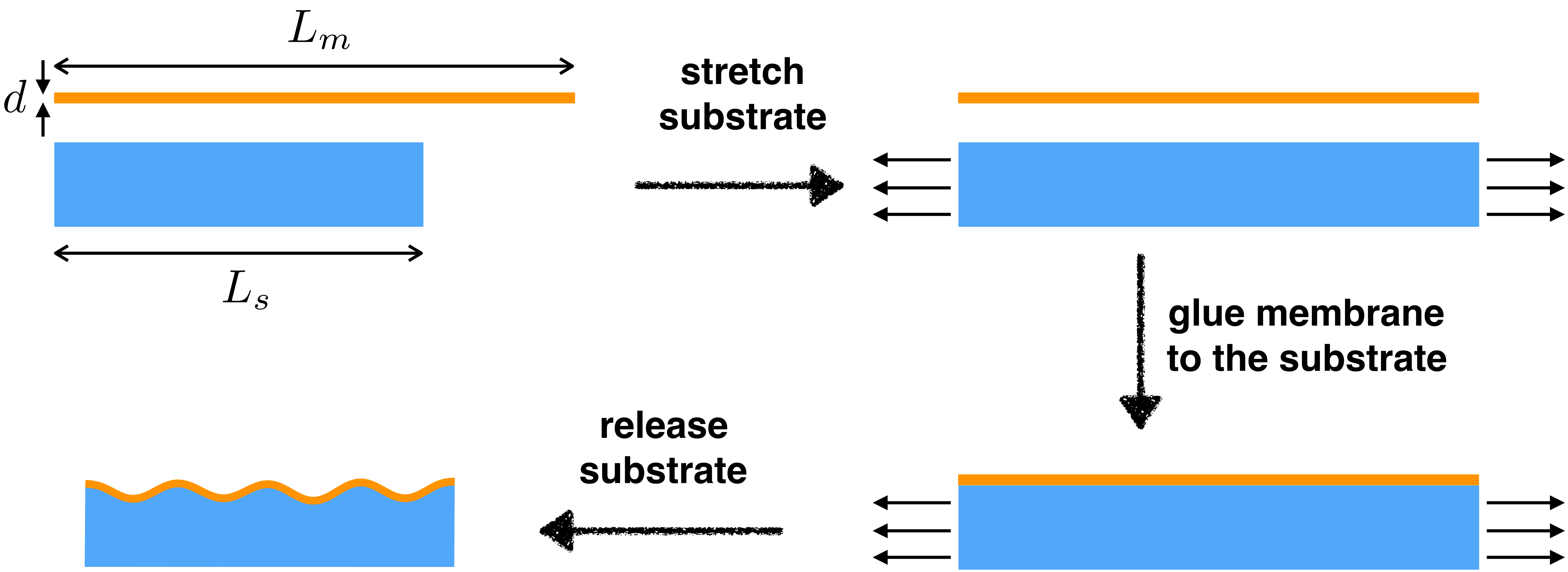
adhesion energy between membrane and substrate → $U = -\Gamma A$

Γ adhesion constant

A contact area

Compression of thin membranes on elastic substrates with finite adhesion

Experimental protocol



$$\epsilon = \frac{L_m - L_s}{L_m}$$ compressive strain

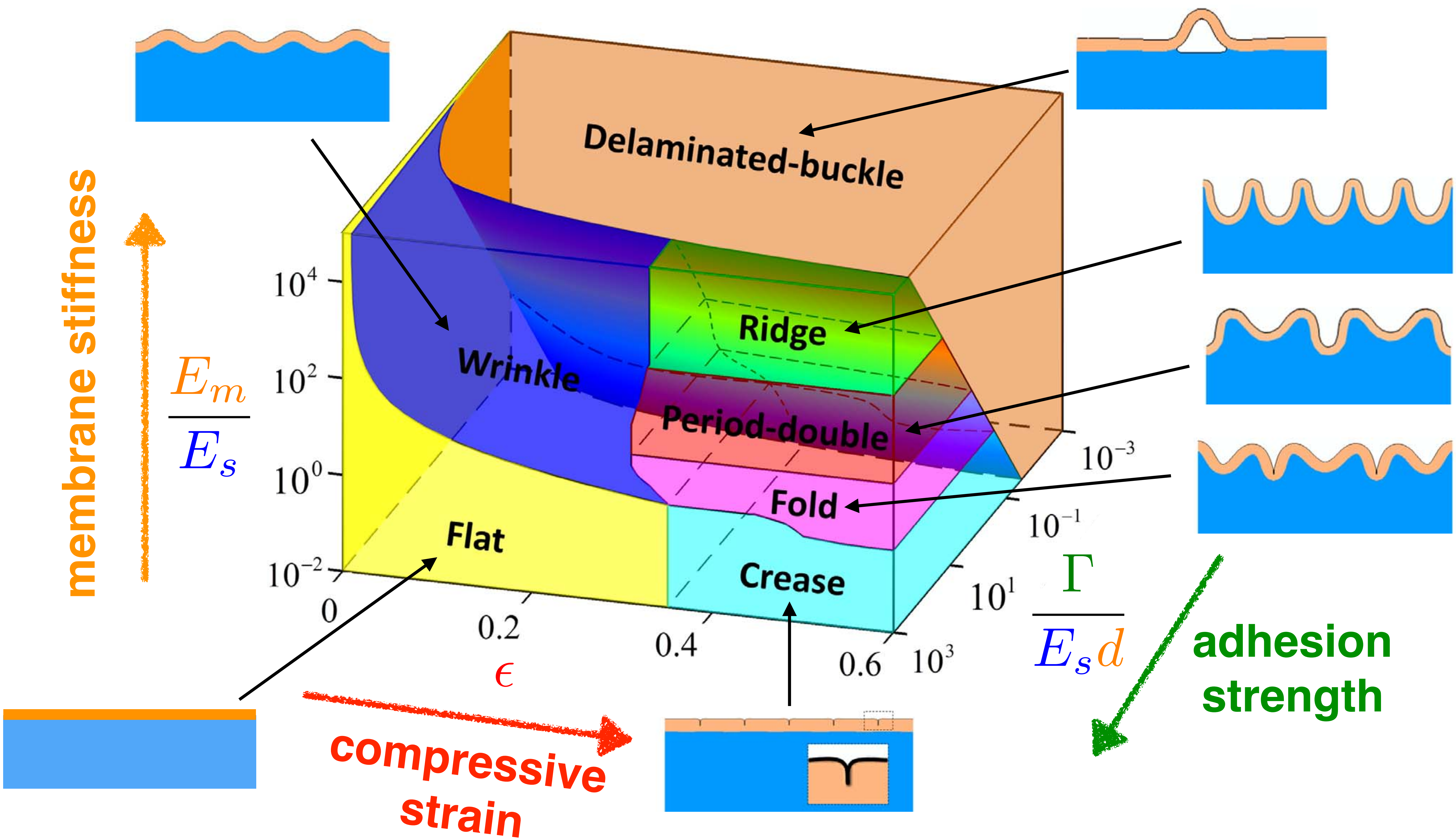
E_m membrane Young's modulus

Γ adhesion constant (strength of glue)

