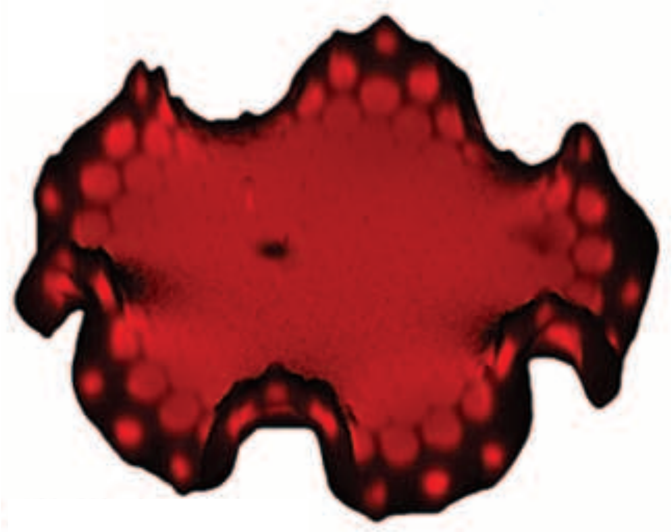
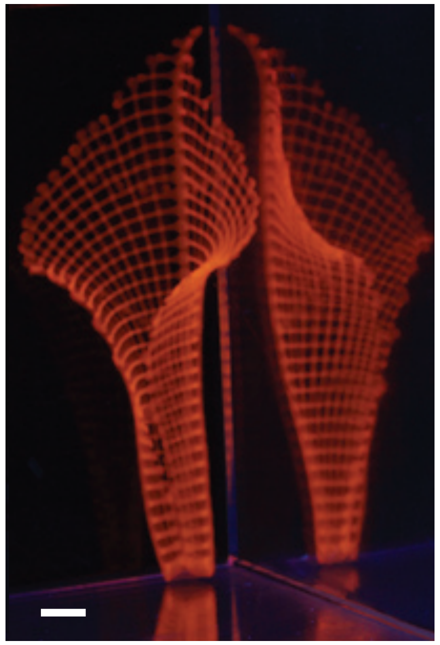
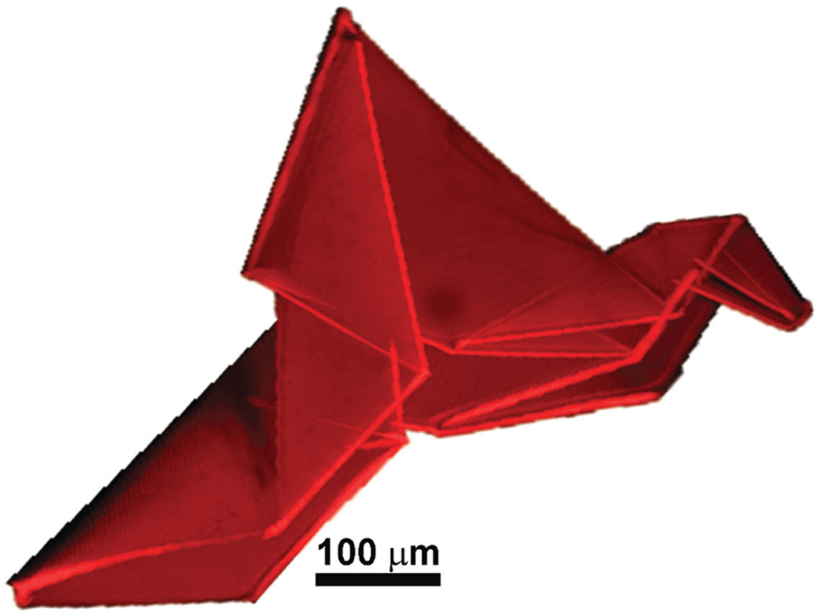
MAE 545: Lecture 10 (3/13)

Shapes of swelling sheets



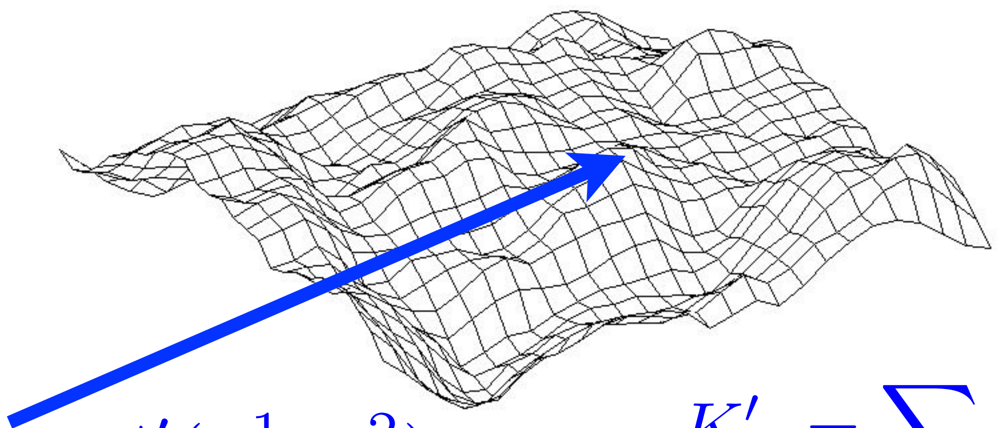


Self-folding origami



Mechanics of growing sheets

Growth defines preferred metric tensor g_{ij},
and preferred curvature tensor K_{ij}.



$\mathbf{0}$ $\vec{r}\,'(x^1, x^2)$

$$g'_{ij} = \frac{\partial \vec{r}\,'}{\partial x^i} \cdot \frac{\partial \vec{r}\,'}{\partial x^j}$$

$$K'_{ij} = \sum_k \left(g'^{-1}\right)_{ik} \left(\vec{n}' \cdot \frac{\partial^2 \vec{r}\,'}{\partial x^k \partial x^j}\right)$$

strain tensors

$$u_{ij} = \frac{1}{2} \sum_k \left(g^{-1}\right)_{ik} \left(g'_{kj} - g_{kj}\right)$$

$$b_{ij} = K'_{ij} - K_{ij}$$

The equilibrium membrane shape $\vec{r}\,'(x^1, x^2)$
corresponds to the minimum of elastic energy:

$$U = \int \left(\sqrt{g}\, dx^1 dx^2\right) \left[\frac{1}{2}\lambda \left(\sum_i u_{ii}\right)^2 + \mu \sum_{i,j} u_{ij} u_{ji} + \frac{1}{2}\kappa \left(\mathrm{tr}(b_{ij})\right)^2 + \kappa_G \det(b_{ij})\right]$$

Growth can independently tune the metric tensor g_{ij} and the curvature tensor K_{ij}, which may not be compatible with any surface shape that would produce zero energy cost!

Zero energy shape exists only when preferred metric tensor g_{ij} and preferred curvature tensor K_{ij} satisfy Gauss-Codazzi-Mainardi relations!

Mechanics of growing membranes

One of the Gauss-Codazzi-Mainardi equations (Gauss's Theorema Egregium) relates the Gauss curvature to metric tensor

$$\det(K'_{ij}) = \mathcal{F}(g'_{ij})$$

The equilibrium membrane shape $\vec{r}\,'(x^1, x^2)$ corresponds to the minimum of elastic energy:

$$U = \int (\sqrt{g}dx^1dx^2) \left[\frac{1}{2}\lambda\left(\sum_i u_{ii}\right)^2 + \mu\sum_{i,j} u_{ij}u_{ji} + \frac{1}{2}\kappa\left(\mathrm{tr}(b_{ij})\right)^2 + \kappa_G \det(b_{ij})\right]$$

scaling with membrane thickness d

$$\lambda, \mu \sim Ed$$

$$\kappa, \kappa_G \sim Ed^3$$

For very thin membranes the equilibrium shape matches the preferred metric tensor to avoid stretching, compressing and shearing. This also specifies the Gauss curvature!

$$g'_{ij} = g_{ij}$$

$$\det(K'_{ij}) = \mathcal{F}(g_{ij})$$

Shaping of gel membranes by differential shrinking

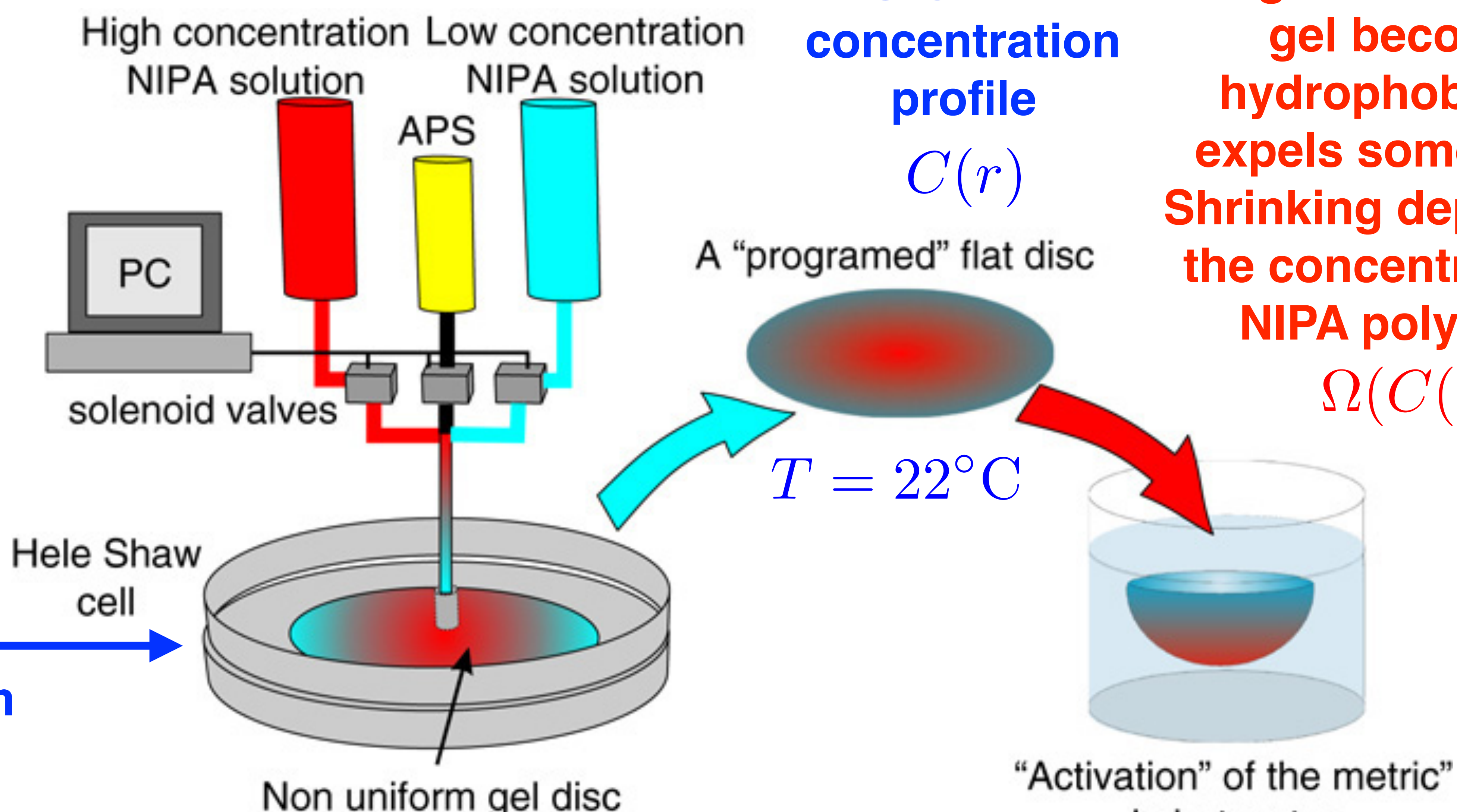
Computer software controls valves to inject a predefined time depend concentration of NIPA polymers in water solution.

Frozen NIPA concentration profile

$C(r)$

At higher temperatures gel becomes hydrophobic and expels some water. Shrinking depends on the concentration of NIPA polymers.

$\Omega(C(r))$

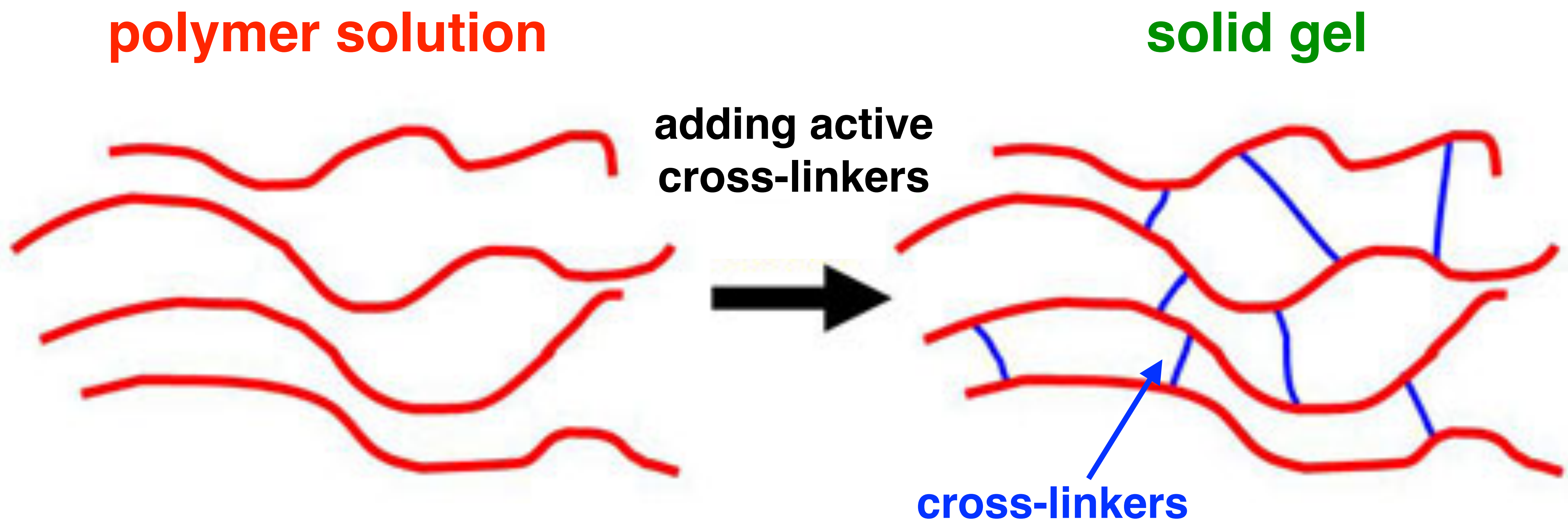


thickness 0.25 or 0.5 mm

Active cross-linkers (APS) polymerize the polymer solution within one minute, before polymers get a chance to diffuse around.

E. Efrati et al., Physica D **235**, 29 (2007)

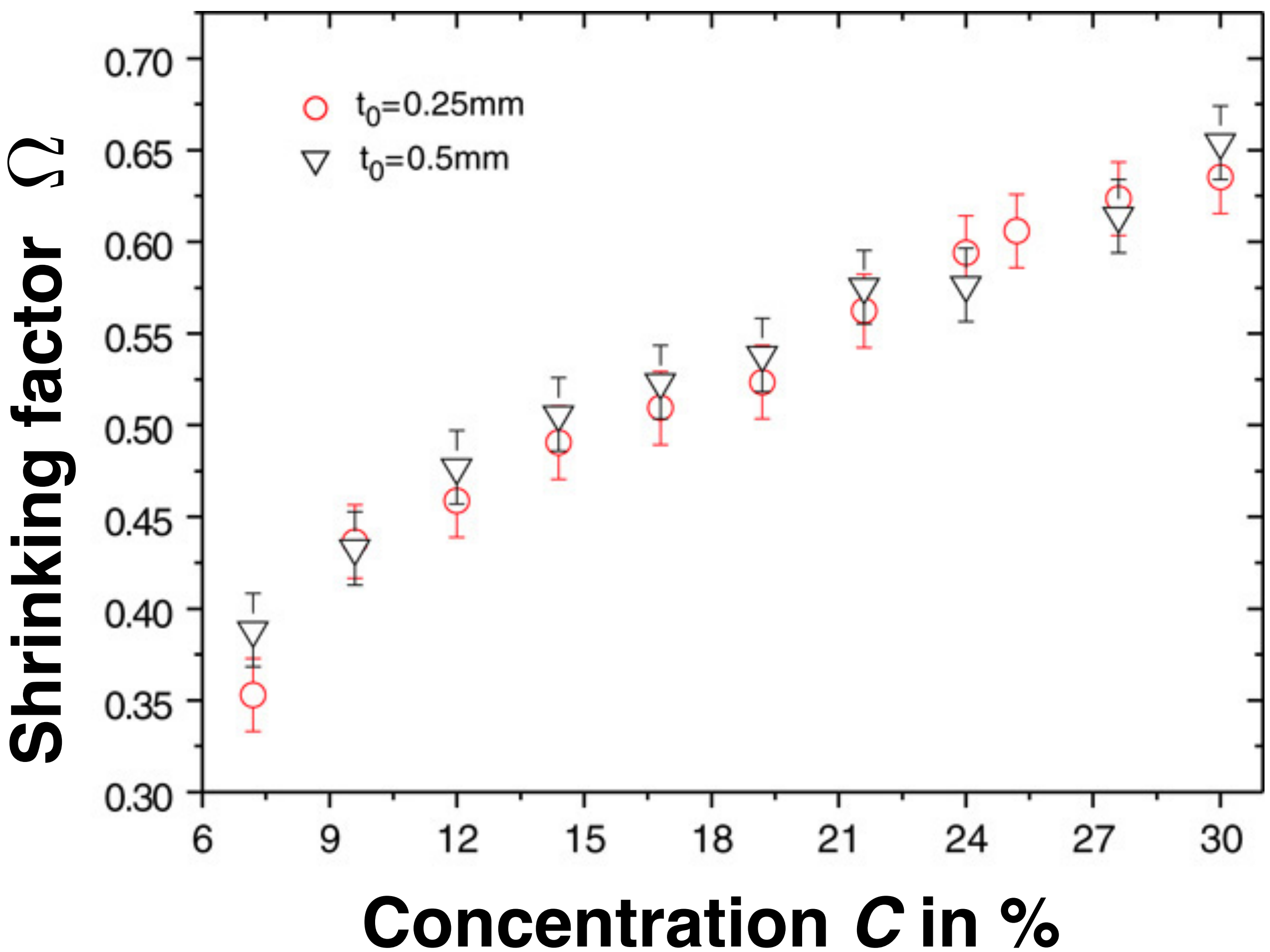
Cross-linking of polymers result in a solid gel



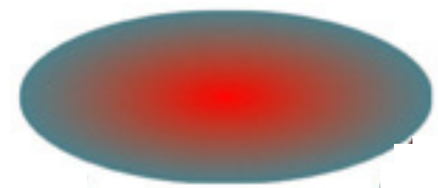
Note: some cross-linkers can be chemically activated by UV light exposure. Duration of UV light exposure controls the degree of cross-linking and therefore the Young's modulus *E* for gels.