E_s substrate Young's modulus

Compression of thin membranes on elastic substrates with finite adhesion

Computationally predicted phase diagram



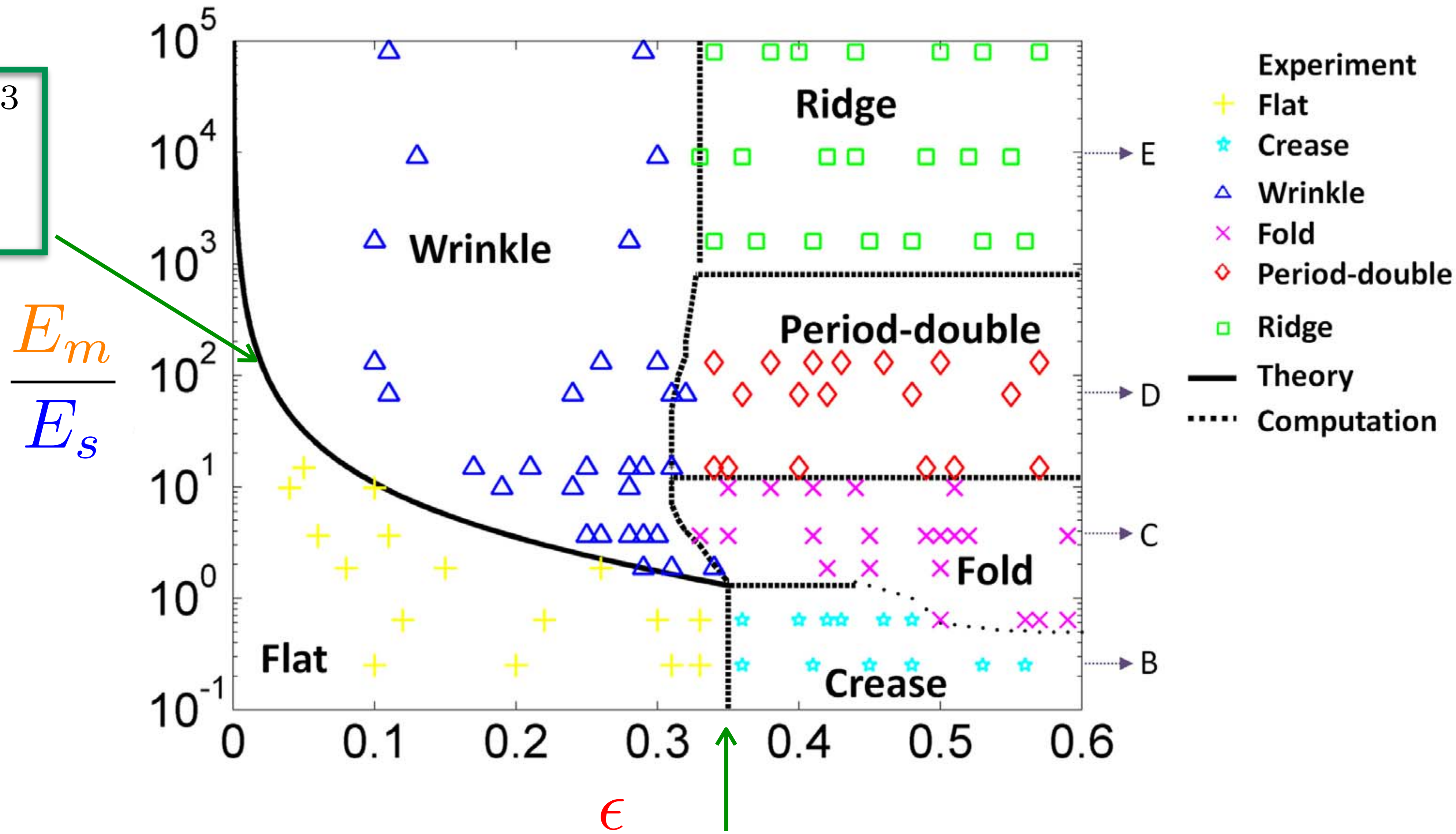
 Q. Wang and X. Zhao, Sci. Rep. **5**, 8887 (2015)

Compression of thin membranes on elastic substrates with finite adhesion

Very strong adhesion ($\Gamma/(E_s d) \gg 1$)

wrinkling transition

$$\epsilon_c \sim \left(\frac{E_s}{E_m}\right)^{2/3}$$



crease transition

$\epsilon_c = 0.35$

Q. Wang and X. Zhao, Sci. Rep. **5**, 8887 (2015)

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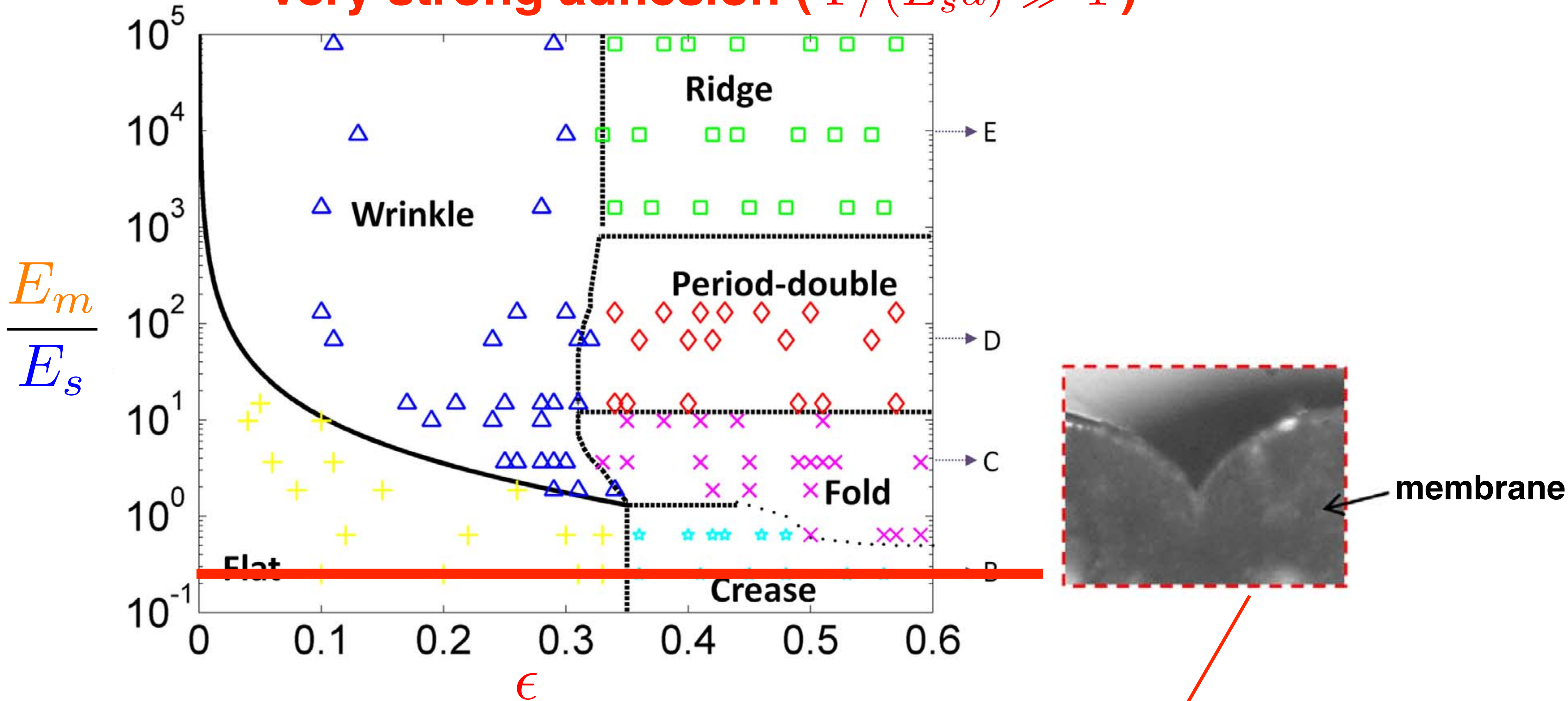
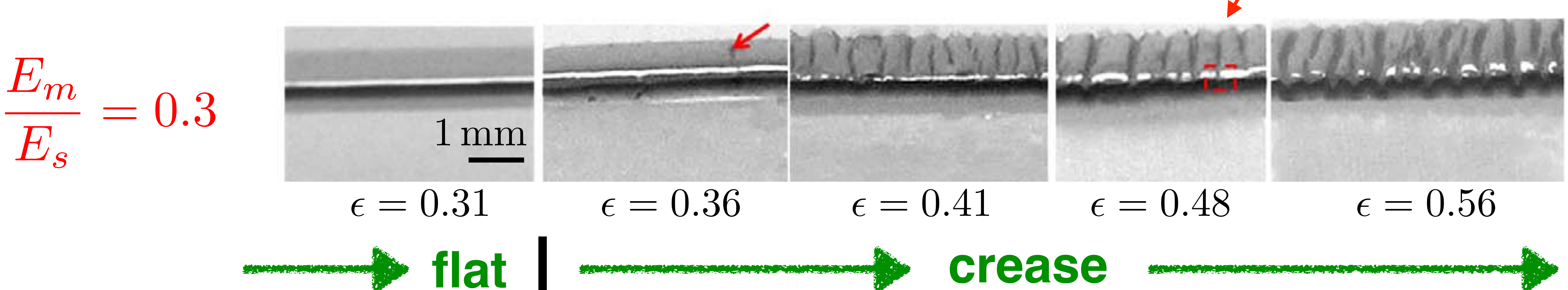
Very strong adhesion ($\Gamma/(E_s d) \gg 1$)





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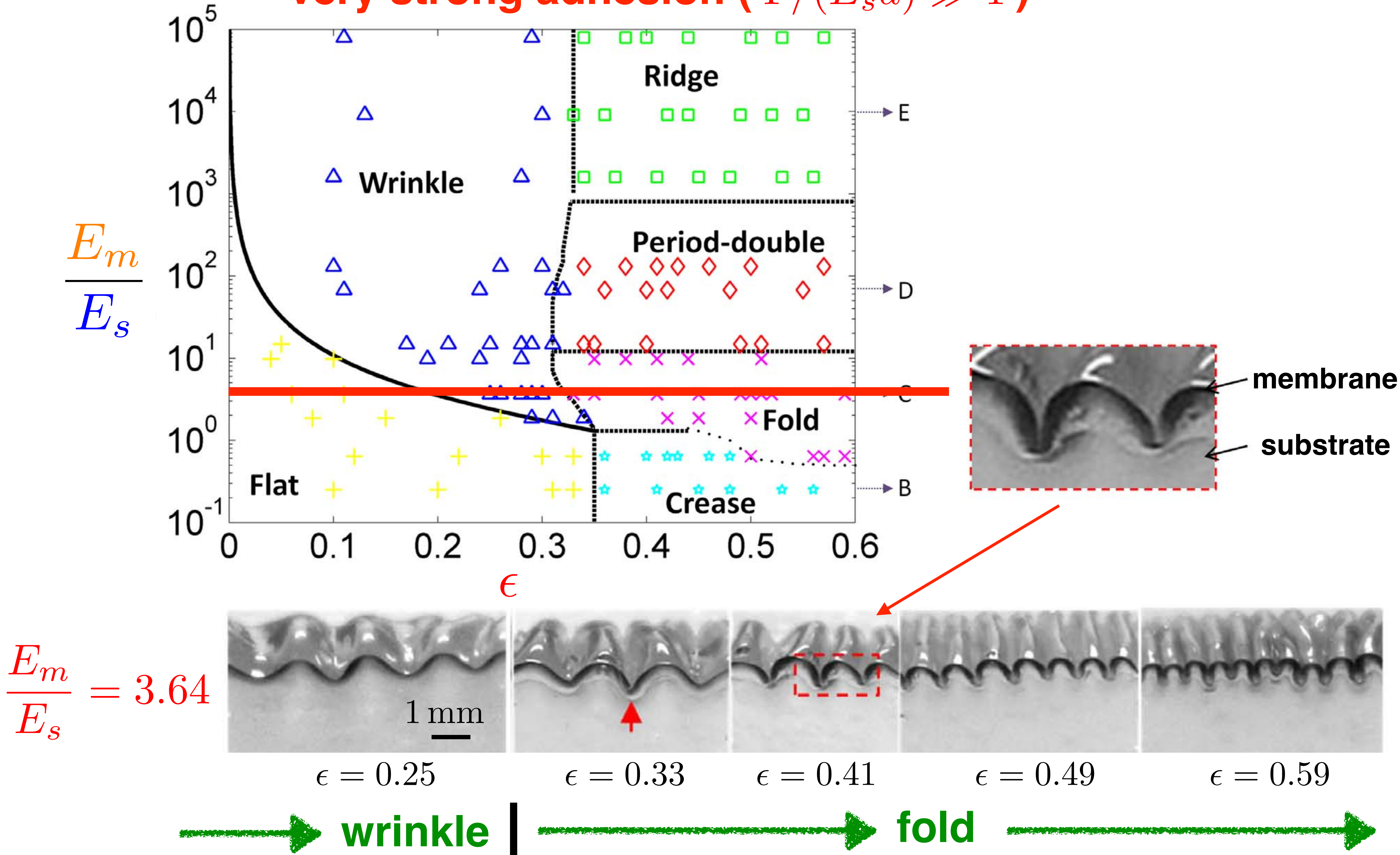