Shaping of gel membranes by differential shrinking

Shrinking of gels at T=45°C



$T = 22°\mathrm{C}$

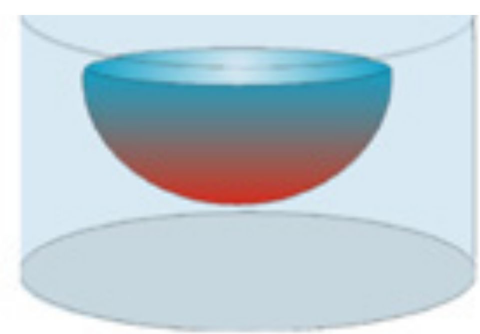


$C(r)$

$$g_{ij} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$$

Shrinking modifies metric

$T = 45°\mathrm{C}$



$$g_{ij} = \begin{pmatrix} \Omega(r), & 0 \\ 0, & \Omega(r) \end{pmatrix}$$

locally isotropic shrinking

Note that shrinking is uniform throughout the gel thickness and there is no preferred curvature

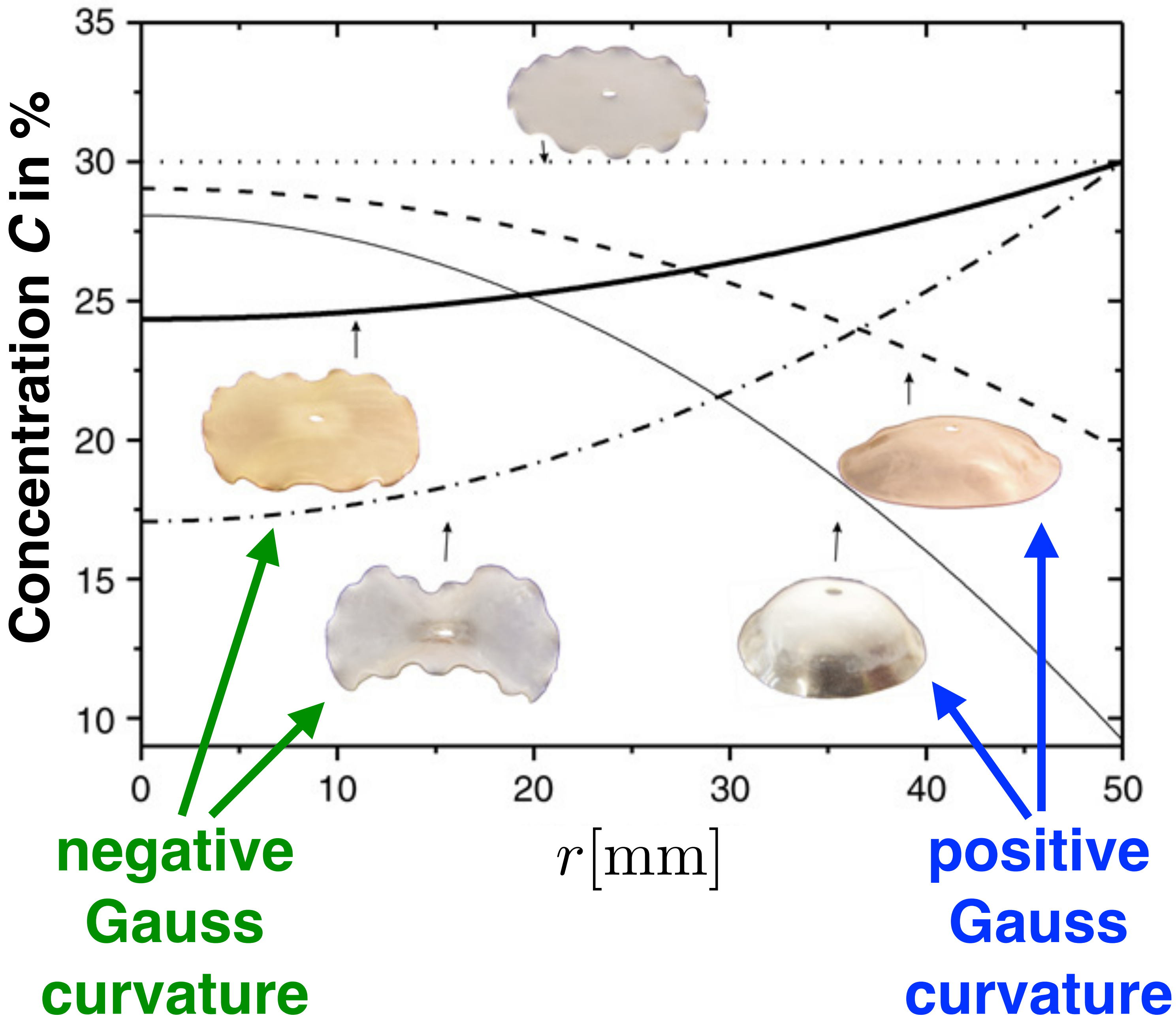
$$K_{ij} = 0$$

For thin membranes the target Gauss curvature is

$$\det(K'_{ij}(r)) = -\frac{\nabla^2(\ln \Omega(r))}{2\Omega(r)}$$

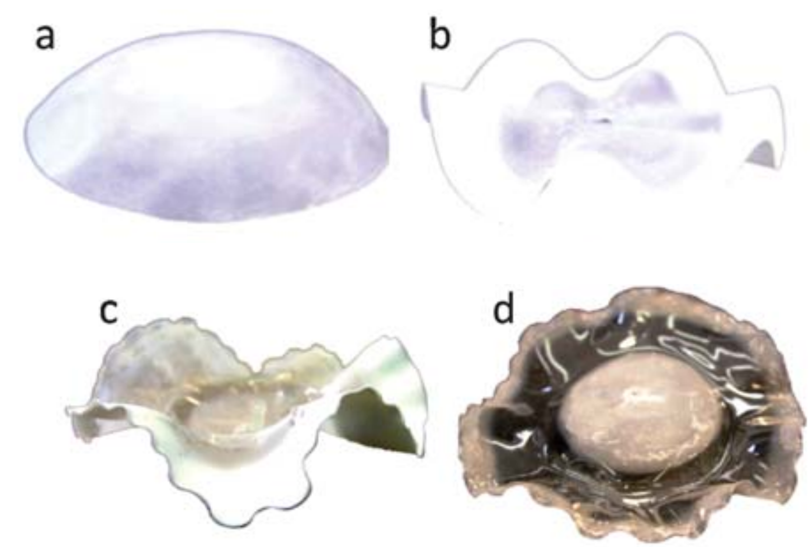
E. Efrati et al., Physica D **235**, 29 (2007)

Shaping of gel membranes by differential shrinking

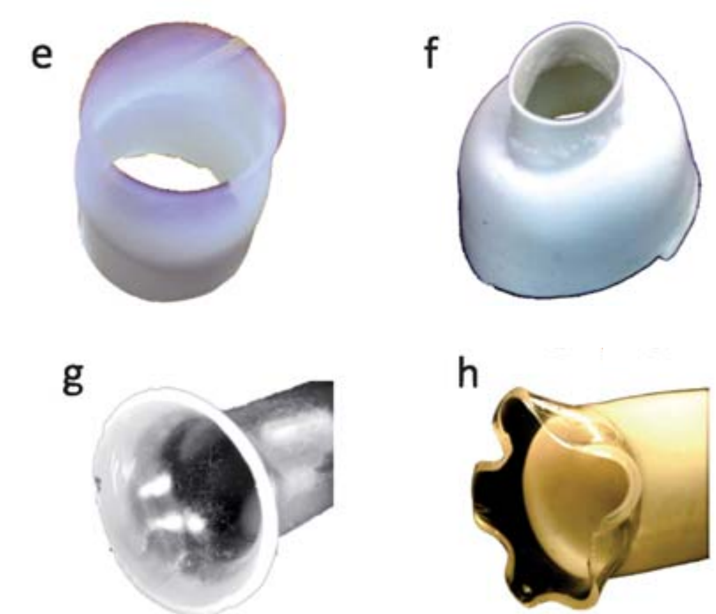


E. Efrati et al., Physica D 235, 29 (2007)

Shrinking of sheets



Shrinking of tubes

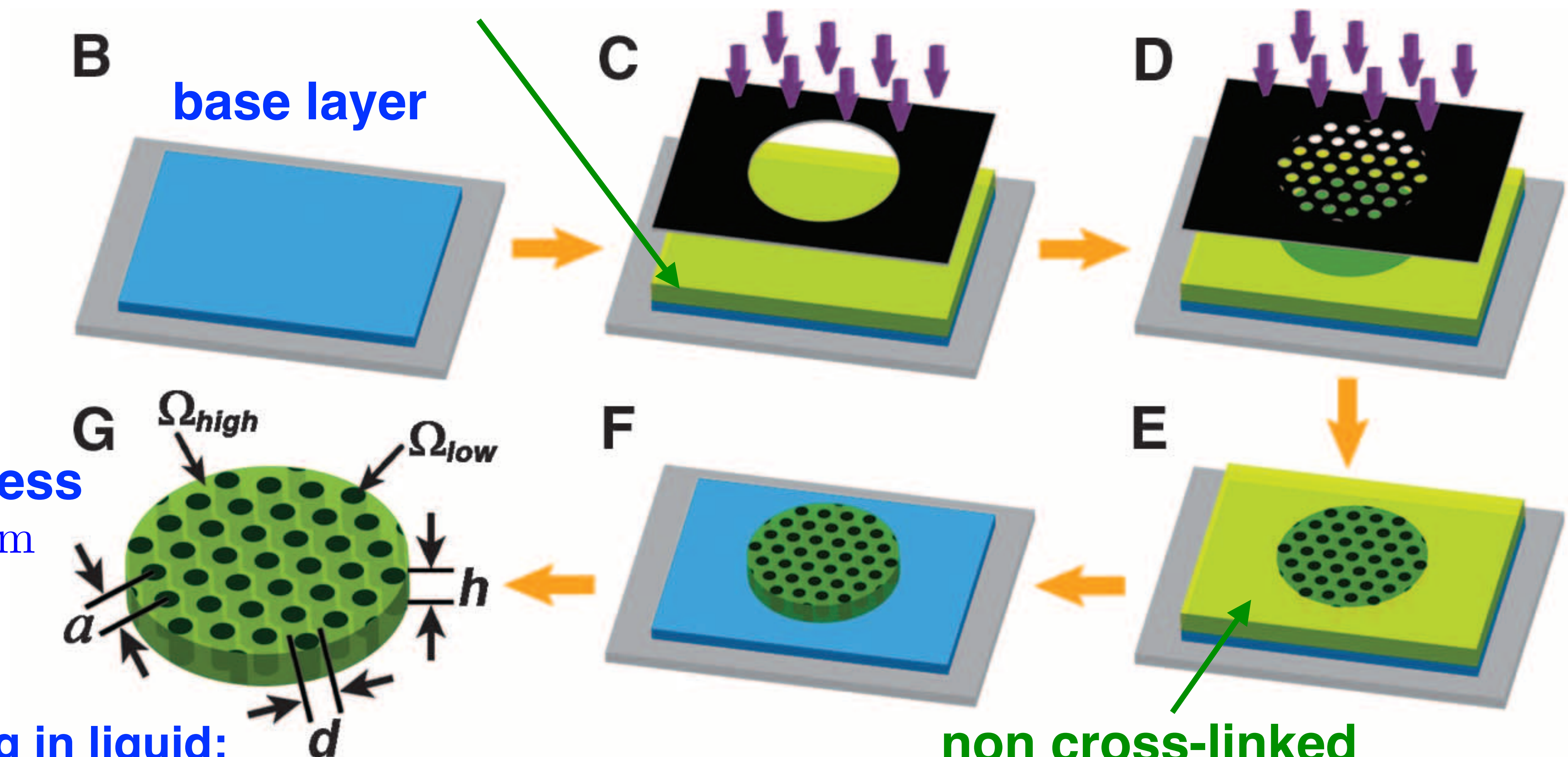


E. Sharon and E. Efrati, Soft Matter **6**, 5693 (2010)

Shaping of gel membrane properties by lithography

thin film of polymer solution with premixed inactive cross-linkers

UV light activates cross-linkers. Time of UV light exposure determines the degree of polymer cross-linking.



thickness
7-17μm

non cross-linked polymer solution is washed away

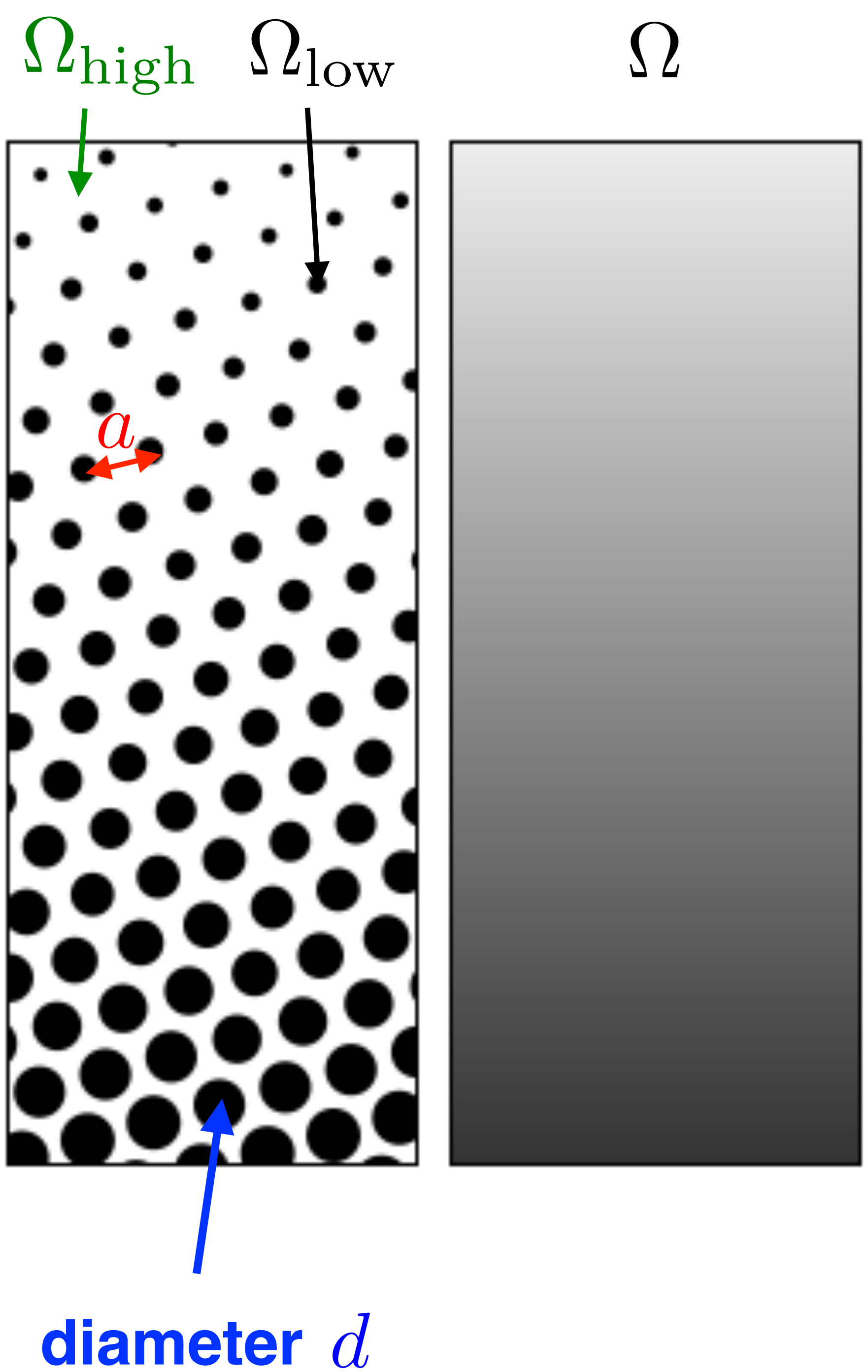
swelling in liquid:

more cross-linked regions swell less Ω_{low}

less cross-linked regions swell more Ω_{high}

J. Kim et al., Science **335**, 1201 (2012)

Halftoning

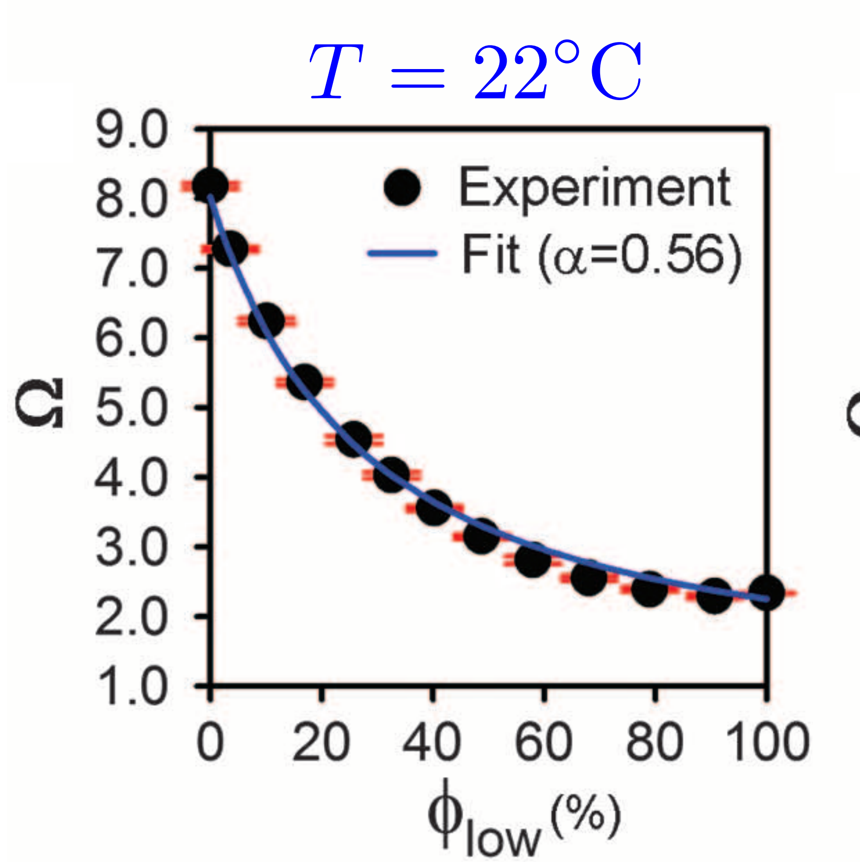


local area fraction of the low swelling regions

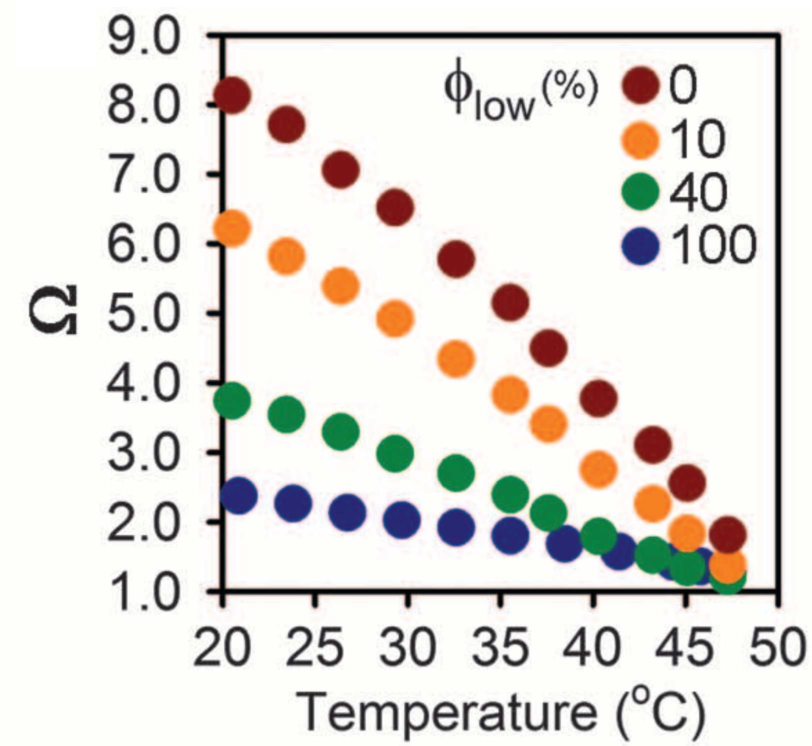
$$\phi_{\text{low}} = \frac{\Delta A_{\text{low}}}{\Delta A_{\text{low}} + \Delta A_{\text{high}}} = \frac{\pi}{2\sqrt{3}} \left(\frac{d}{a}\right)^2$$

Effective swelling Ω can be estimated from local force balance as

$$\frac{\phi_{\text{low}} + \alpha(1-\phi_{\text{low}})}{\Omega^{1/2}} = \frac{\phi_{\text{low}}}{\Omega_{\text{low}}^{1/2}} + \frac{\alpha(1-\phi_{\text{low}})}{\Omega_{\text{high}}^{1/2}}$$