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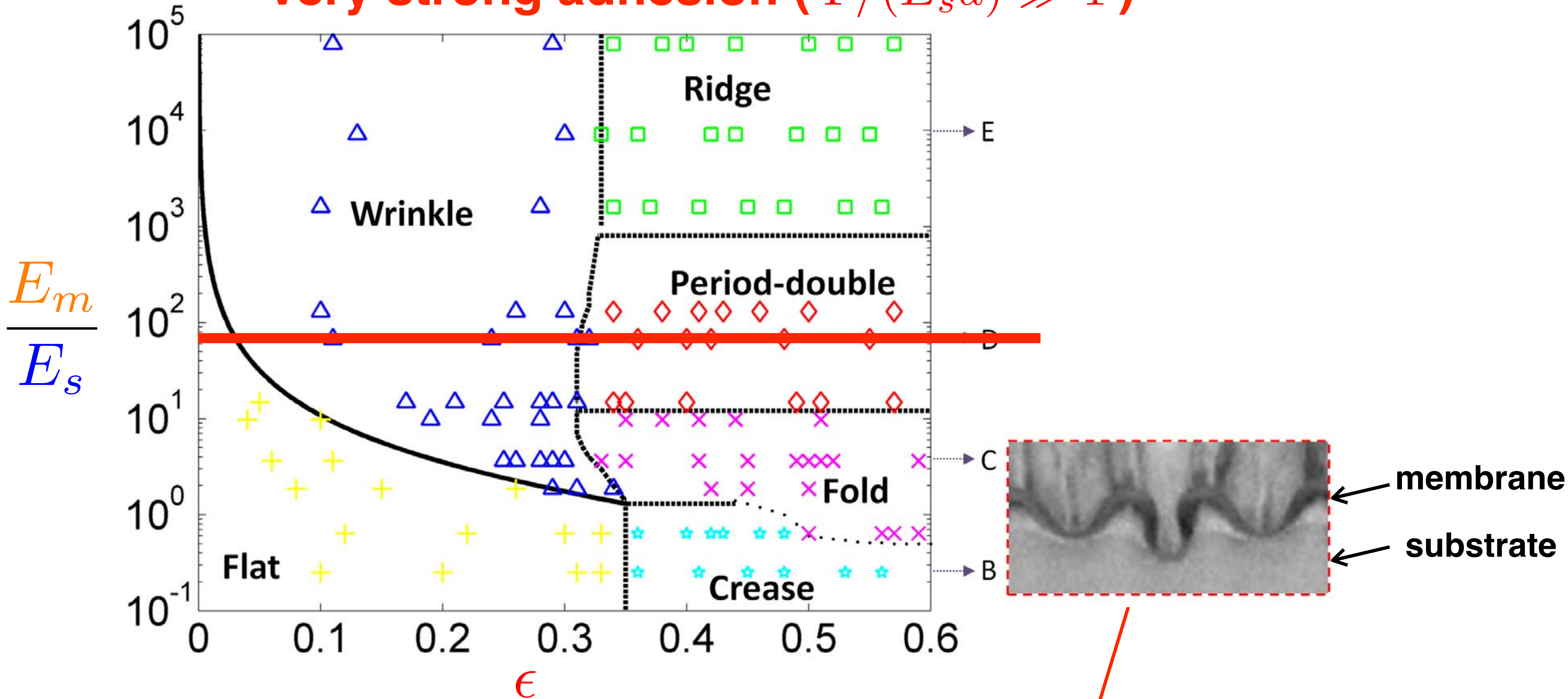
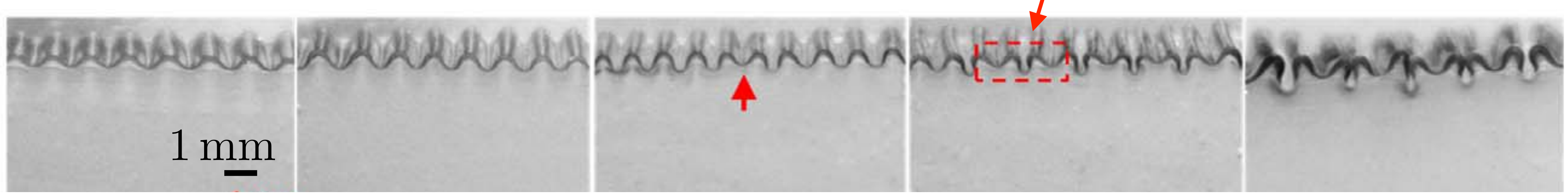


$\frac{E_m}{E_s} = 67.24$



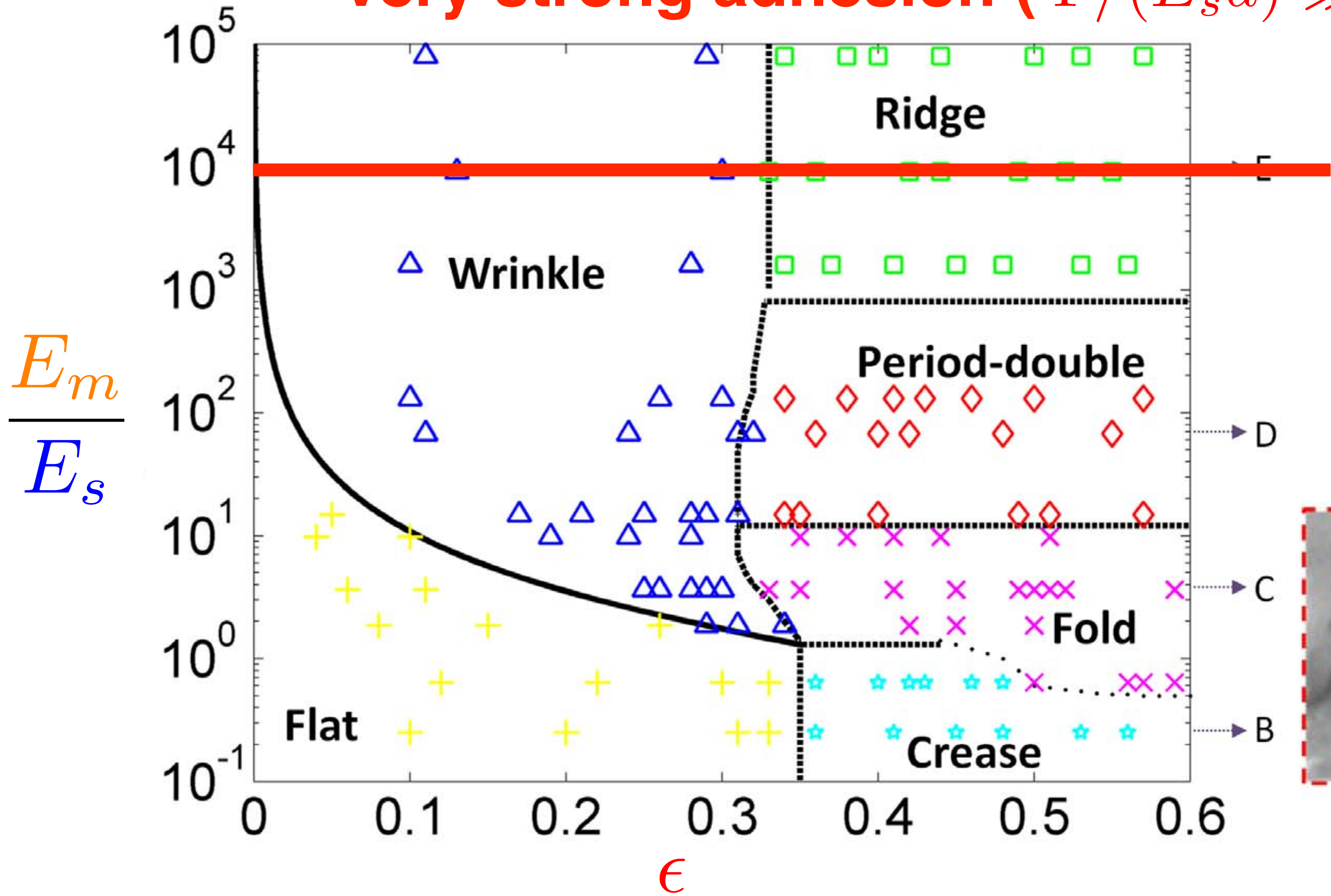
$\epsilon = 0.11$ $\epsilon = 0.24$ $\epsilon = 0.31$ $\epsilon = 0.36$ $\epsilon = 0.48$