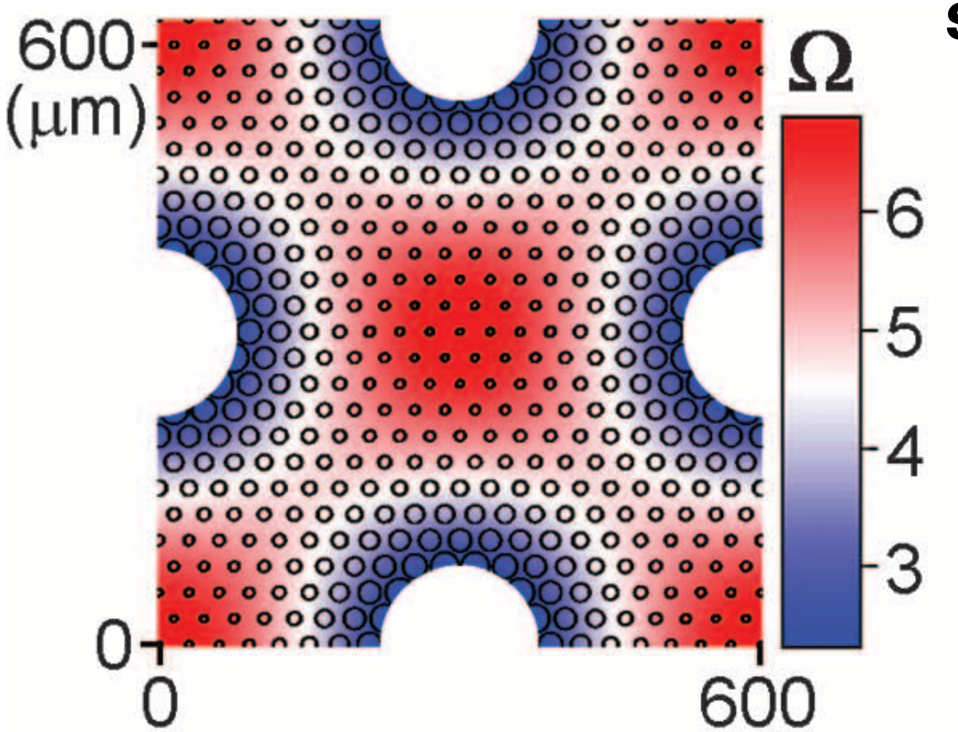


swelling depends on *T*

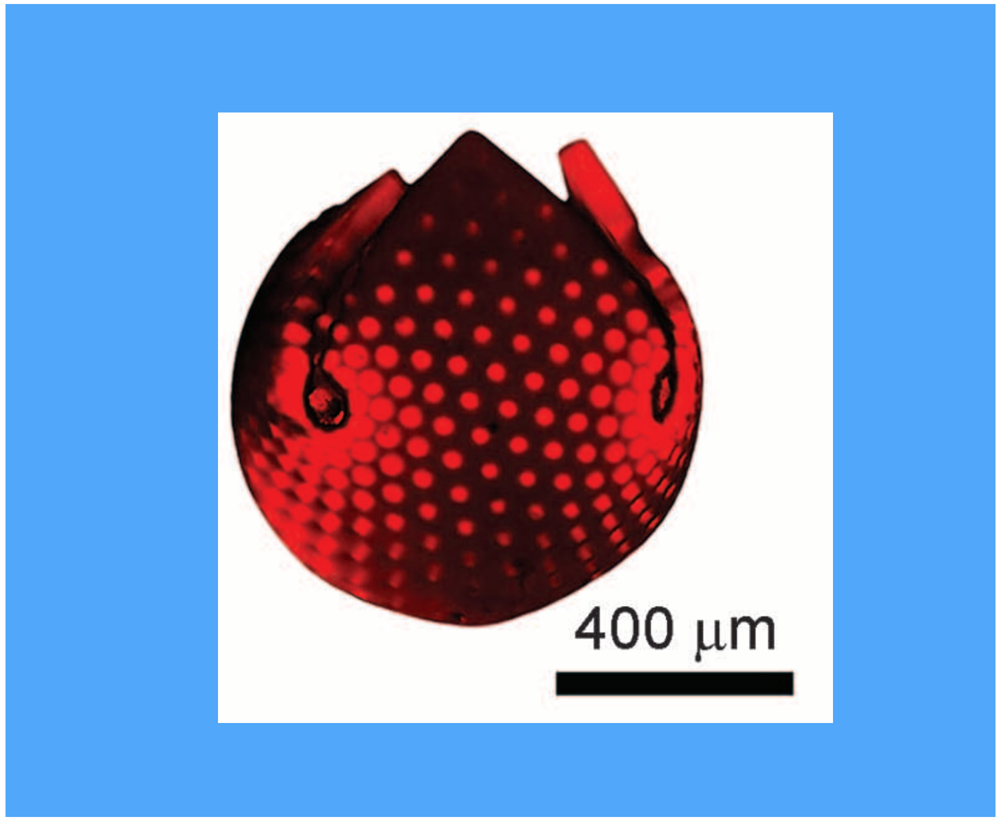


J. Kim et al., Science **335**, 1201 (2012)

Shaping of gel membrane properties by halftone lithography



Differential swelling in liquid deforms square membrane to a closed sphere



metric tensor

$$g_{ij} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$$

locally isotropic swelling

$$g_{ij} = \begin{pmatrix} \Omega(x,y), & 0 \\ 0, & \Omega(x,y) \end{pmatrix}$$

For thin membranes the target Gauss curvature is

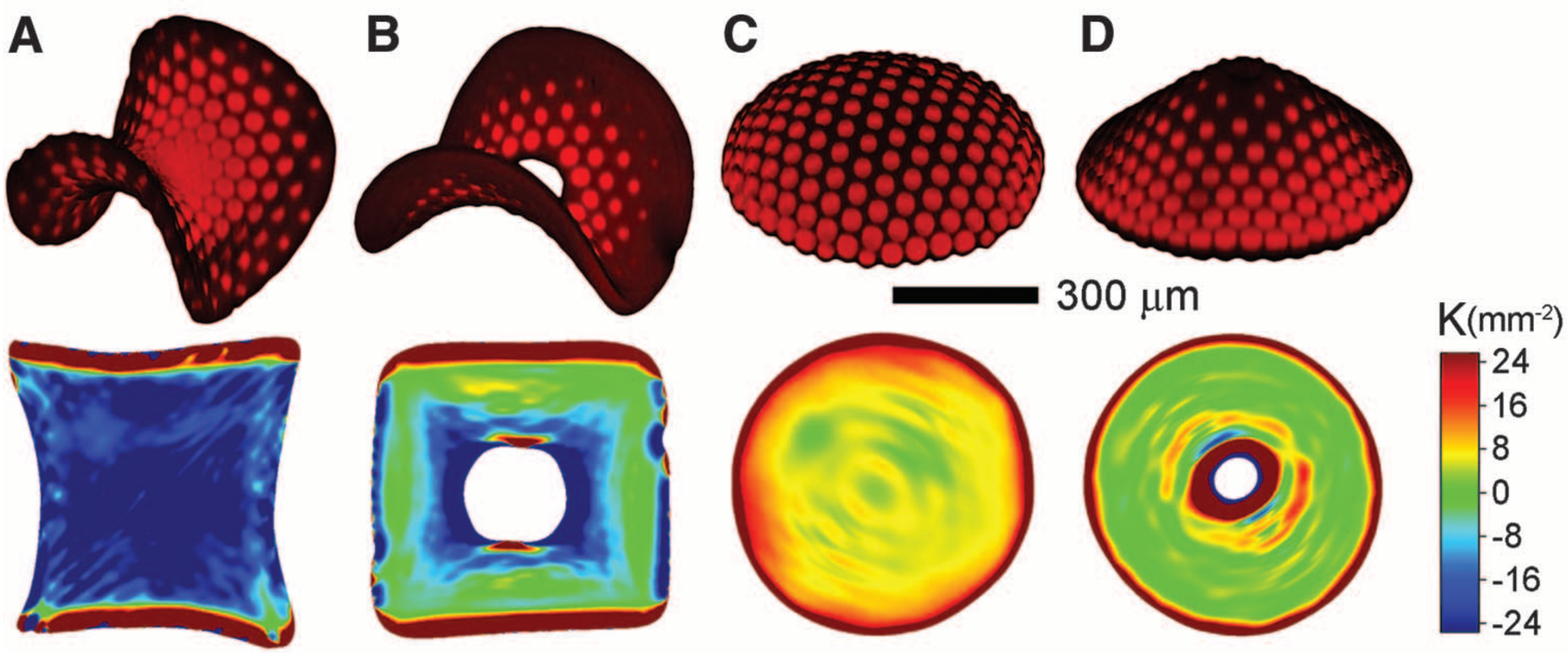
$$\det(K'_{ij}(x,y)) = -\frac{\nabla^2(\ln\Omega(x,y))}{2\Omega(x,y)}$$

Inverse problem can be solved with conformal maps.

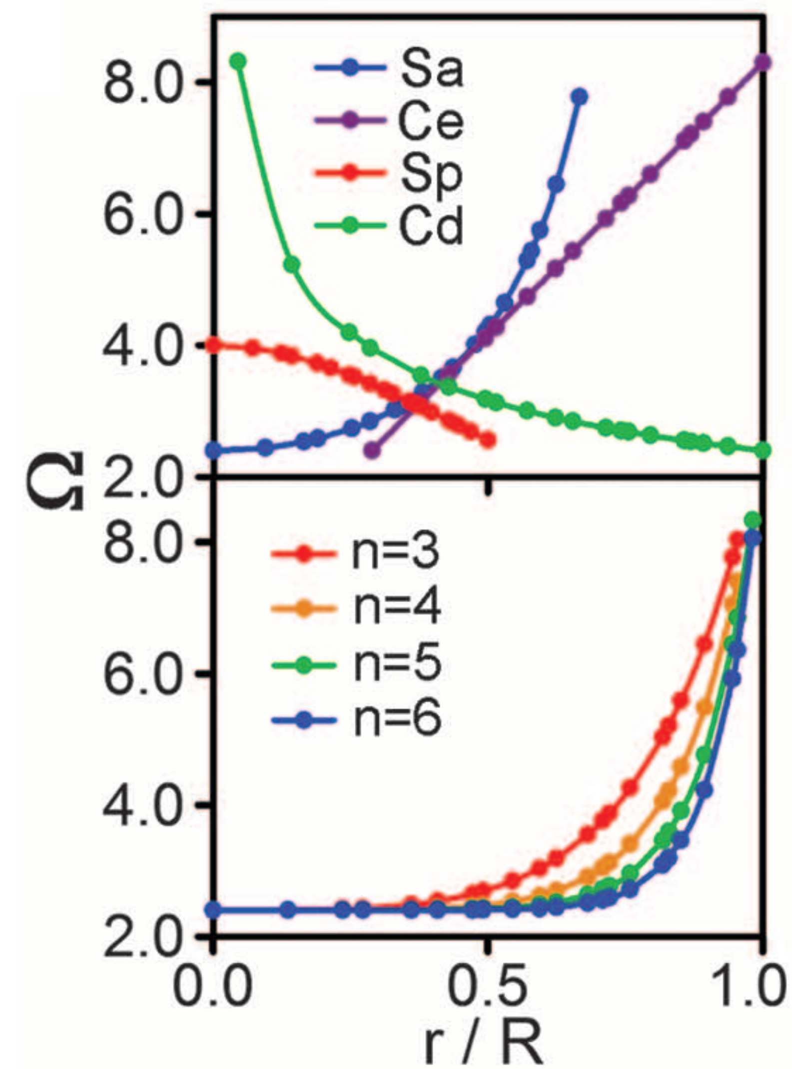
J. Kim et al., Science **335**, 1201 (2012)

Shaping of gel membrane properties by halftone lithography

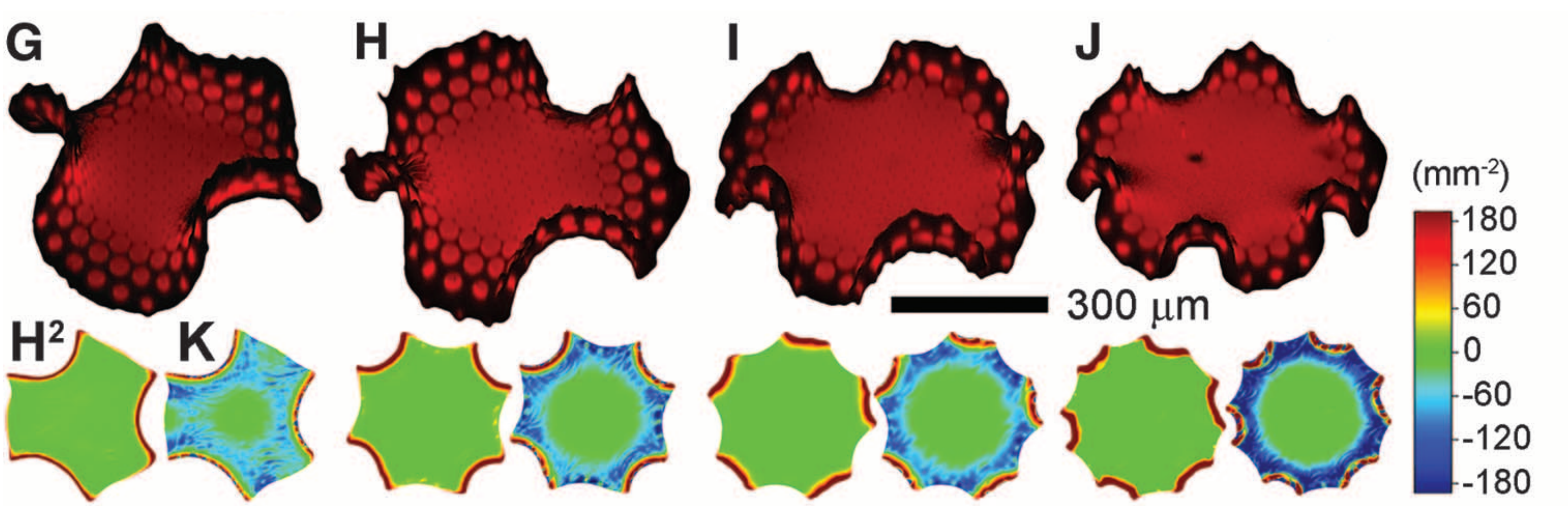
saddle (Sa) | cone with excess angle (Ce) | spherical cap (Sp) | cone with deficit angle (Cd)



swelling profiles



Enneper's minimal surfaces (H=0)

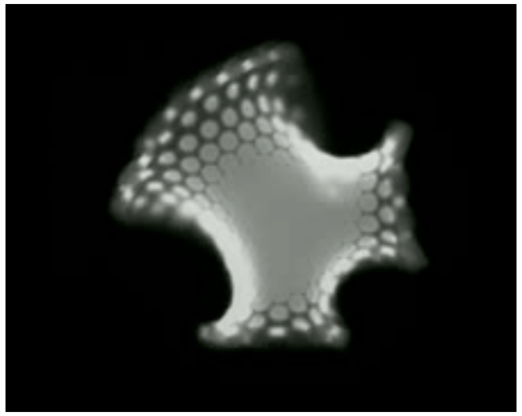


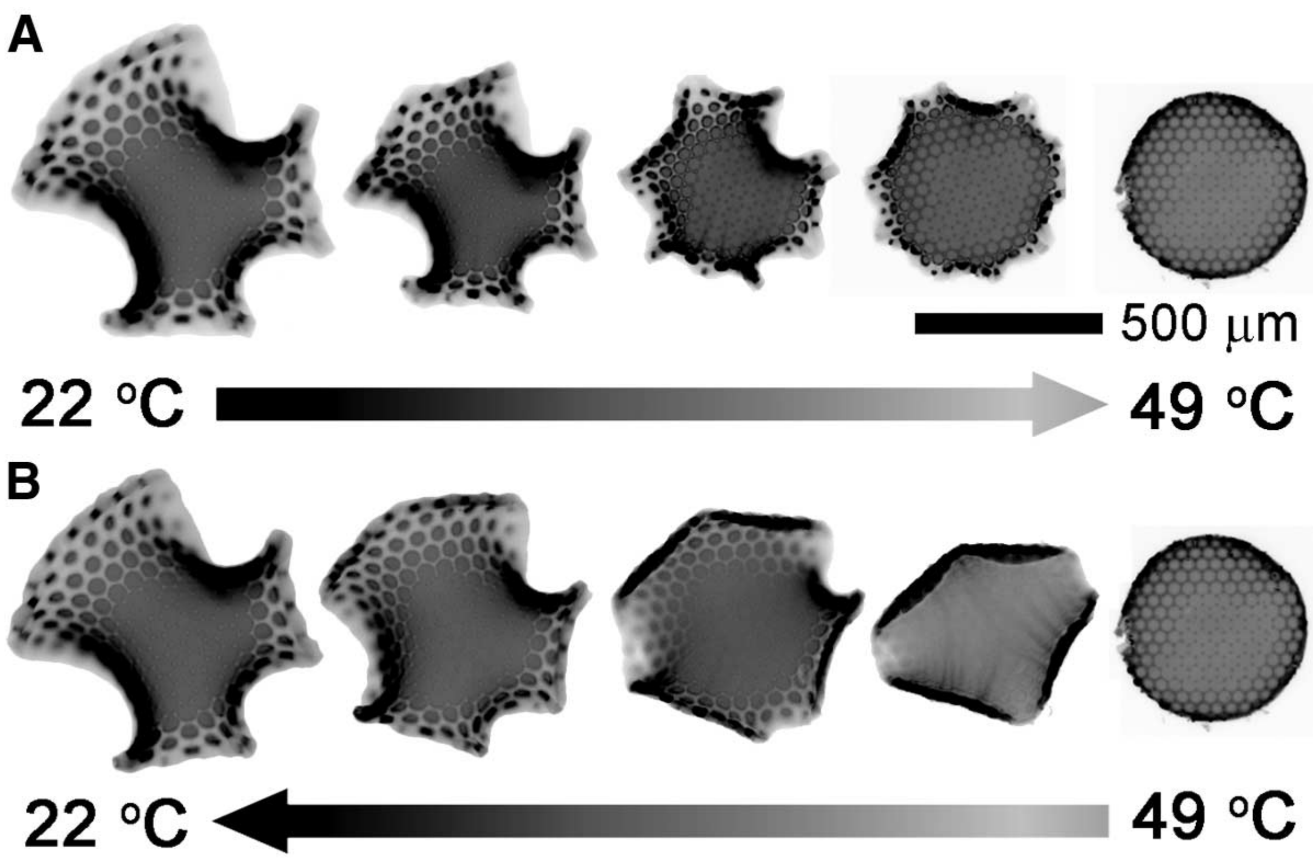
H - mean curvature

K - Gauss curvature

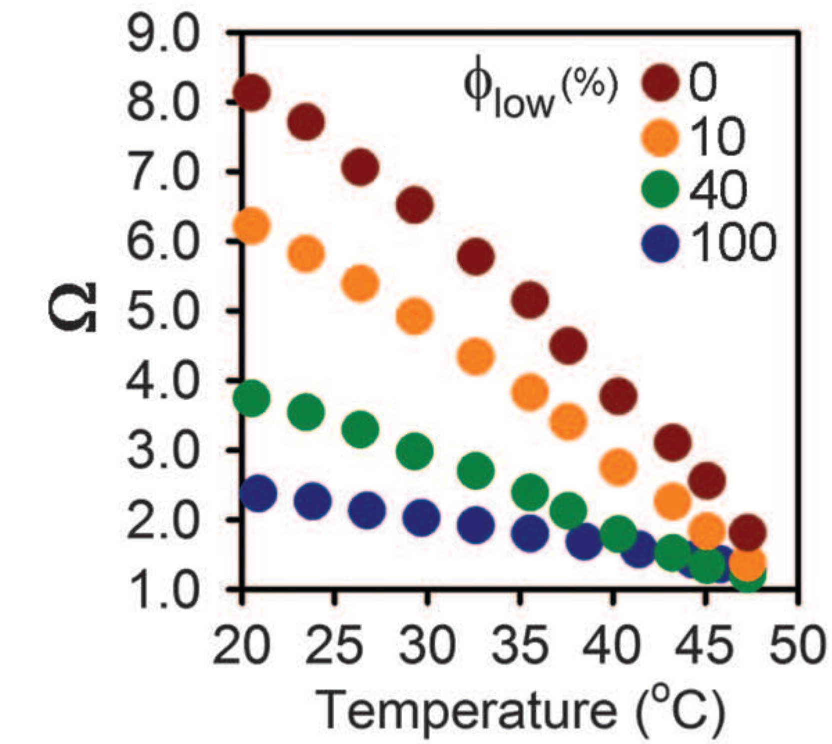
J. Kim et al., Science **335**, 1201 (2012)

Temperature controls swelling and thus the deformed shape





swelling depends on *T*

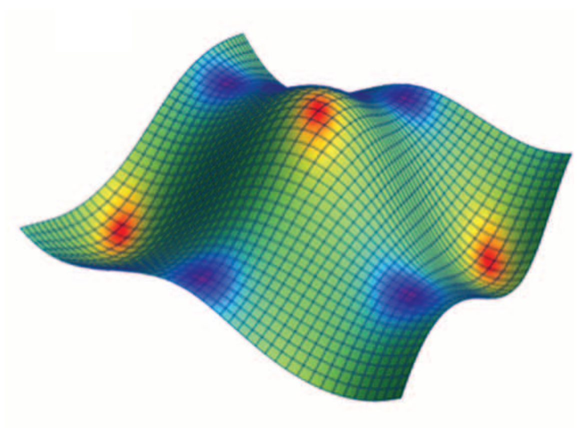


Note different intermediate shapes!
By slowly varying the temperature we stay in a local energy minimum!

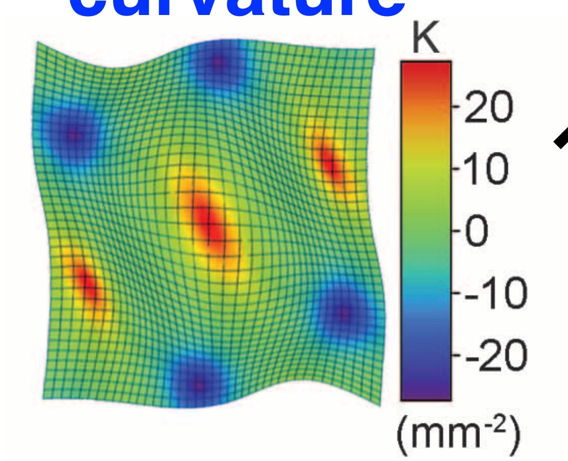
J. Kim et al., Science **335**, 1201 (2012)

Gaussian curvature does not uniquely specify the shape!

target shape

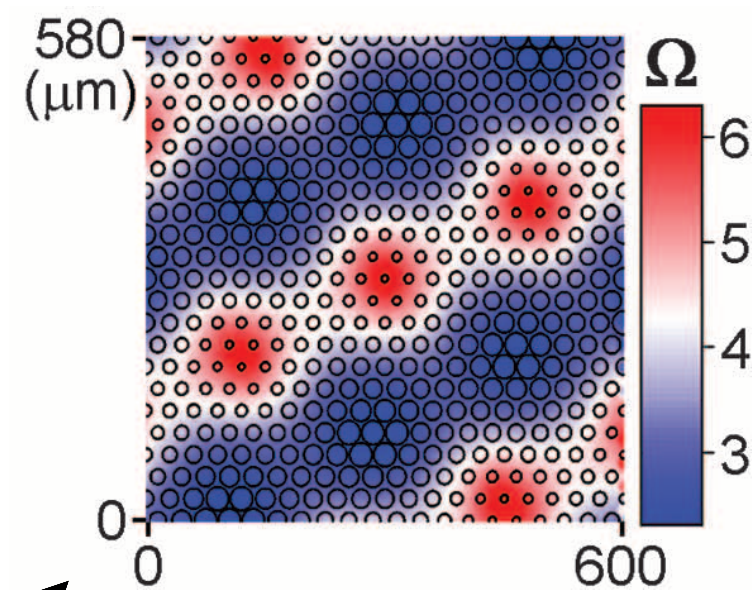


target Gauss curvature

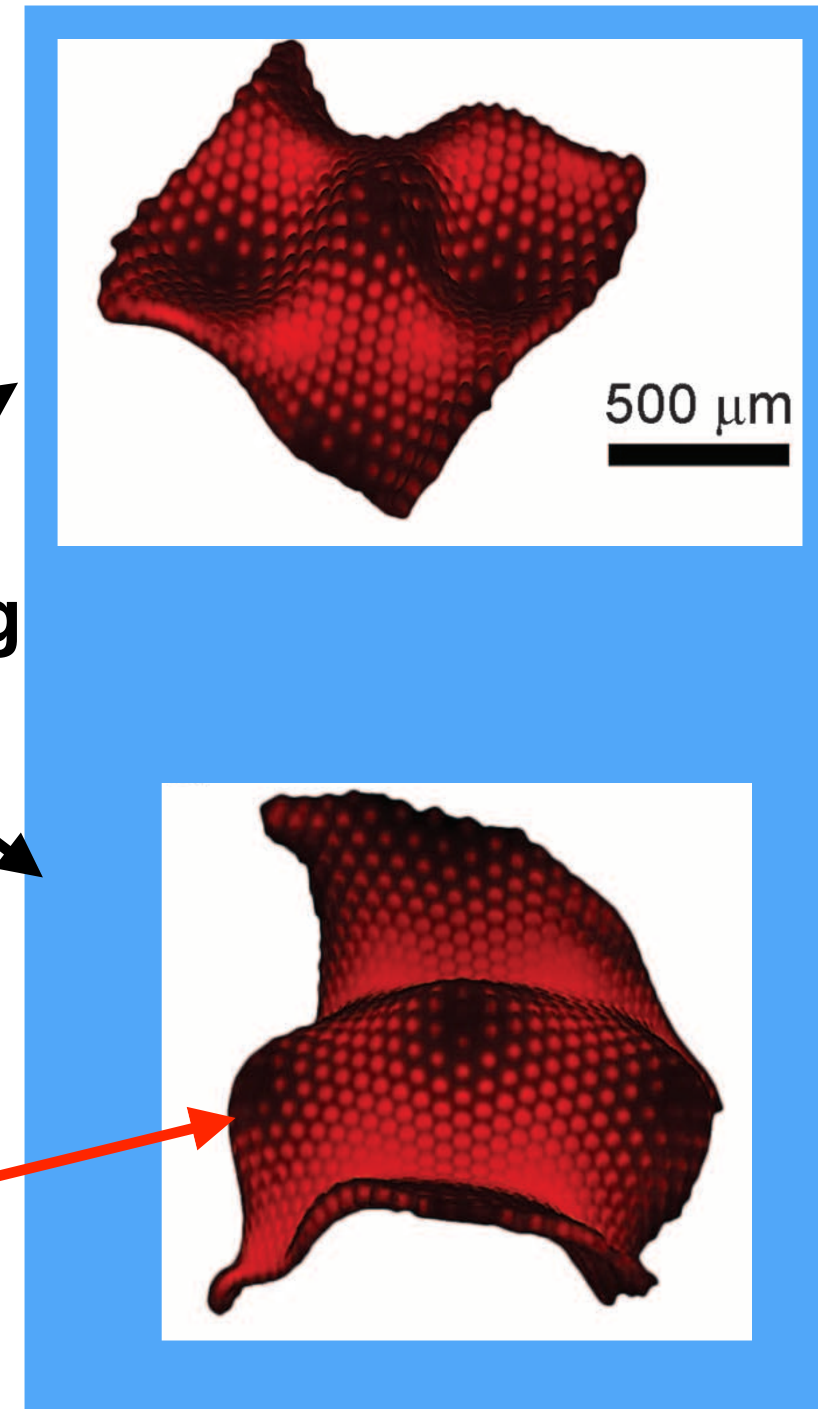


conformal map

swelling pattern



swelling

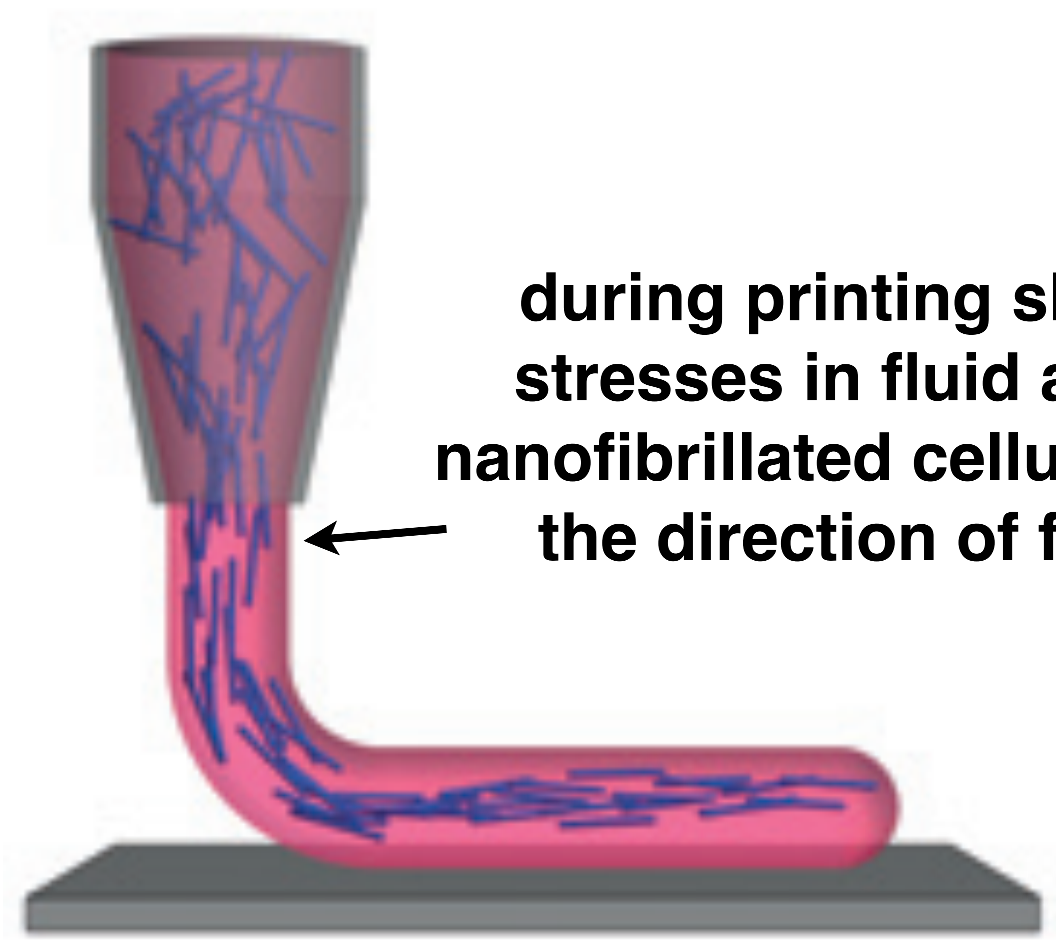


this bump buckled on the wrong side

J. Kim et al., Science **335**, 1201 (2012)

3D printing anisotropic hydrogels

3D printed solution includes polymers, inactive cross-linkers and nanofibrillated cellulose



during printing shear stresses in fluid align nanofibrillated cellulose in the direction of flow

This procedure produces anisotropic elastic material with Young's moduli:

direction of fibers $E_{\|} \sim 40\,\mathrm{kPa}$

orthogonal direction $E_{\perp} \sim 20\,\mathrm{kPa}$

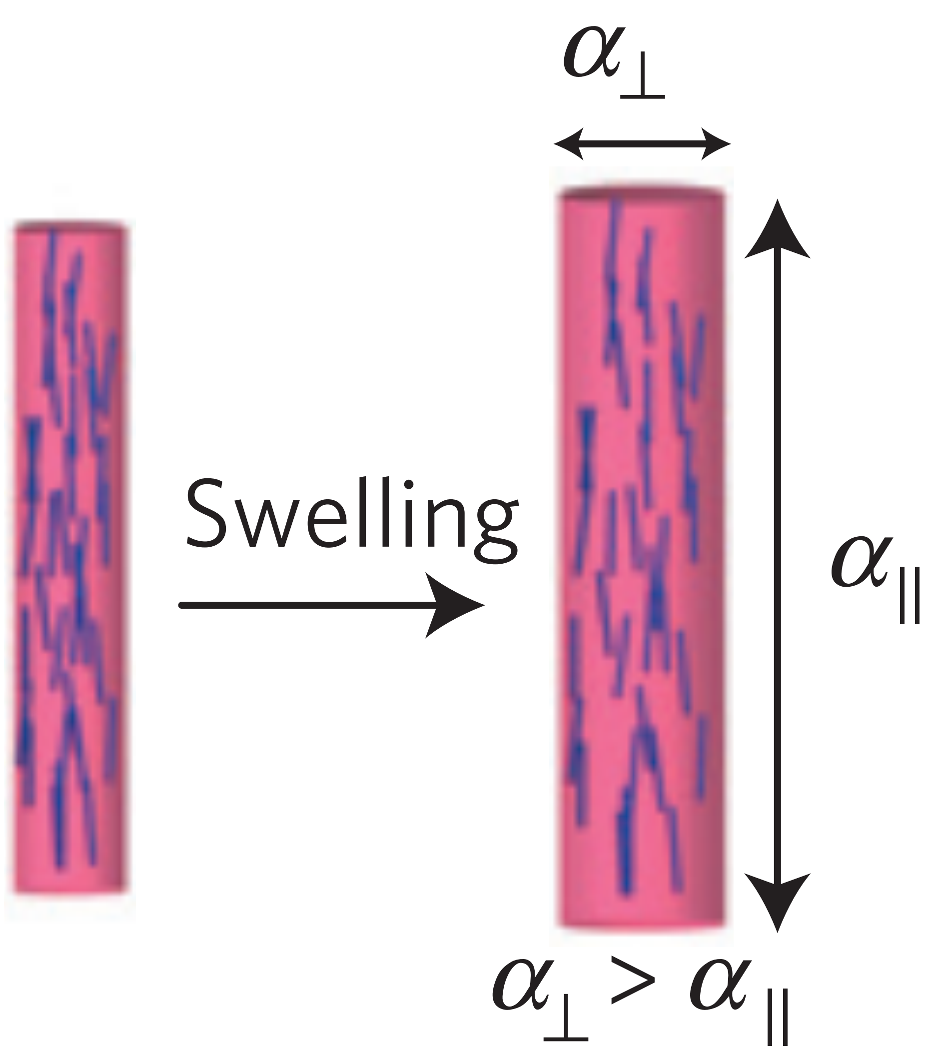
After printing the cross-linkers are activated with UV light.

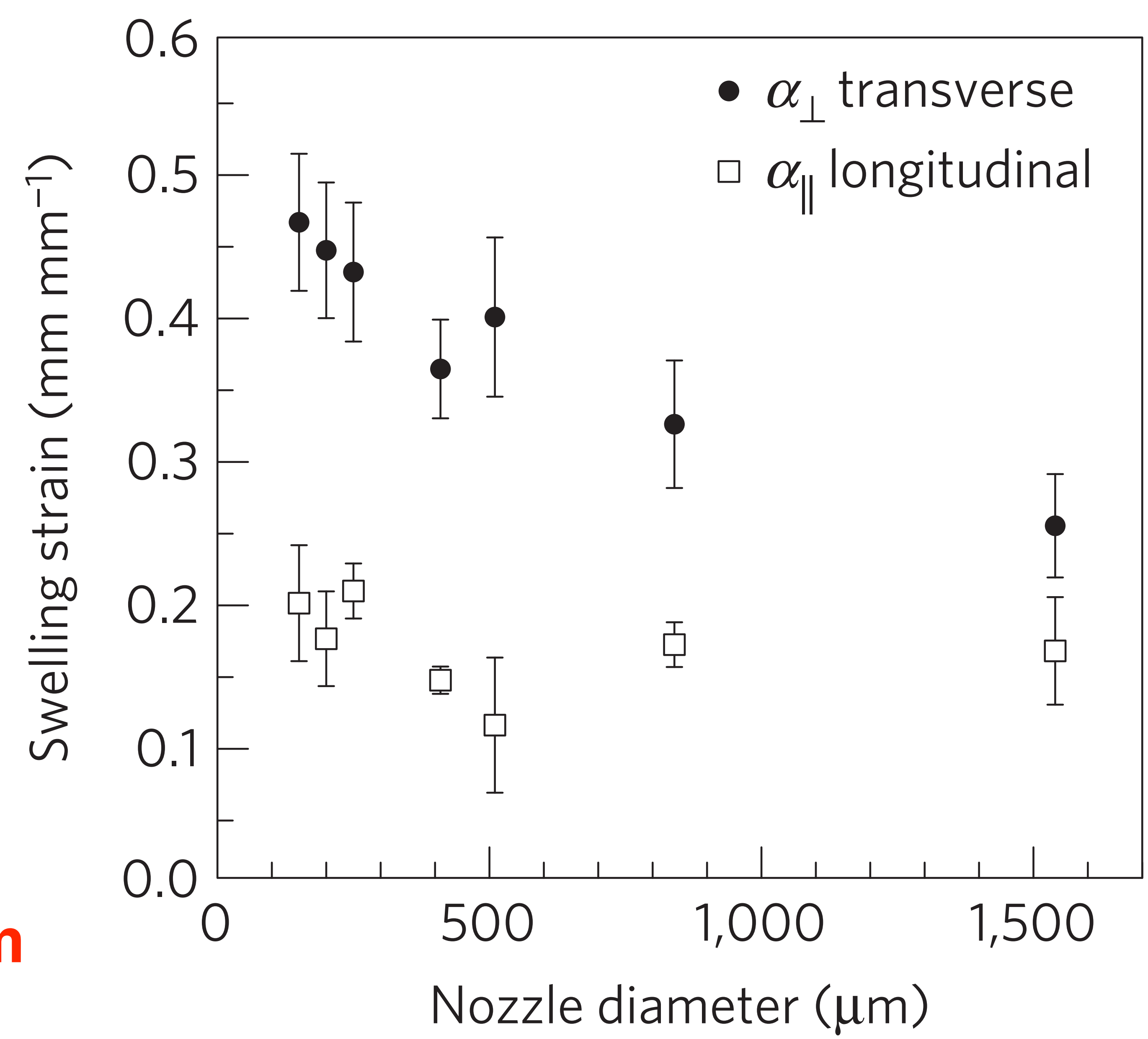
A. S. Gladman et al., Nat. Materials **15**, 413 (2016)

Anisotropic swelling of hydrogels

After the hydrogel is immersed in water it swells due to absorption of water.

Swelling is larger in direction orthogonal to nanofibrillated cellulose.



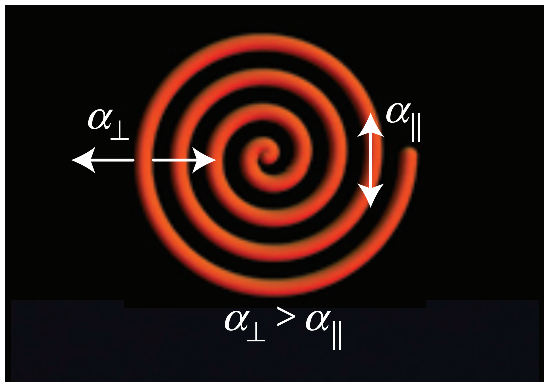


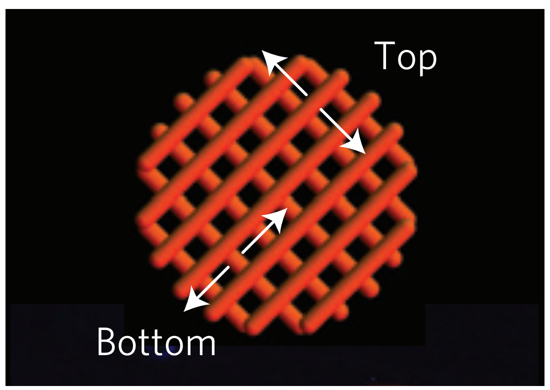
The inspiration for this came from plants, where the anisotropy in swelling upon changes in humidity is due to directed fibers.