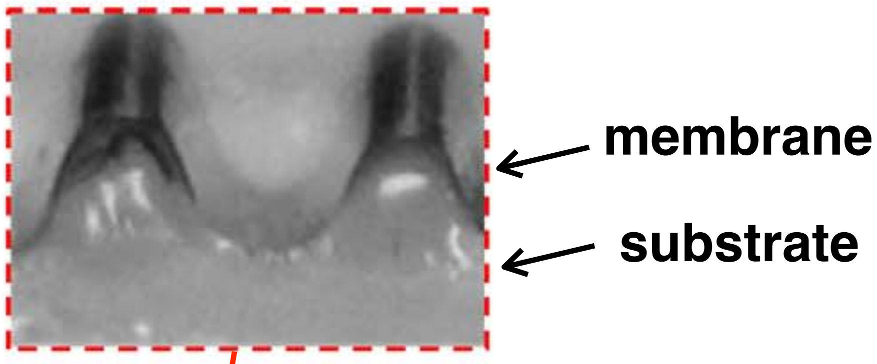
wrinkle → | period-double →

Q. Wang and X. Zhao, Sci. Rep. **5**, 8887 (2015)

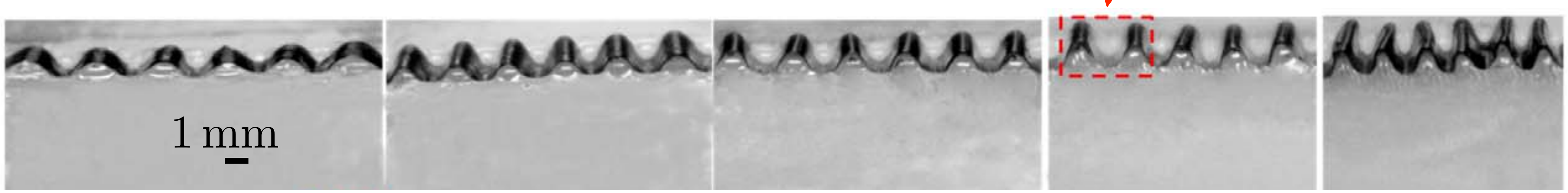
Compression of thin membranes on elastic substrates with finite adhesion

Very strong adhesion ($\Gamma/(E_s d) \gg 1$)





$$\frac{E_m}{E_s} = 9110$$



$\epsilon = 0.13$ $\epsilon = 0.30$ $\epsilon = 0.33$ $\epsilon = 0.42$ $\epsilon = 0.52$

→ **wrinkle** → | → **ridge** →

Q. Wang and X. Zhao, Sci. Rep. **5**, 8887 (2015)

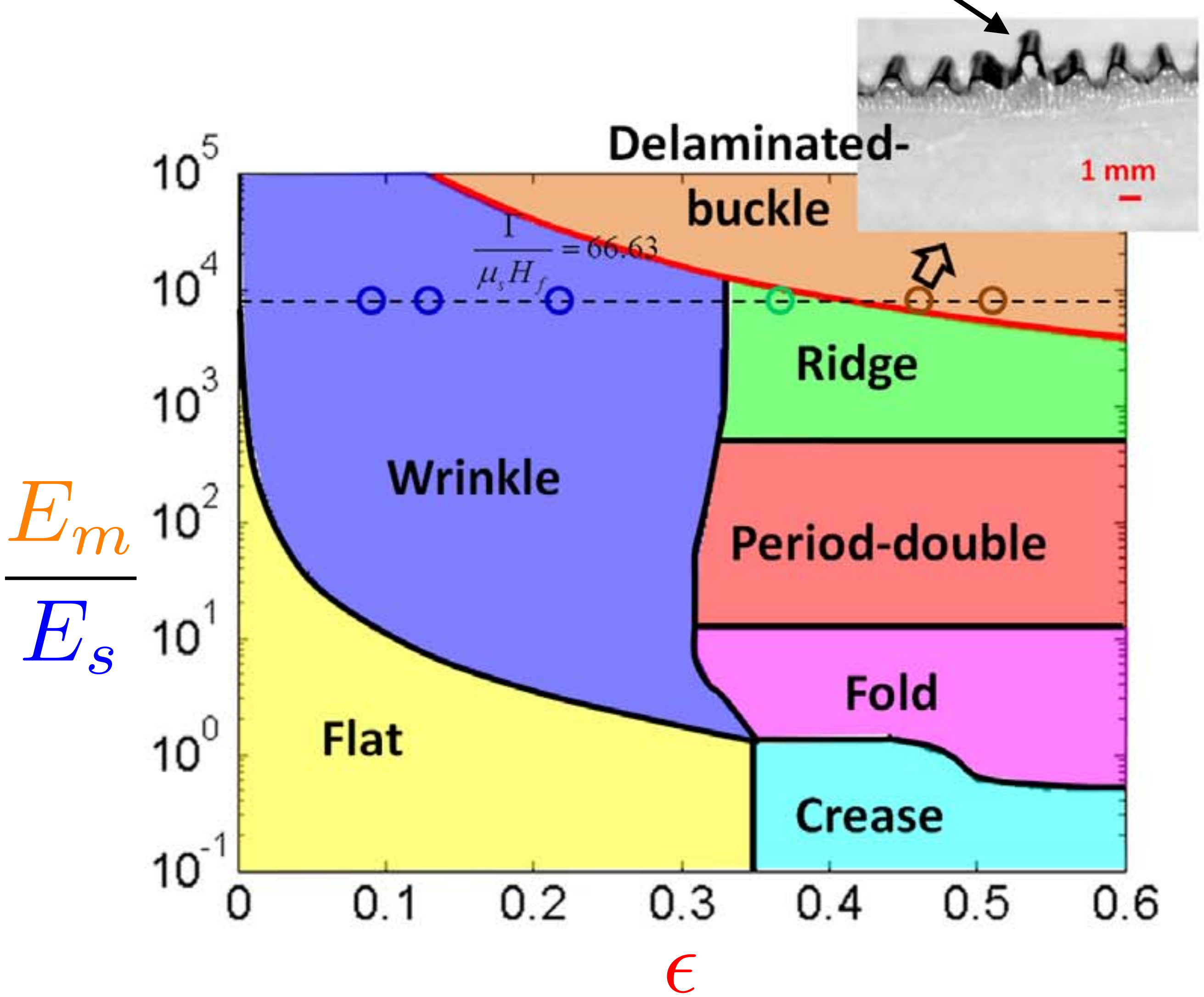
Compression of thin membranes on elastic substrates with finite adhesion