A. S. Gladman et al., Nat. Materials **15**, 413 (2016)

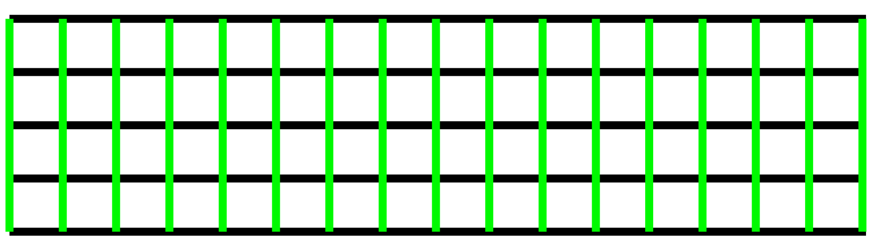
Shaping hydrogels via anisotropic swelling

3D printed patterns of hydrogels

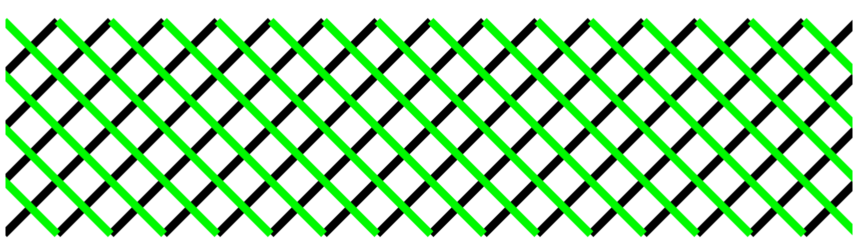




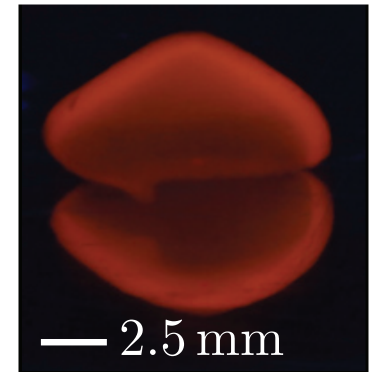
bottom layer **top layer**



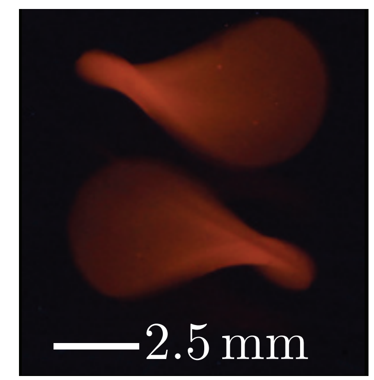
bottom layer **top layer**



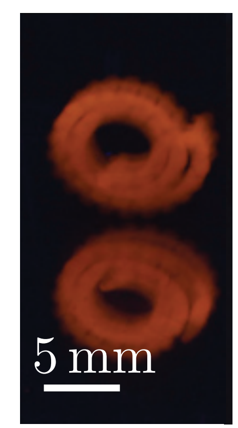
transformed shapes after swelling



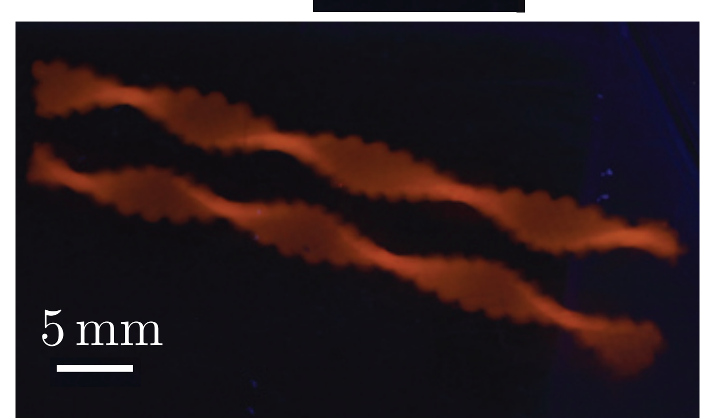
positive Gauss curvature



negative Gauss curvature



bending of long strip



twisting of long strip

(similar to drying seedpods)

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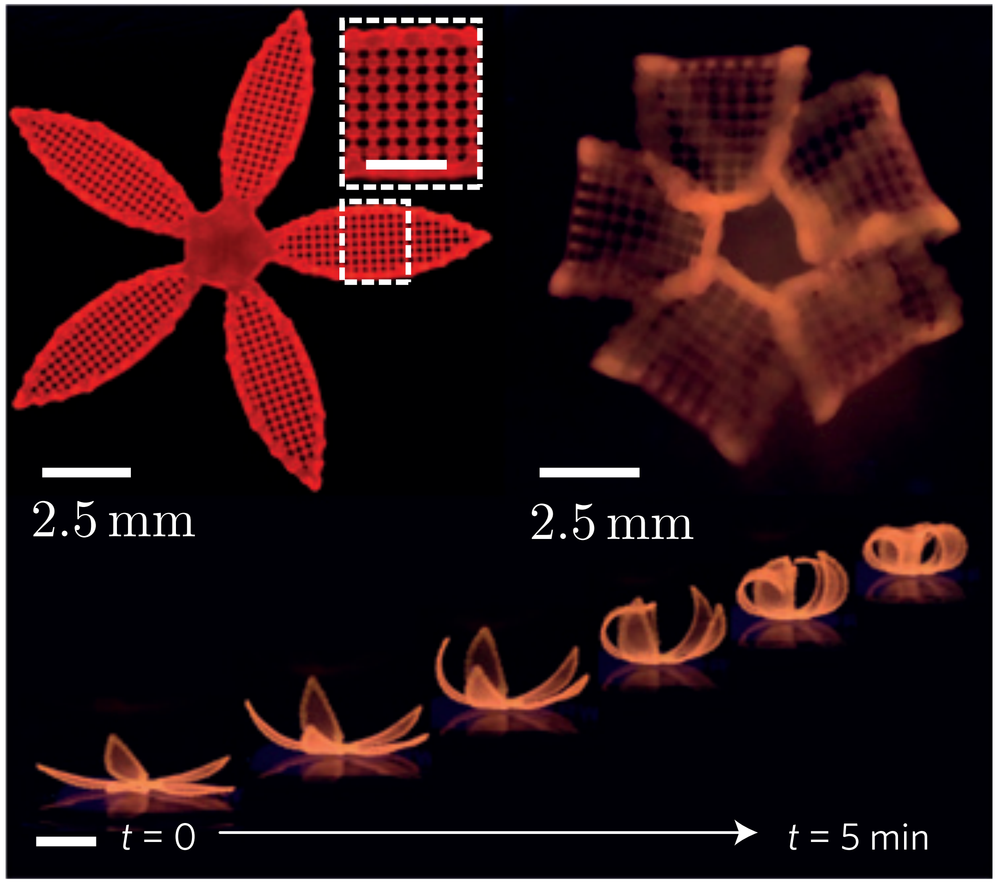
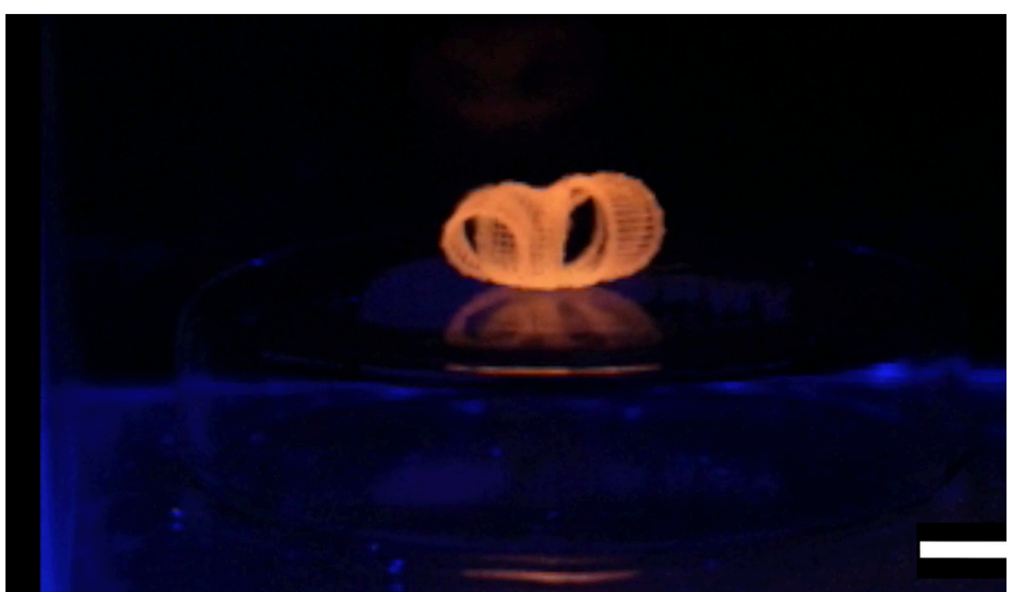
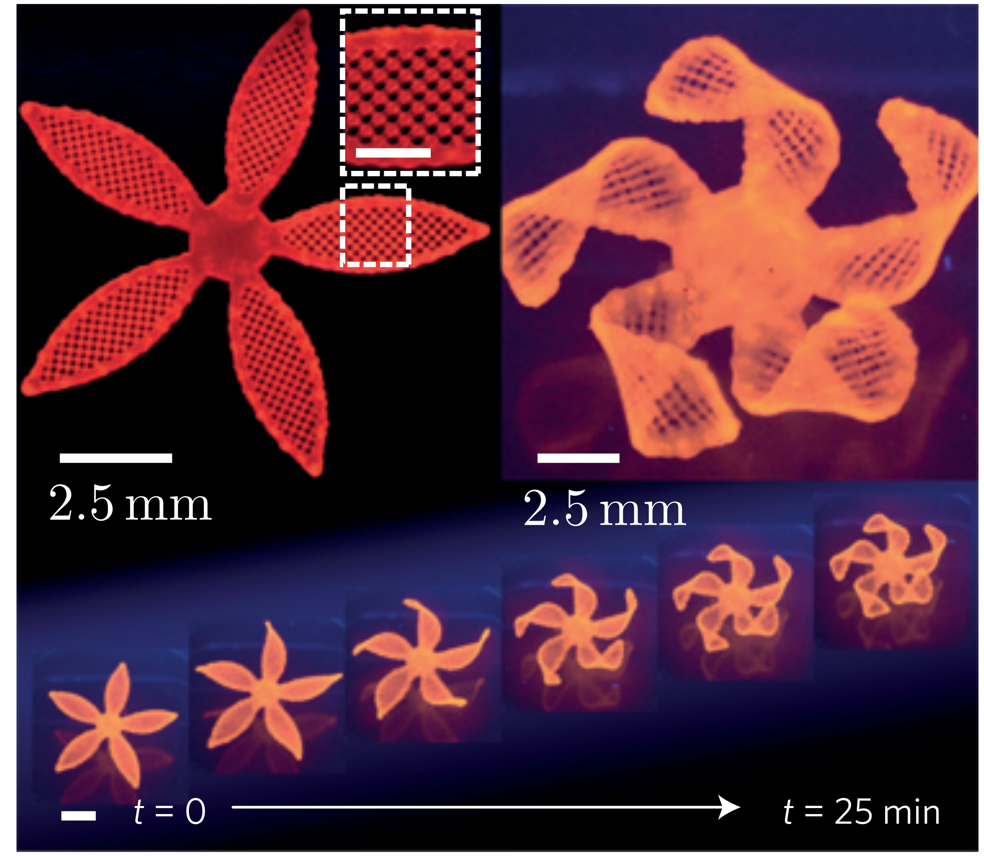
Shaping hydrogels via anisotropic swelling

“curling of leaves”





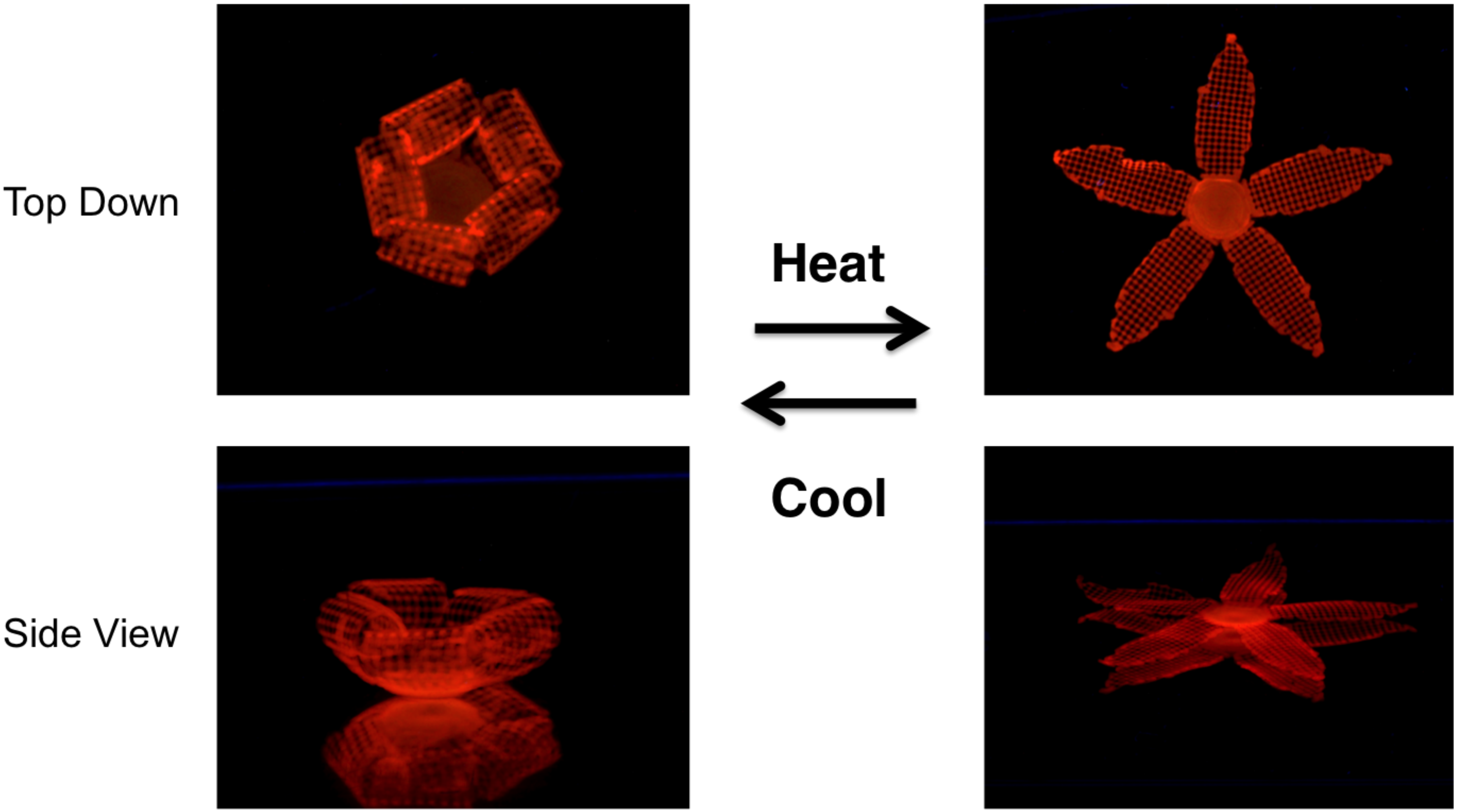
“twisting of leaves”



A. S. Gladman et al., <u>Nat. Materials</u> **15**, 413 (2016)

Shaping hydrogels via anisotropic swelling

The degree of swelling can be controlled via temperature!



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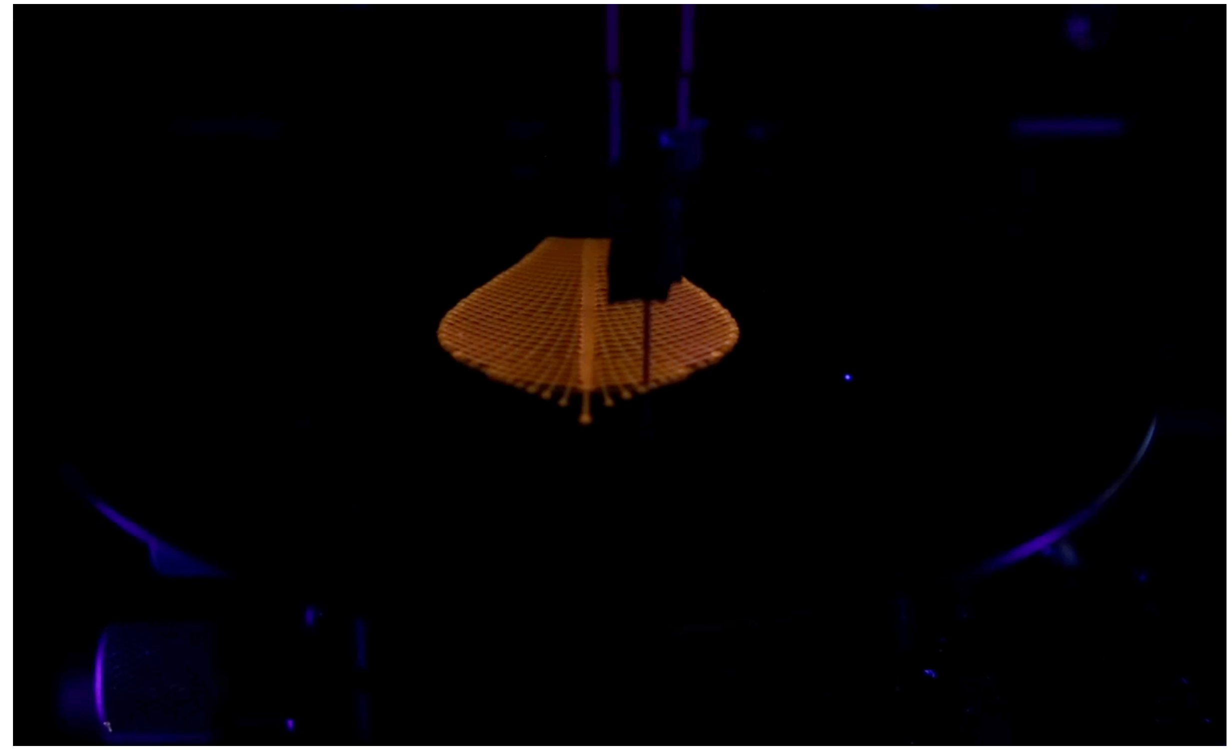
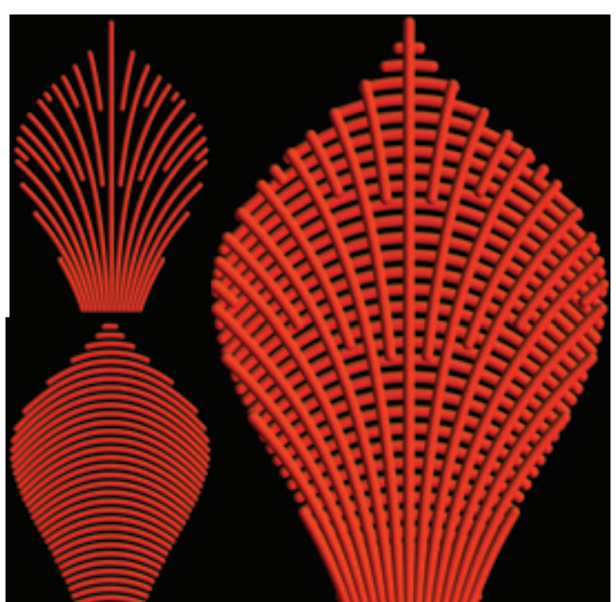
Shaping hydrogels via anisotropic swelling

target shape: calla lily flower

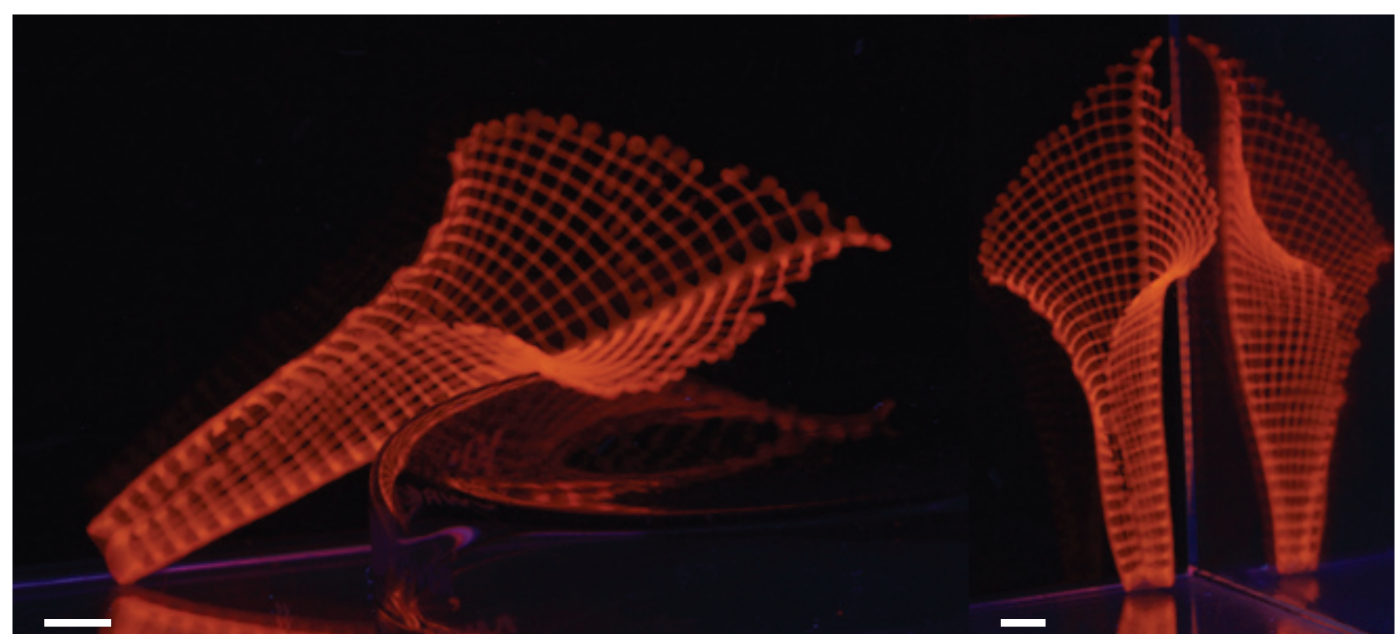
3D printer in action



3D printed hydrogel



swollen hydrogel

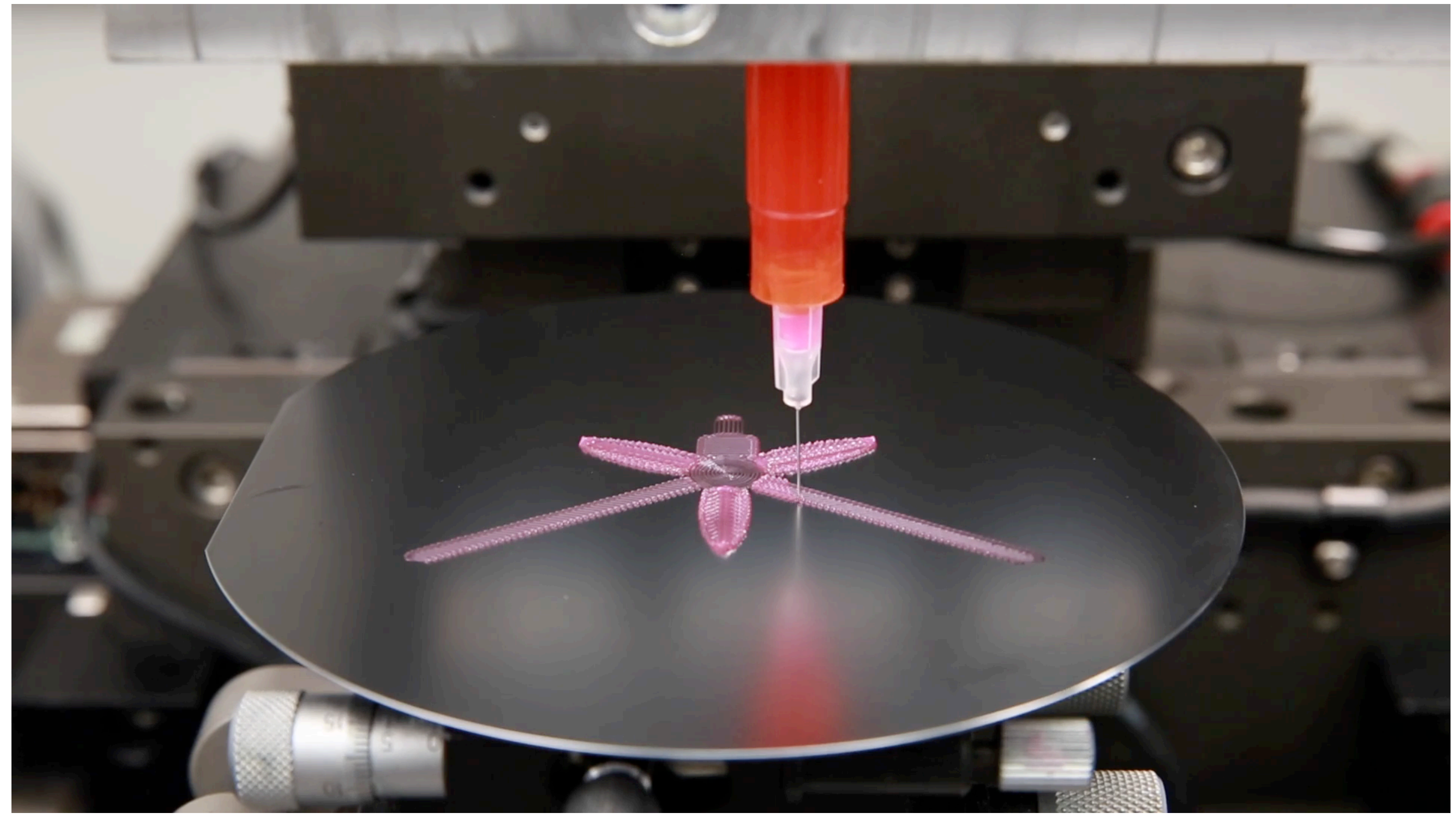


A. S. Gladman et al., <u>Nat. Materials</u> **15**, 413 (2016)

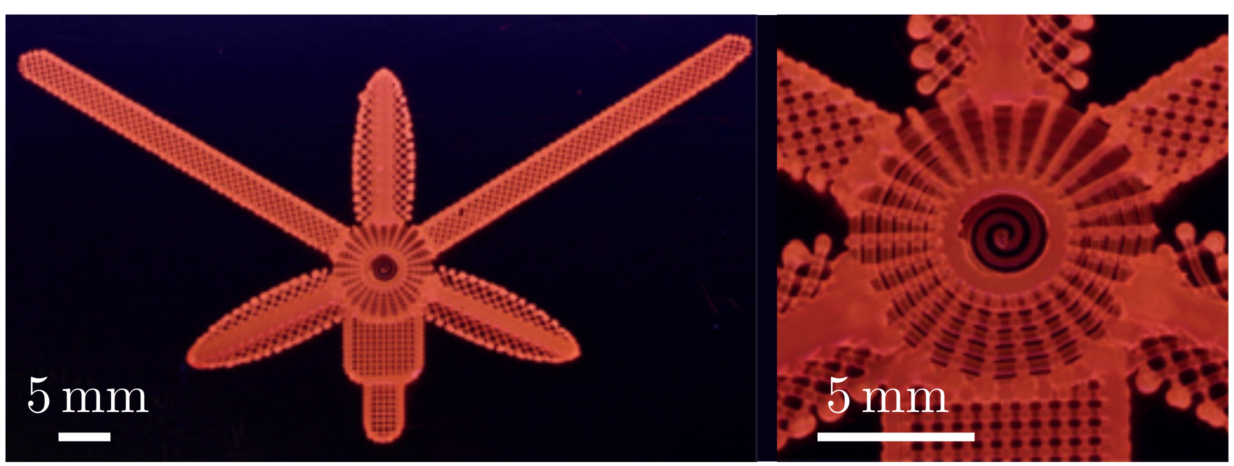
Shaping hydrogels via anisotropic swelling

target shape:
orchid *Dendrobium helix*

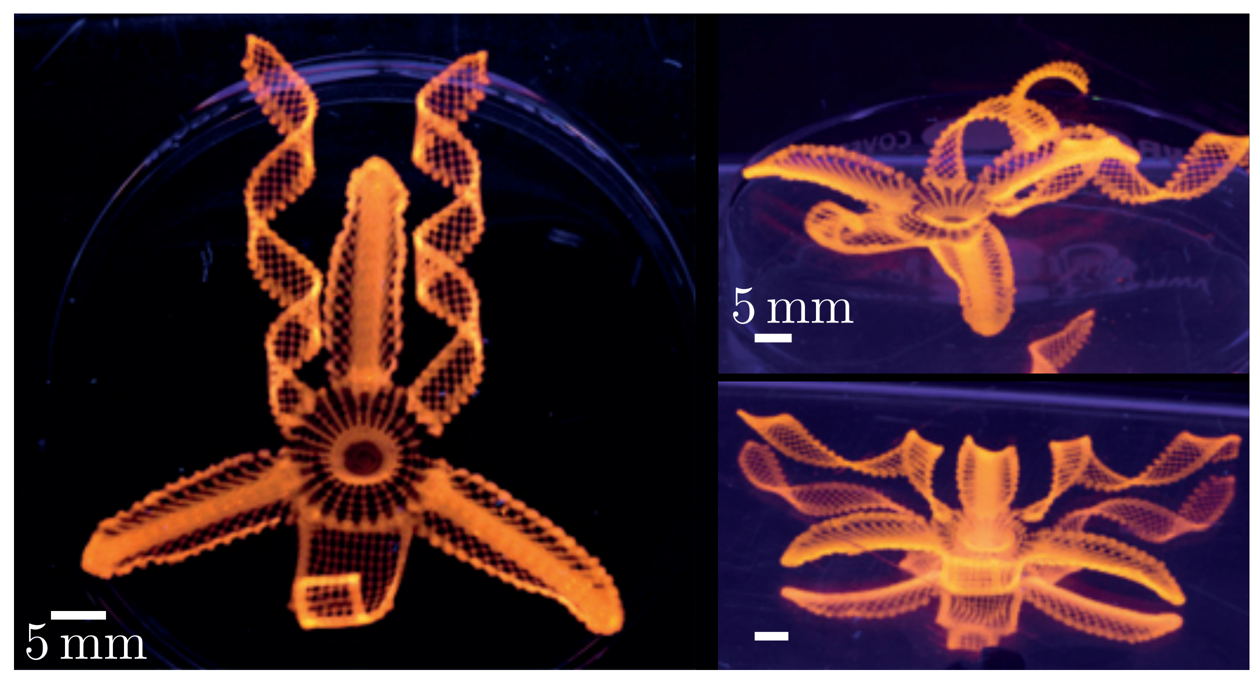
3D printer in action



3D printed hydrogel



swollen hydrogel



A. S. Gladman et al., Nat. Materials **15**, 413 (2016)

Mimosa pudica = “Touch-me-not plant”



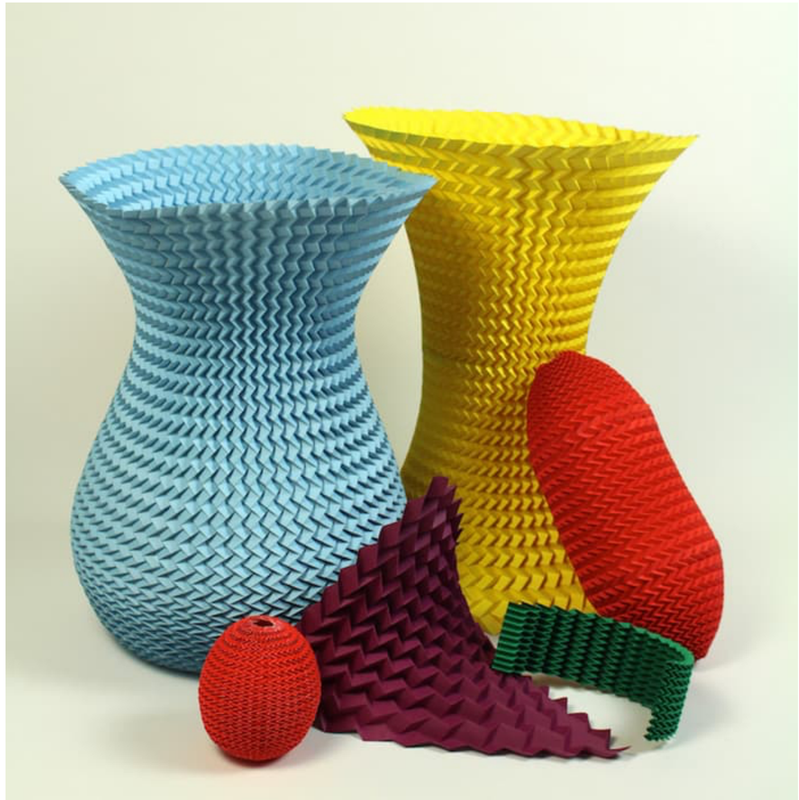
In response to touch plant releases certain chemicals and changes the osmotic environment for cells near the base of touched leaves. As a consequence these cells lose water and their shrinking causes the folding of leaves.


https://www.youtube.com/watch?v=nPf3FbR6eQE

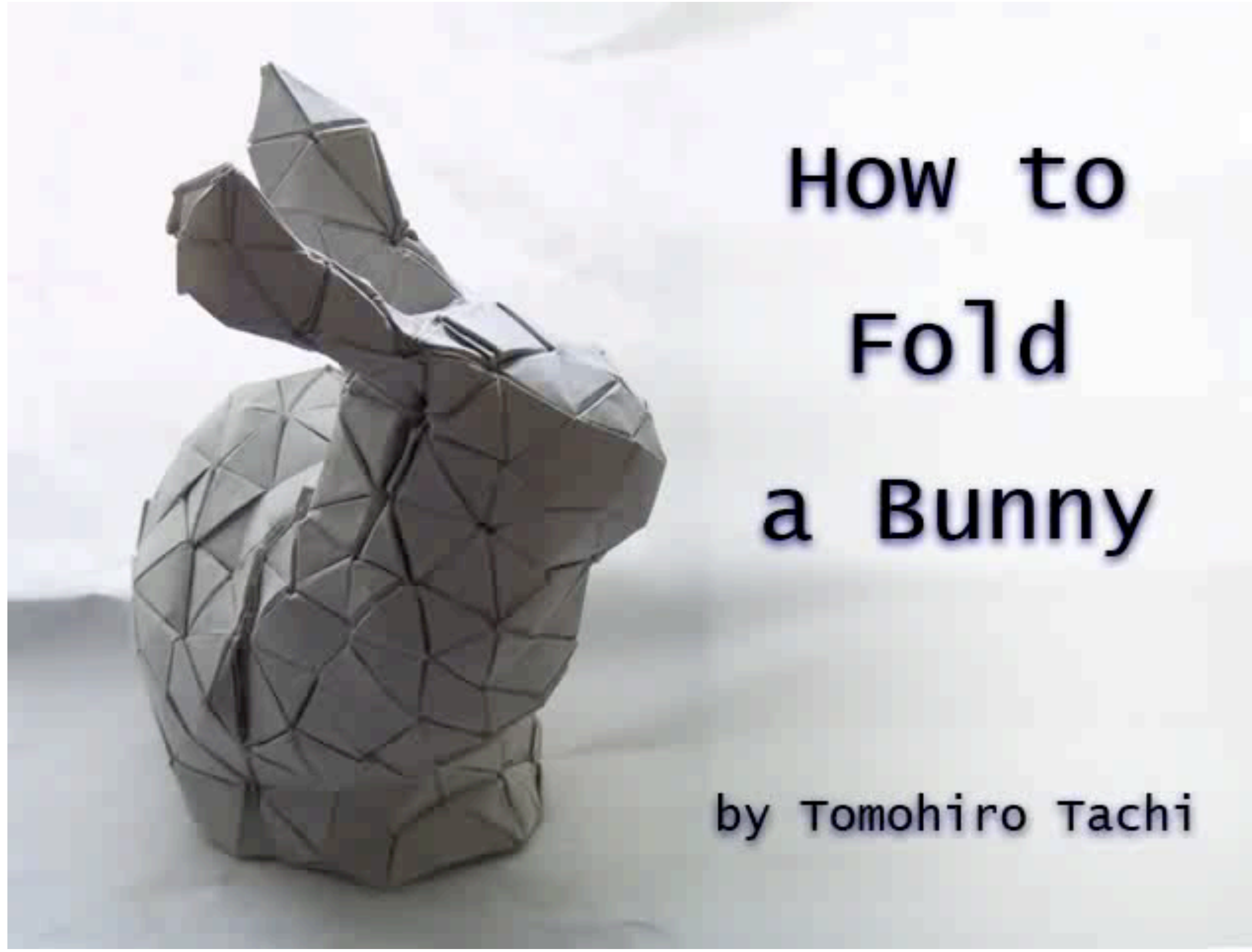
https://www.youtube.com/watch?v=g0LFBM3hOLs

Origami

Japanese for ori=fold, gami=paper



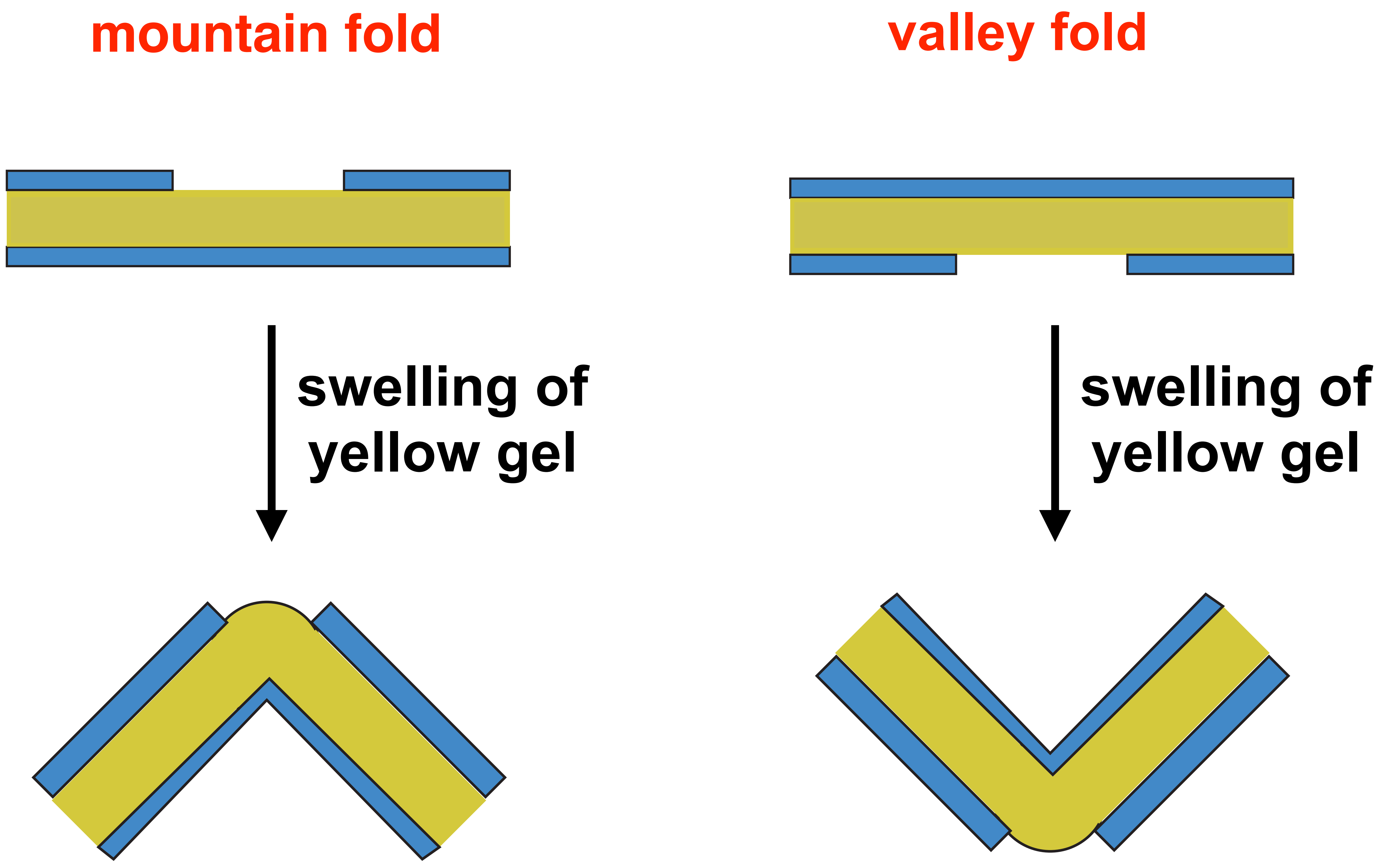
Folding a Bunny



https://www.youtube.com/watch?v=GAnW-KU2yn4

Can we make a self-folding origami?