Strong adhesion

$$\frac{\Gamma}{E_s d} = 66.63$$

all phases are present

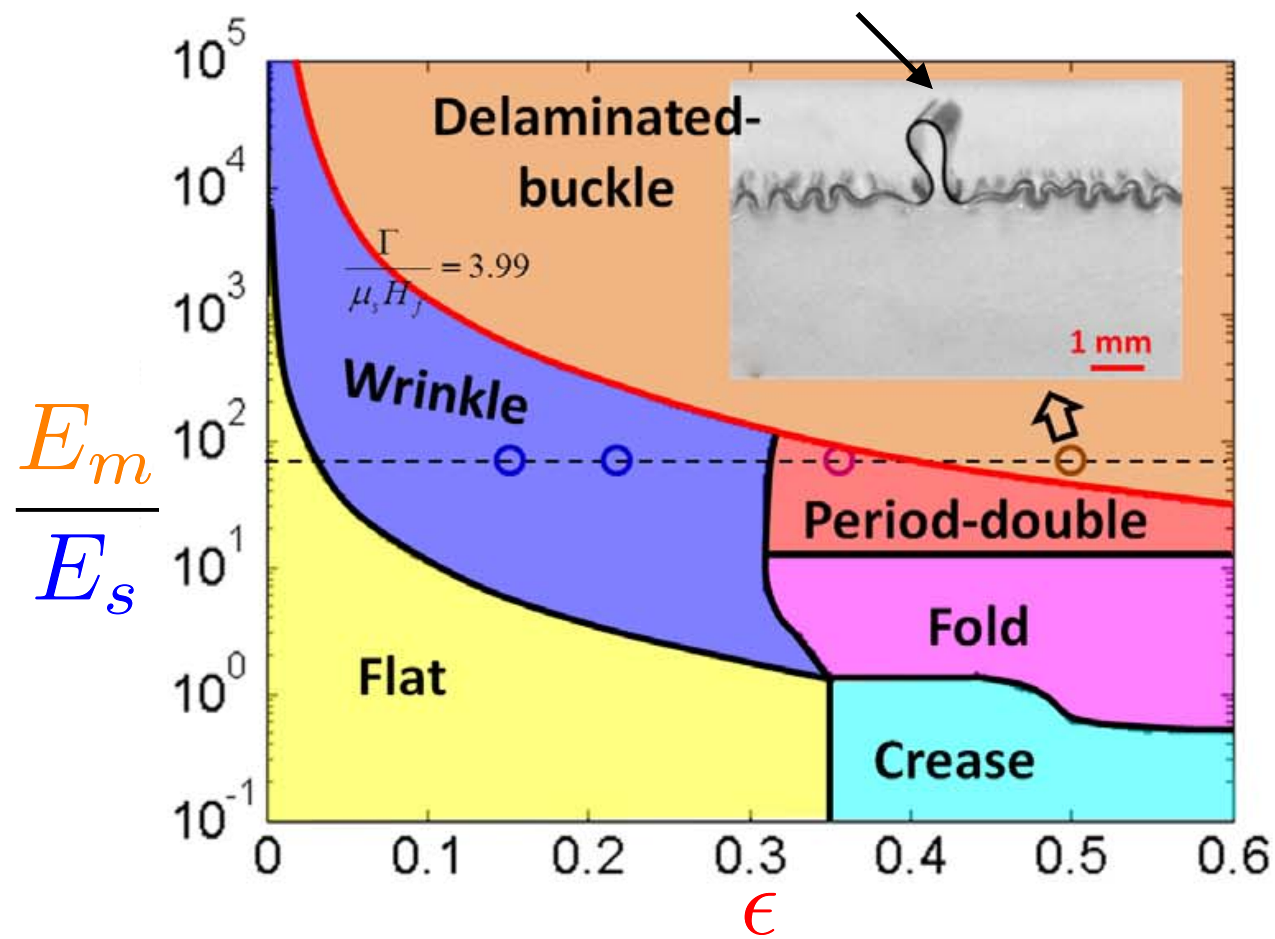
delamination/buckling of ridges



$$\frac{\Gamma}{E_s d} = 3.99$$

the region of "Delaminated-buckle" phase expands, the "Ridge" phase disappears

delamination/buckling of wrinkles