Making a fold with swelling of gels

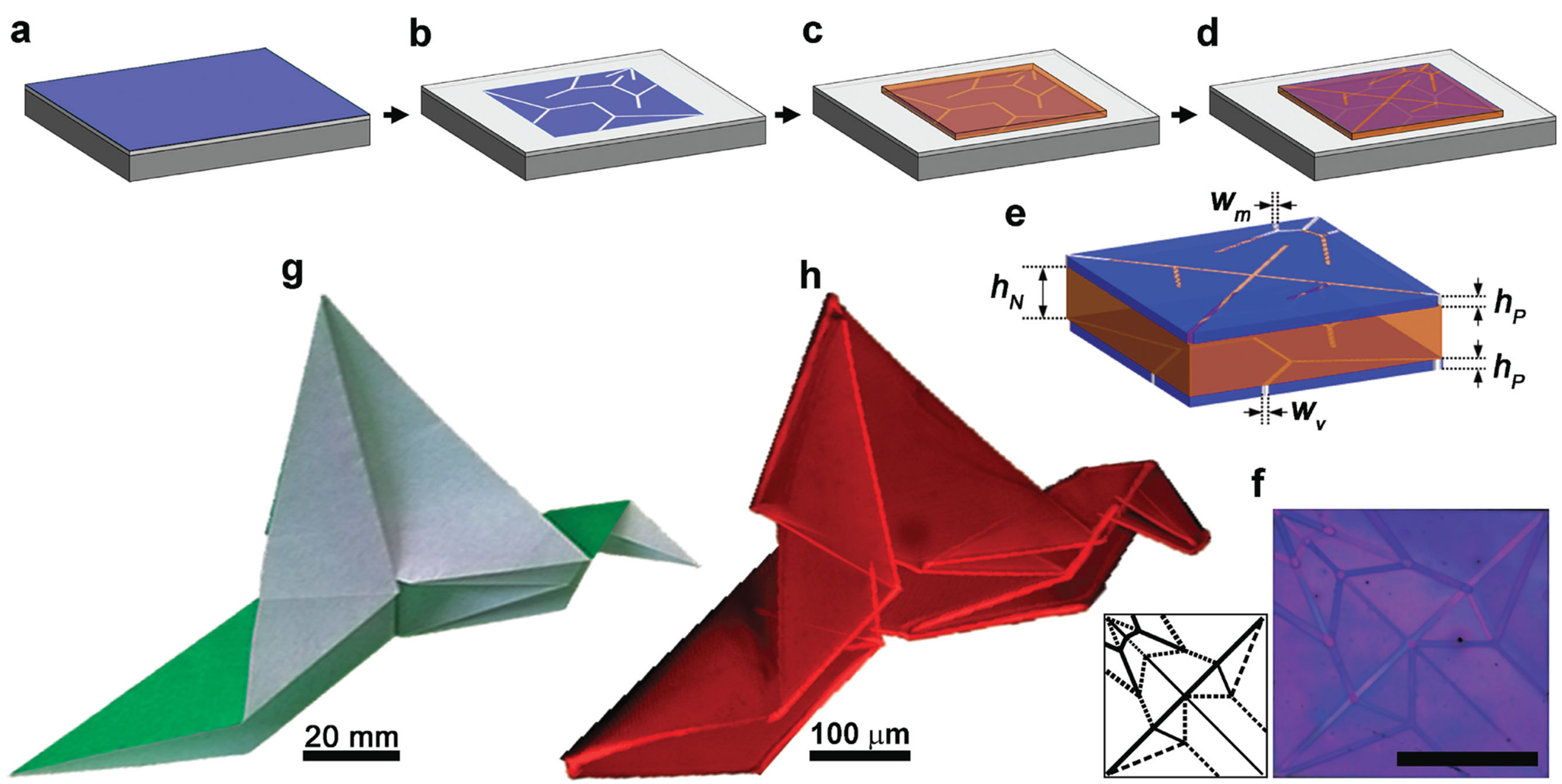


Self folding origami with gel swelling

pattern of valley folds

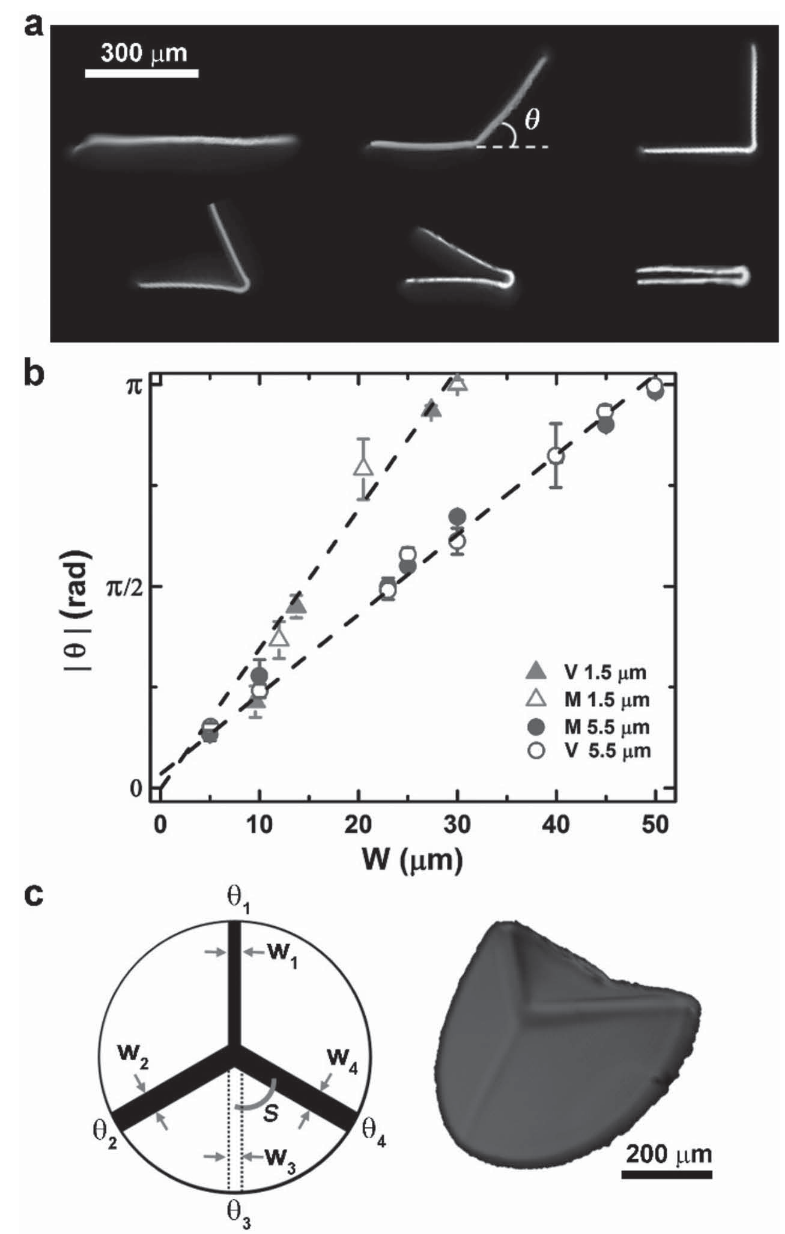
intermediate layer

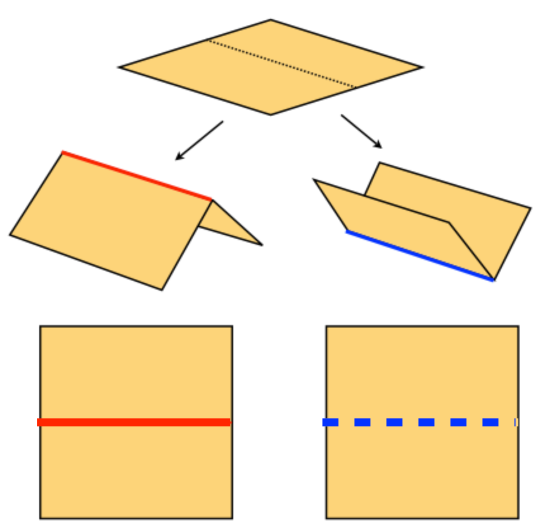
pattern of mountain folds



Randlett's flapping bird

width of the "cuts" determines the folding angle



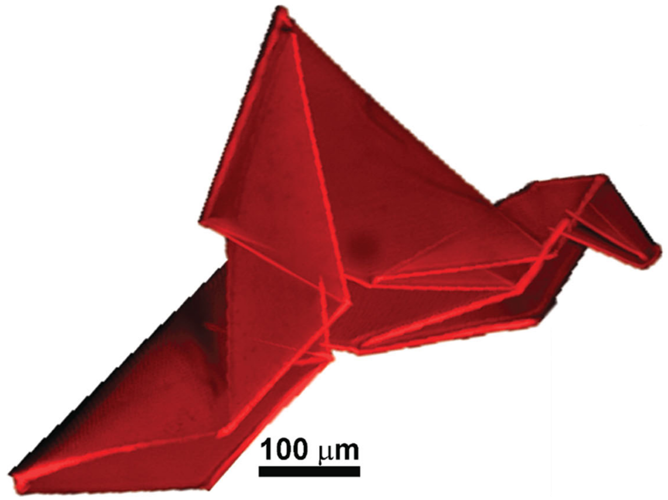


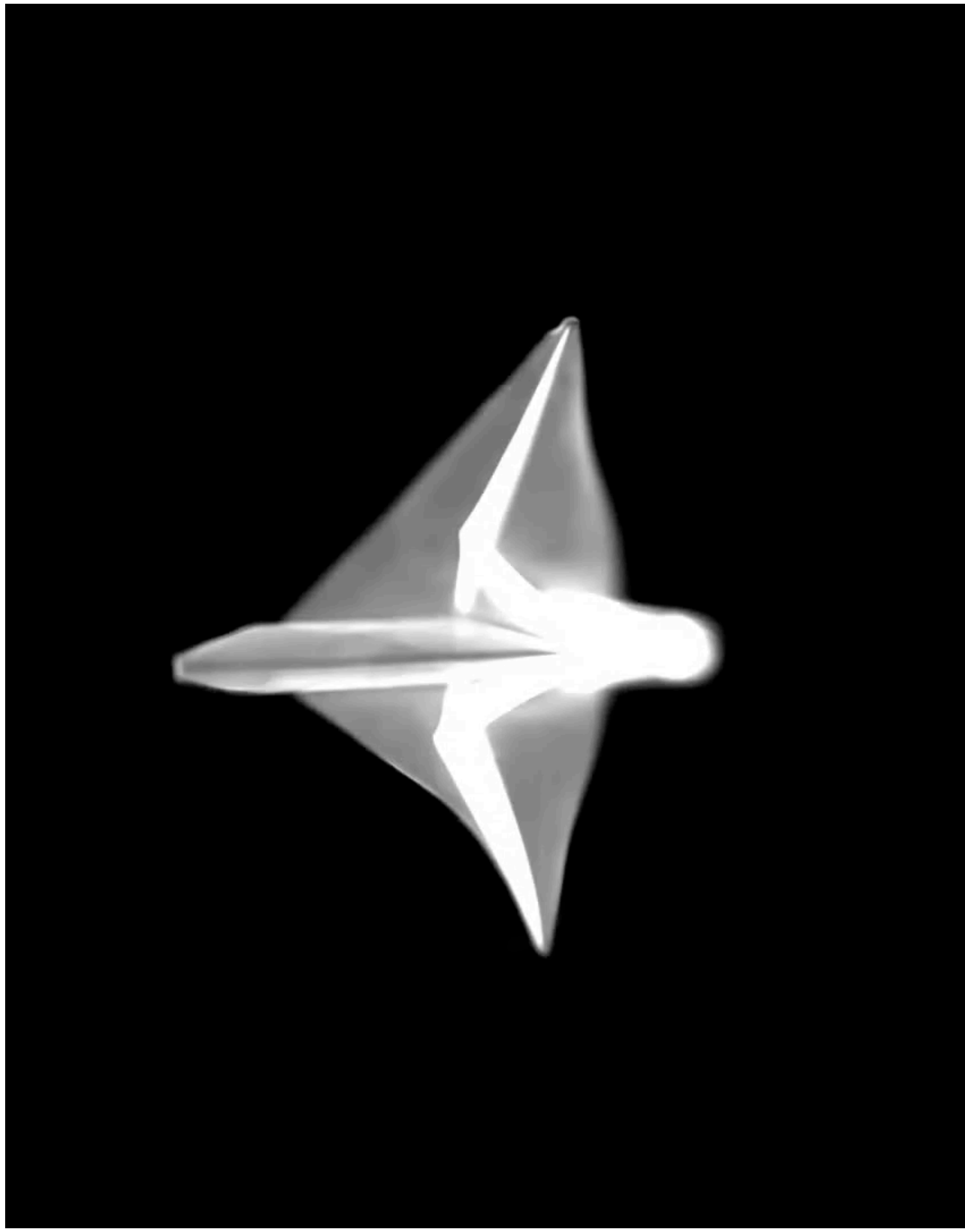
mountain

valley

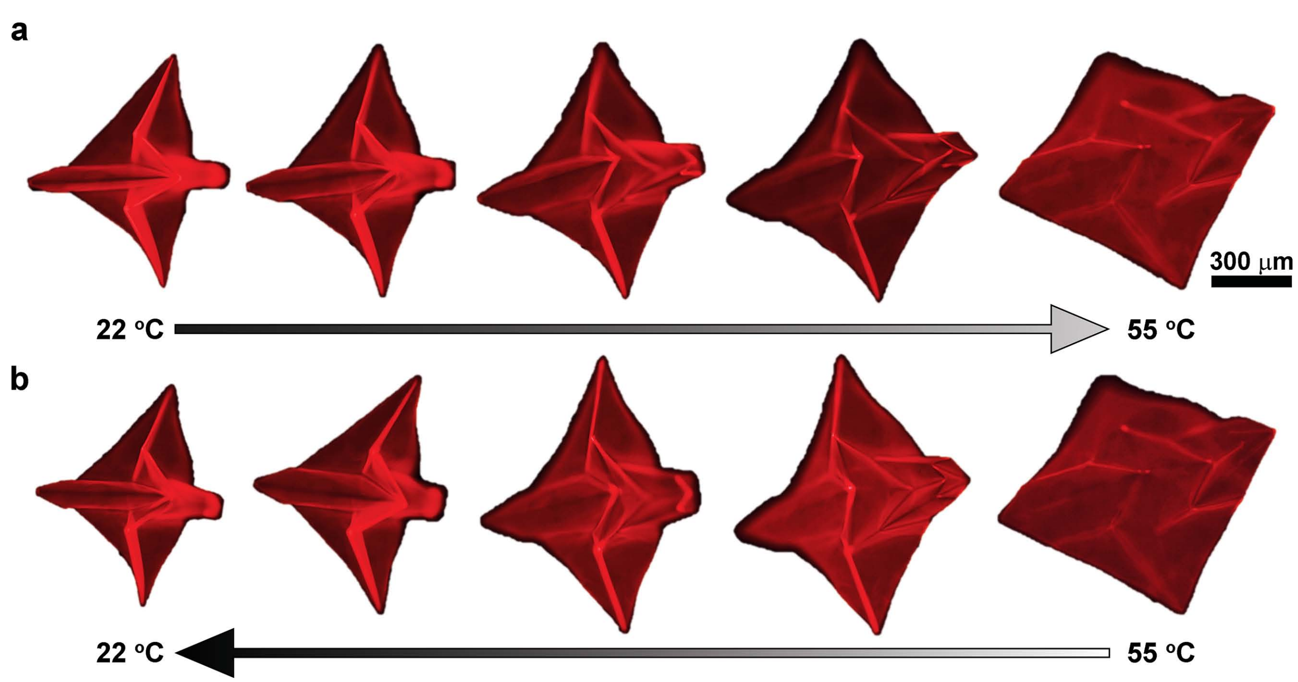
J.-H. Na et al., Adv. Mater. **27**, 79 (2015)

Temperature controls swelling and thus the folding of origami





Top view of self-folding origami

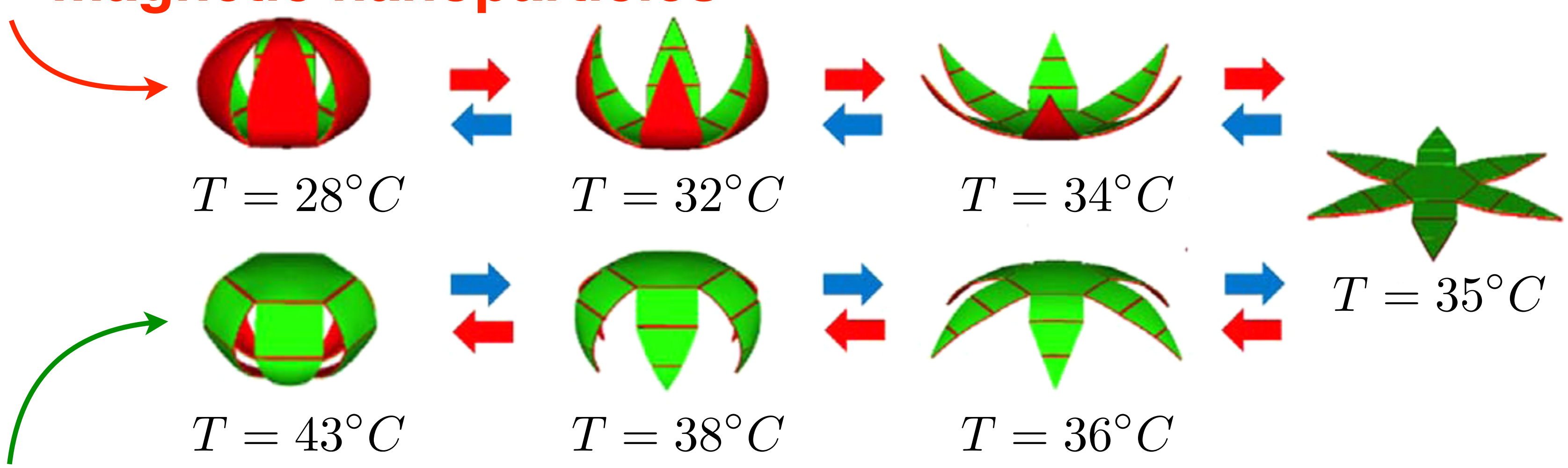


J.-H. Na et al., Adv. Mater. **27**, 79 (2015)

Biodegradable microgrippers for robotic surgeries

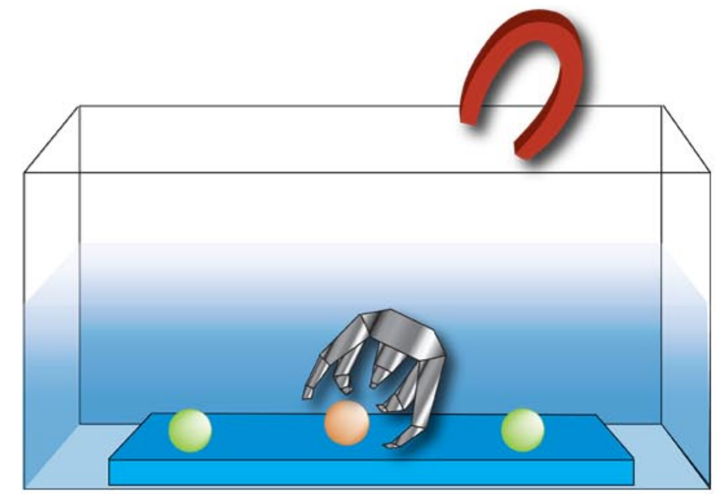
Temperature regulates opening/ closing of microgrippers