Q. Wang and X. Zhao, Sci. Rep. **5**, 8887 (2015)

Compression of thin membranes on elastic substrates with finite adhesion

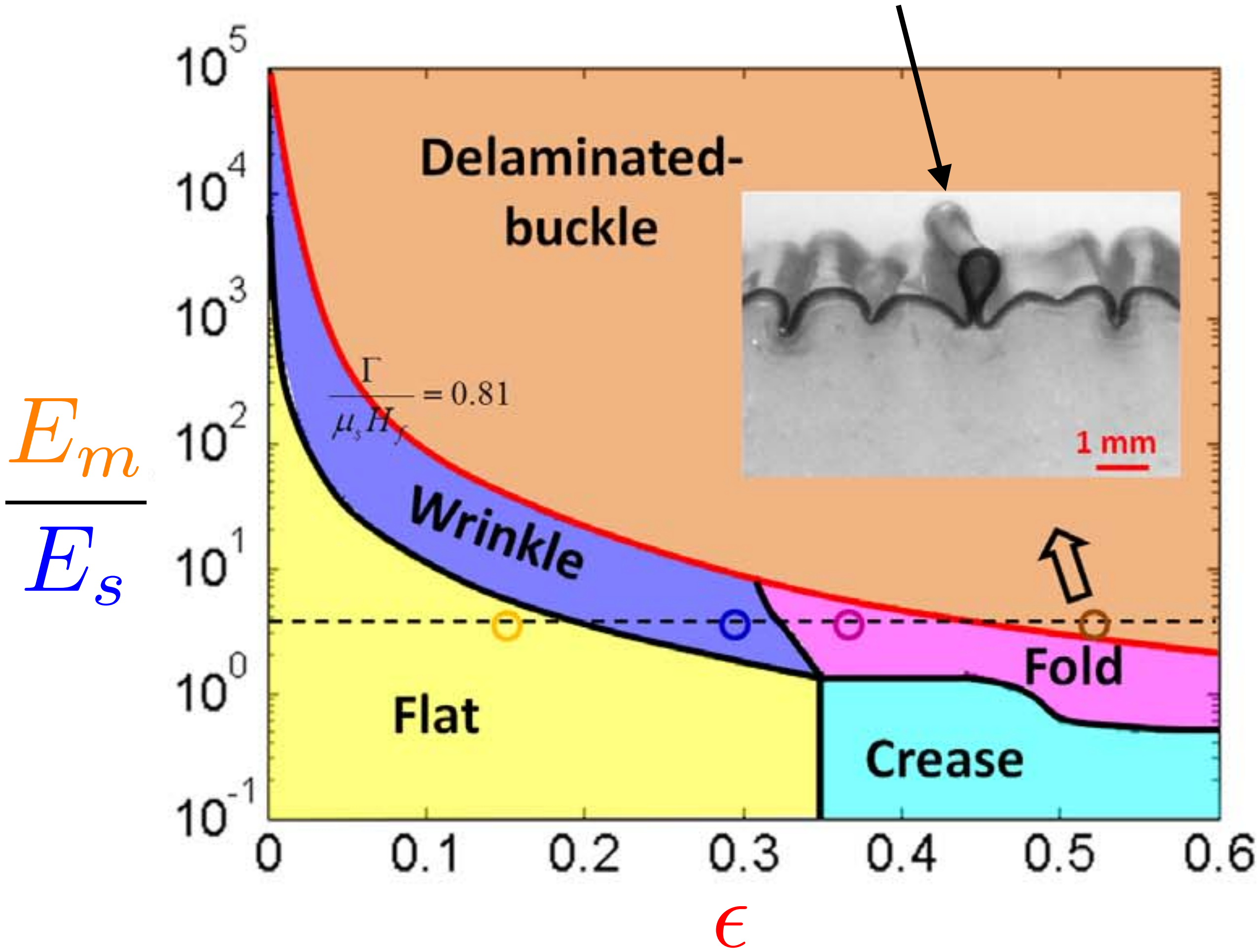
Moderate adhesion

$$\frac{\Gamma}{E_s d} = 0.81$$

$$\frac{\Gamma}{E_s d} = 0.46$$

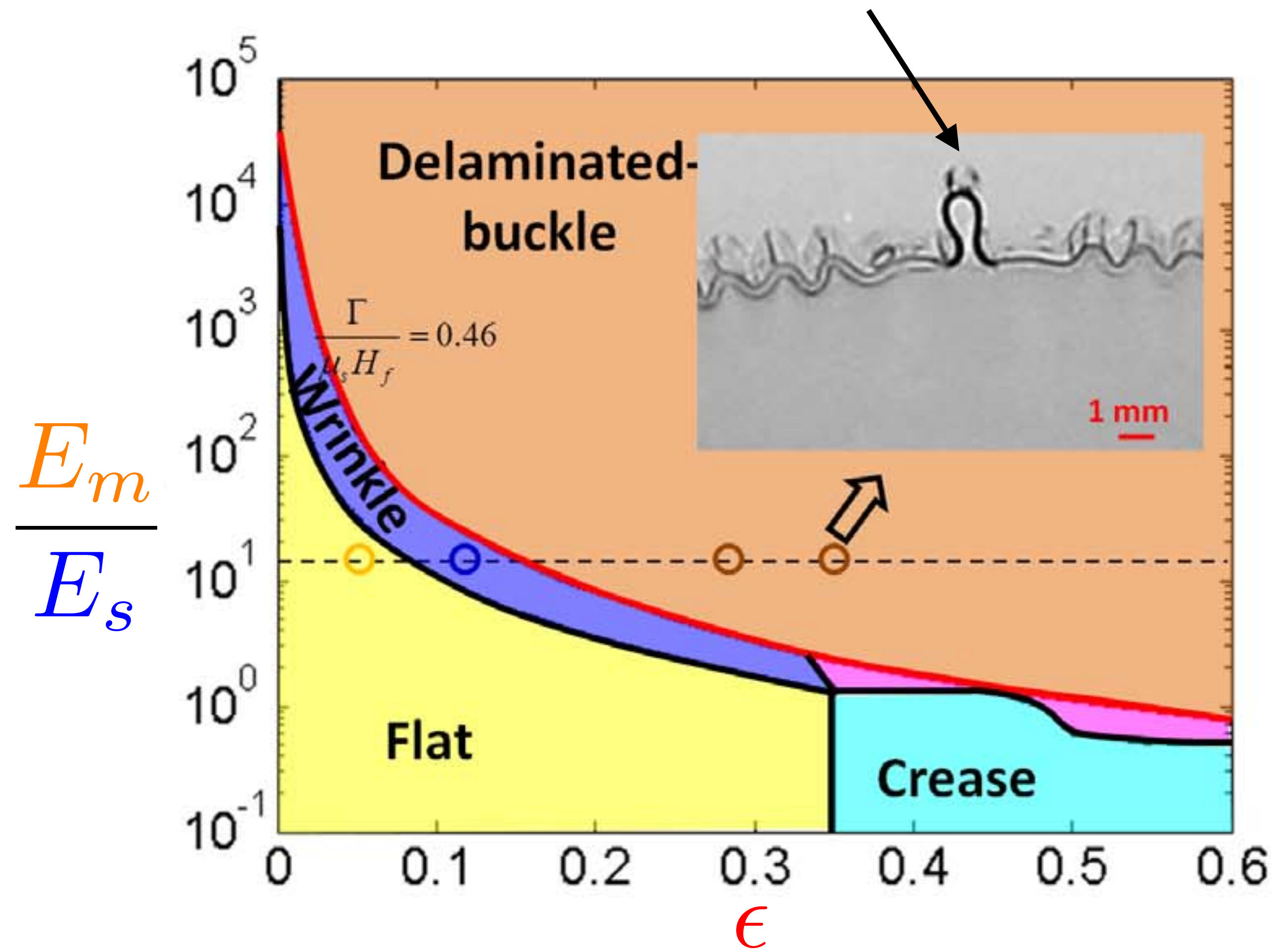
"Ridge" and "Period-double" phases disappear

delamination/buckling of folds



"Ridge" and "Period-double" phases disappear

delamination/buckling of wrinkles



Q. Wang and X. Zhao, Sci. Rep. **5**, 8887 (2015)

Compression of thin membranes on elastic substrates with finite adhesion

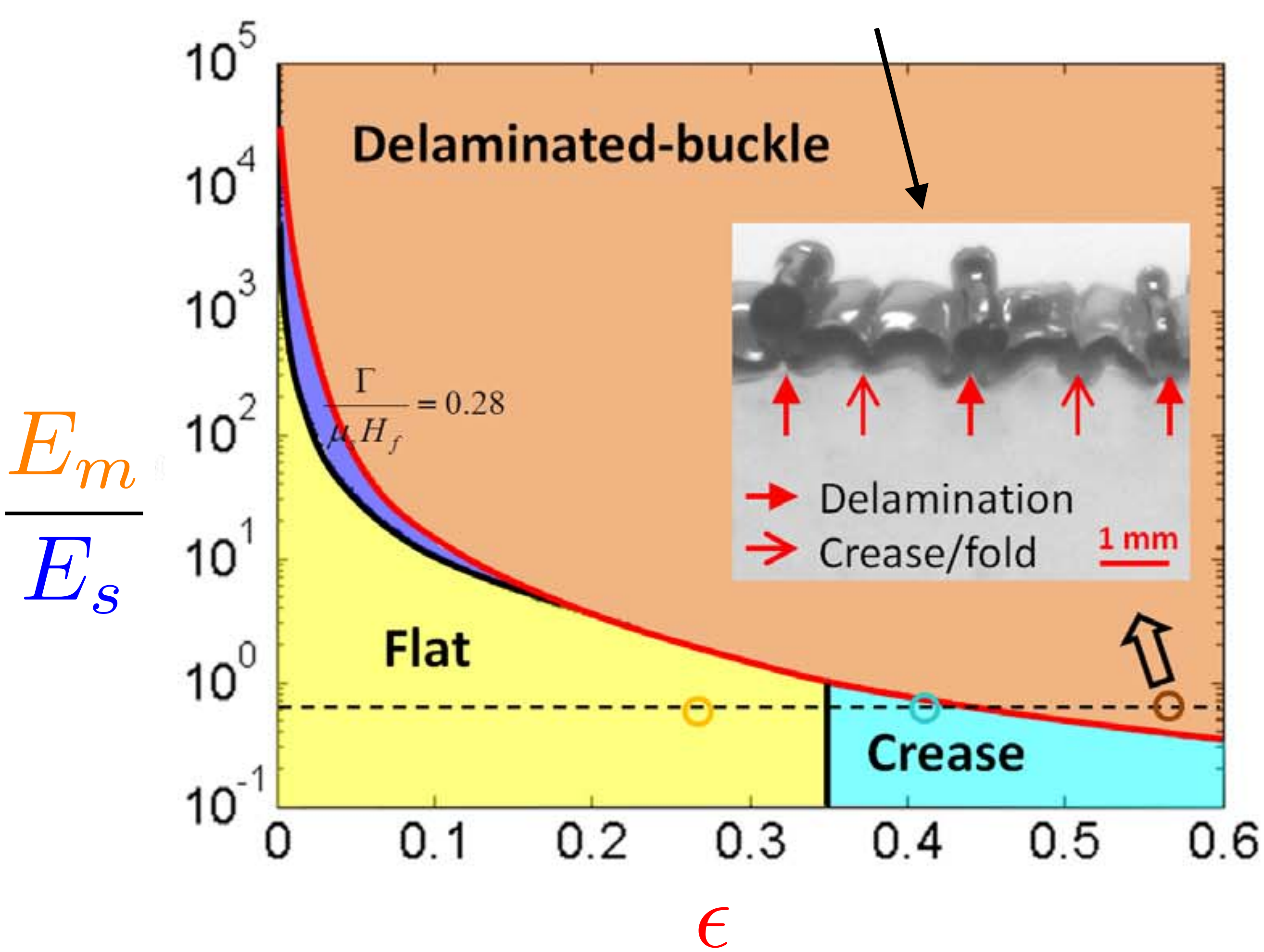
Weak adhesion

$$\frac{\Gamma}{E_s d} = 0.28$$

$$\frac{\Gamma}{E_s d} = 0.13$$

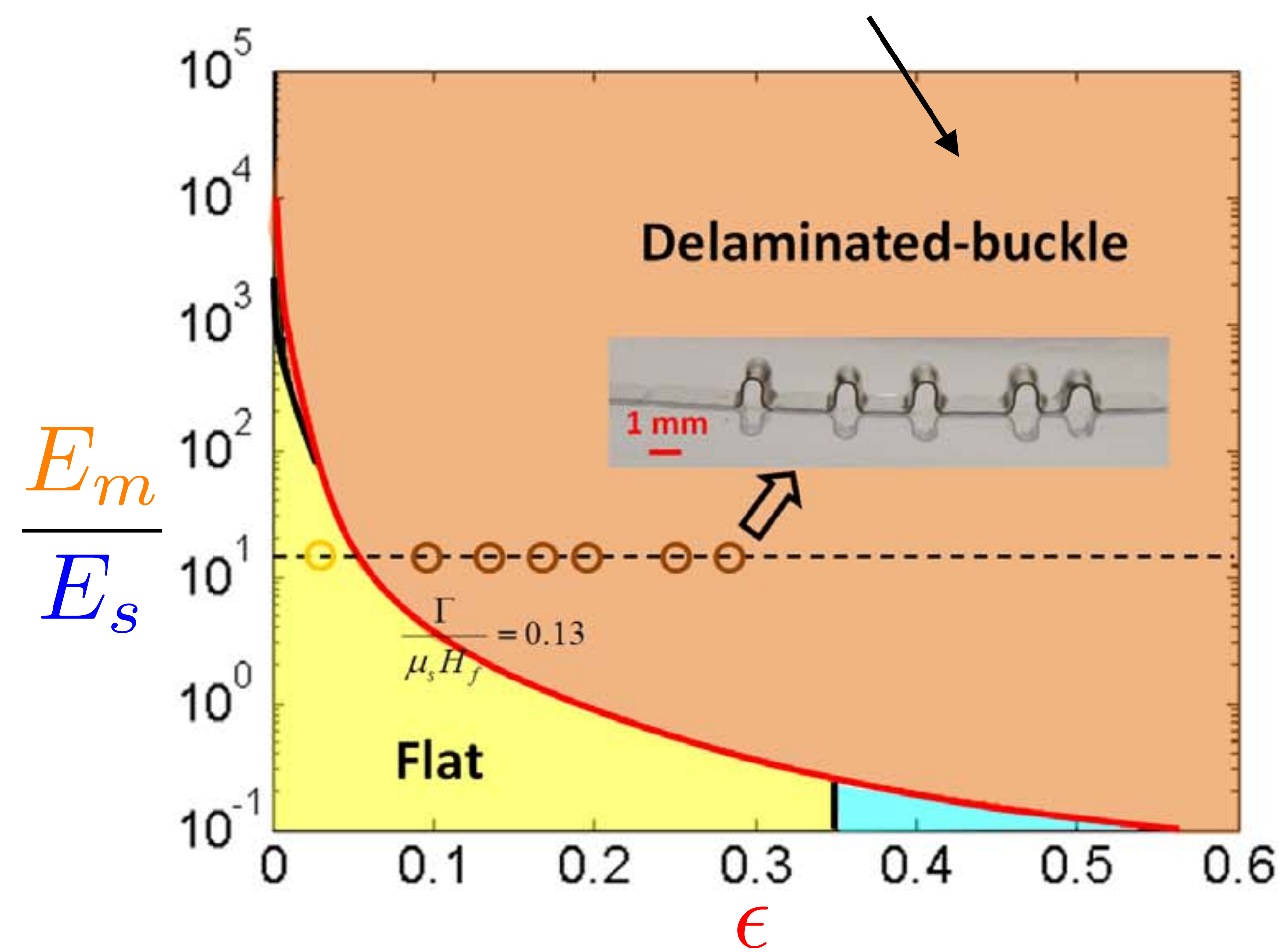
"Ridge", "Period-double" and "Fold" phases disappear

delamination/buckling of creases



delaminatied/buckled phase almost completely takes over the other phases

delamination/buckling of flat phase



Q. Wang and X. Zhao, Sci. Rep. **5**, 8887 (2015)