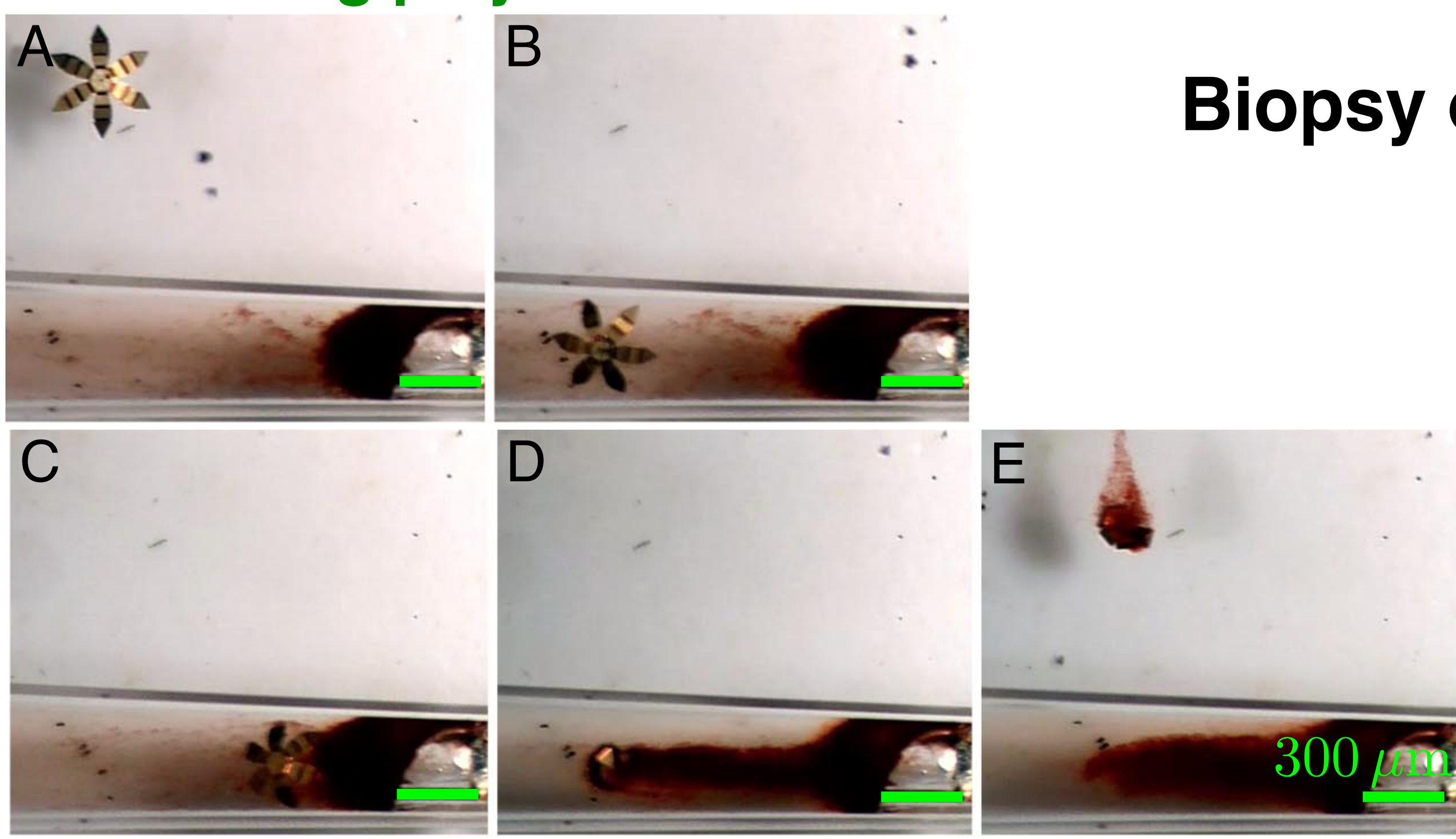
swelling hydrogel containing magnetic nanoparticles



non-swelling polymer

Position of microgrippers can be controlled with magnets

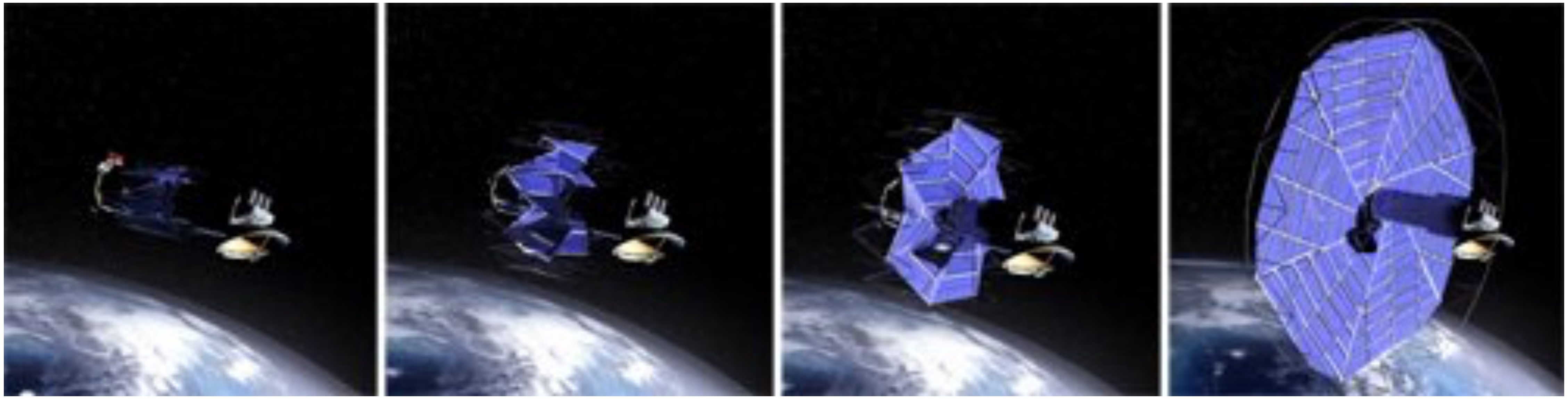


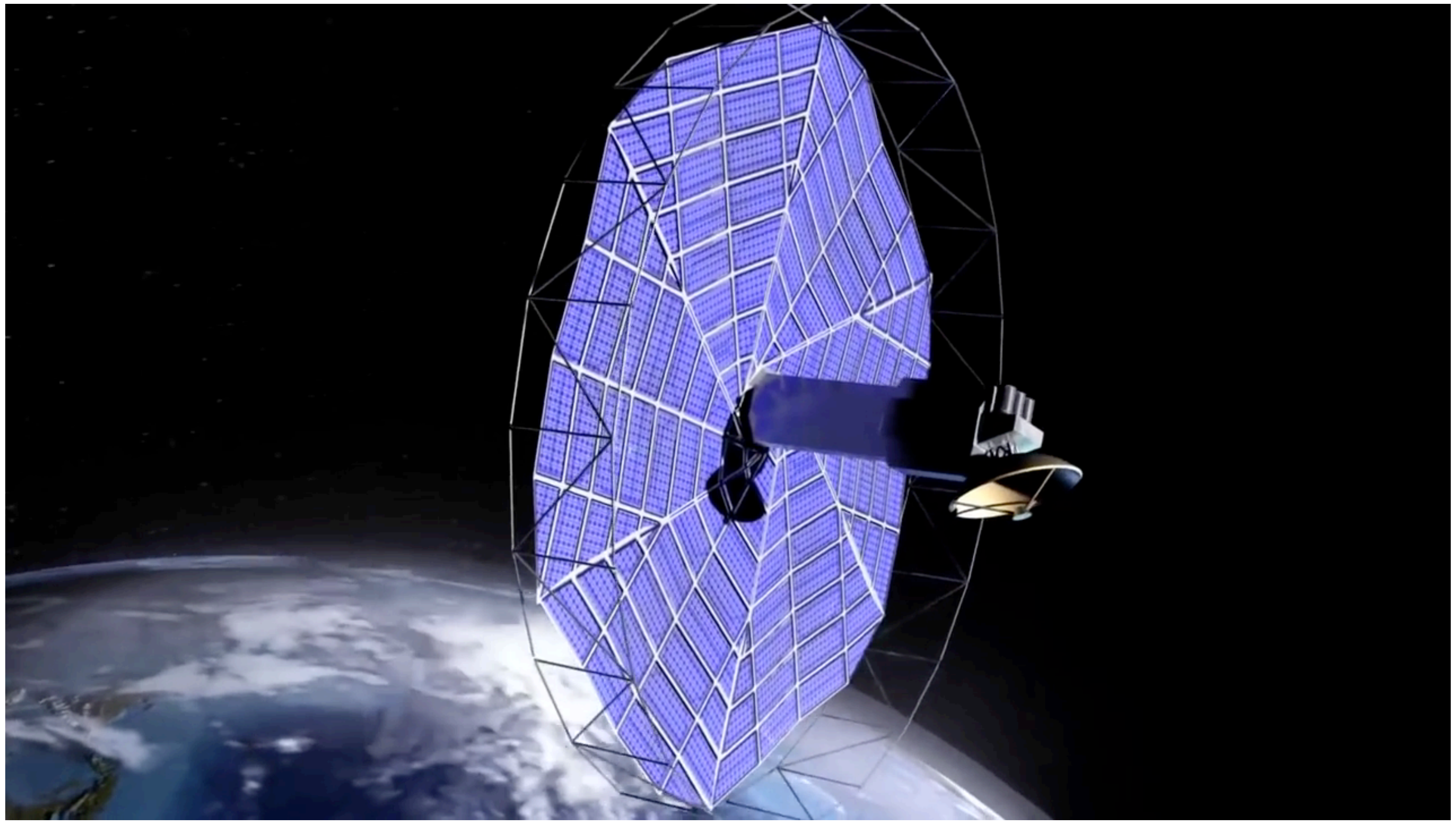


Biopsy of biological tissues

J.C. Breger et al., ACS Appl. Mater. Interfaces **7**, 3398 (2015)
T.G. Leong et al., PNAS **106**, 703 (2009)

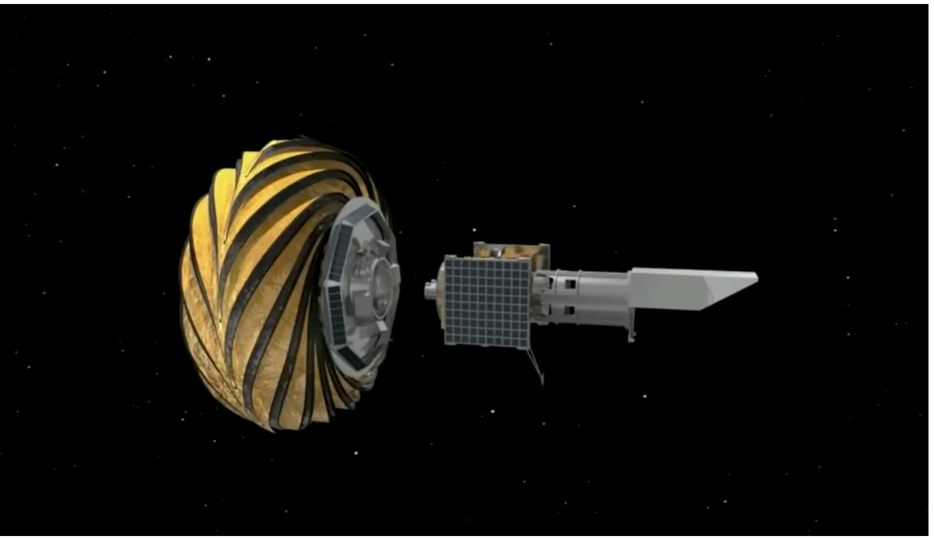
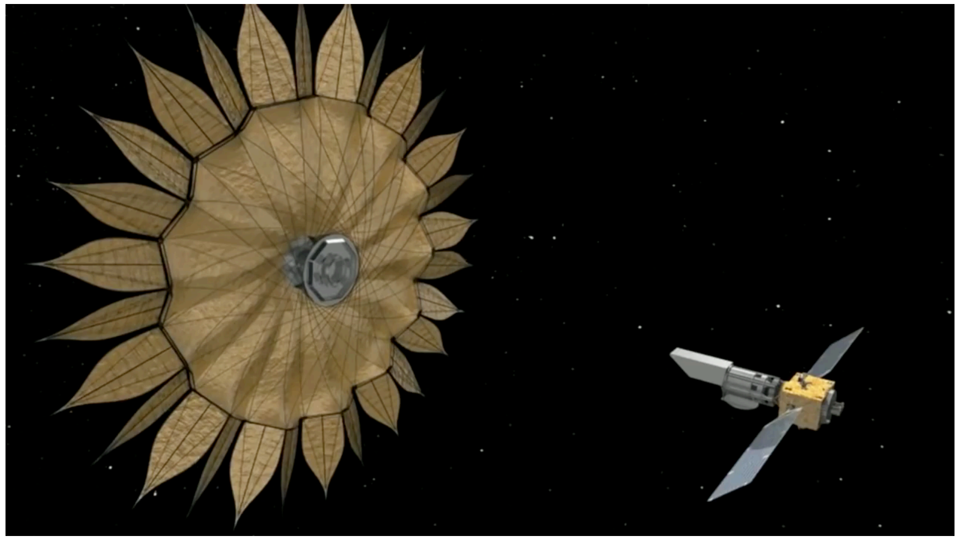
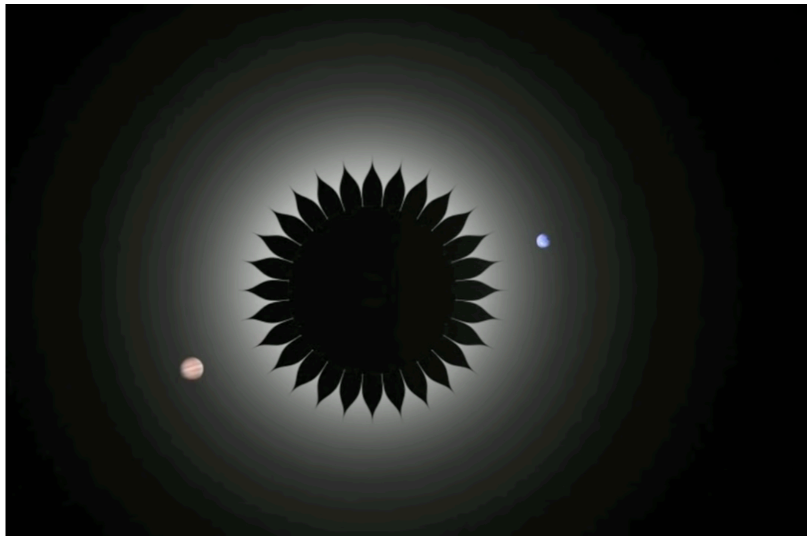
Origami for satellite solar panels

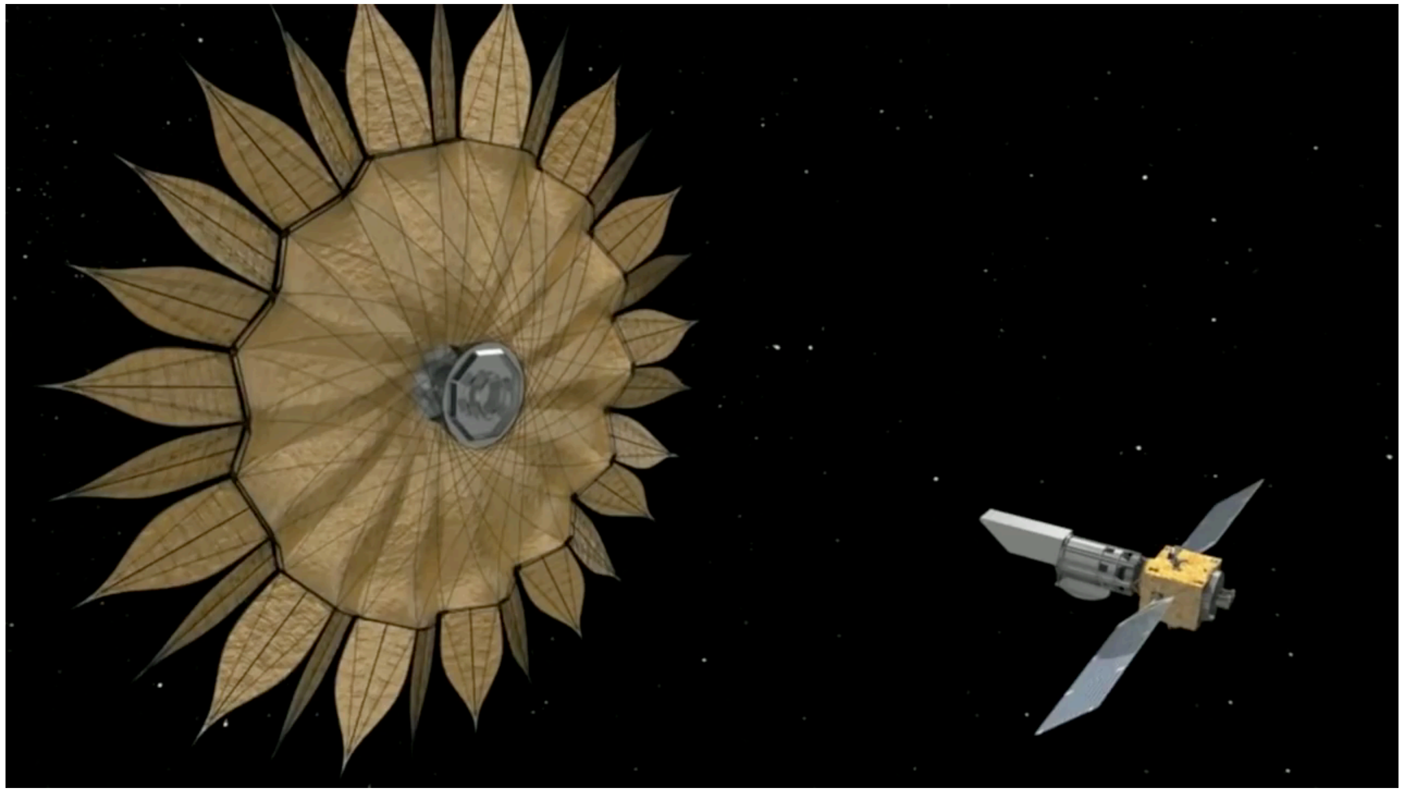




https://www.youtube.com/watch?v=3E12uju1vgQ

Origami for shielding telescopes for detection of exoplanets



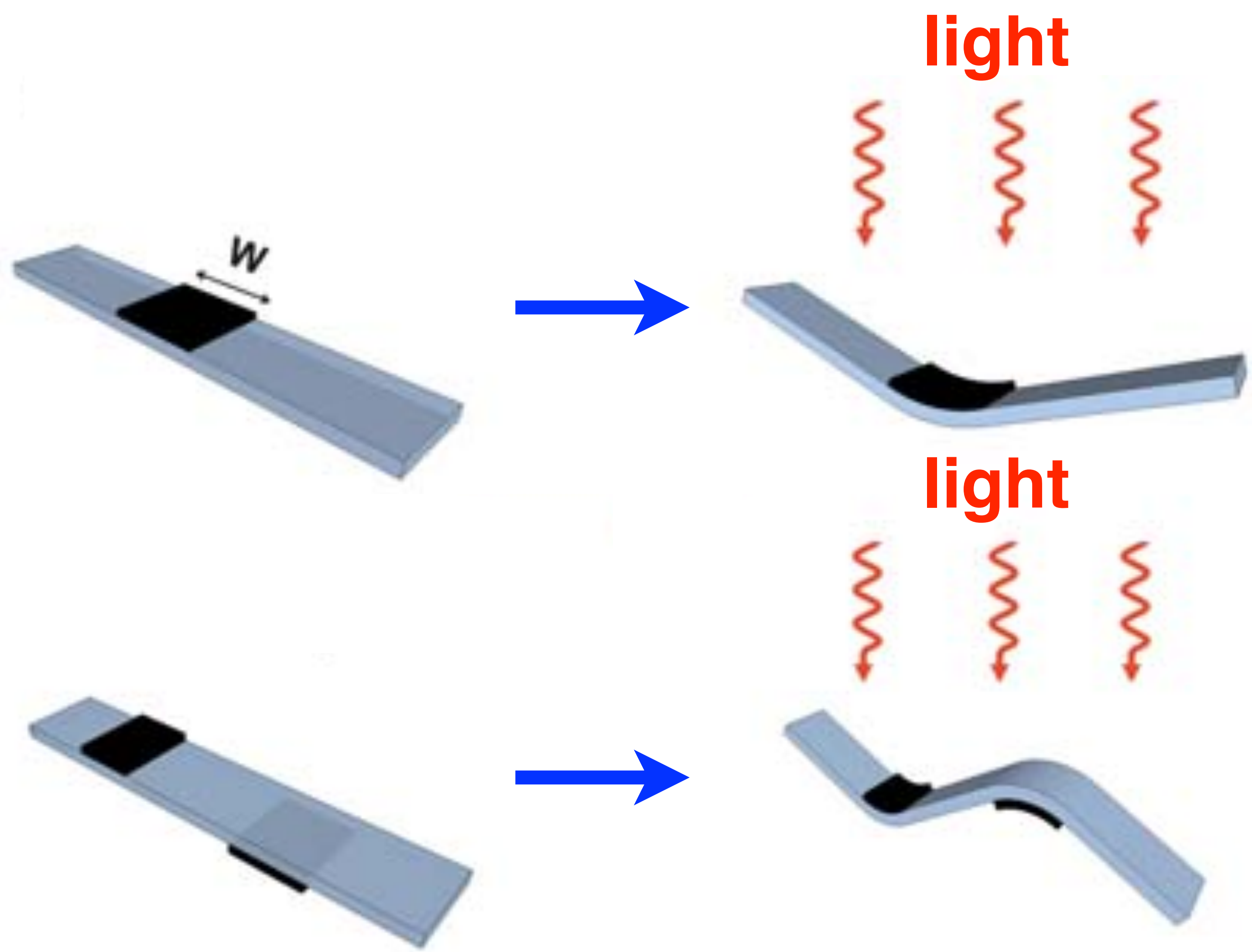
https://www.ted.com/speakers/jeremy_kasdin

Shield is used to block the strong light coming from a star, which enables the telescope to detect faint signals from planets orbiting the star.

Shrinky-Dinks

Shrinky-Dinks are sheets made of optically transparent, pre-strained polystyrene that shrink if heated to the glass transition temperature.

Localized heating and shrinking of Shrinky-Dinks can be achieved with patterning of black ink that absorb light.

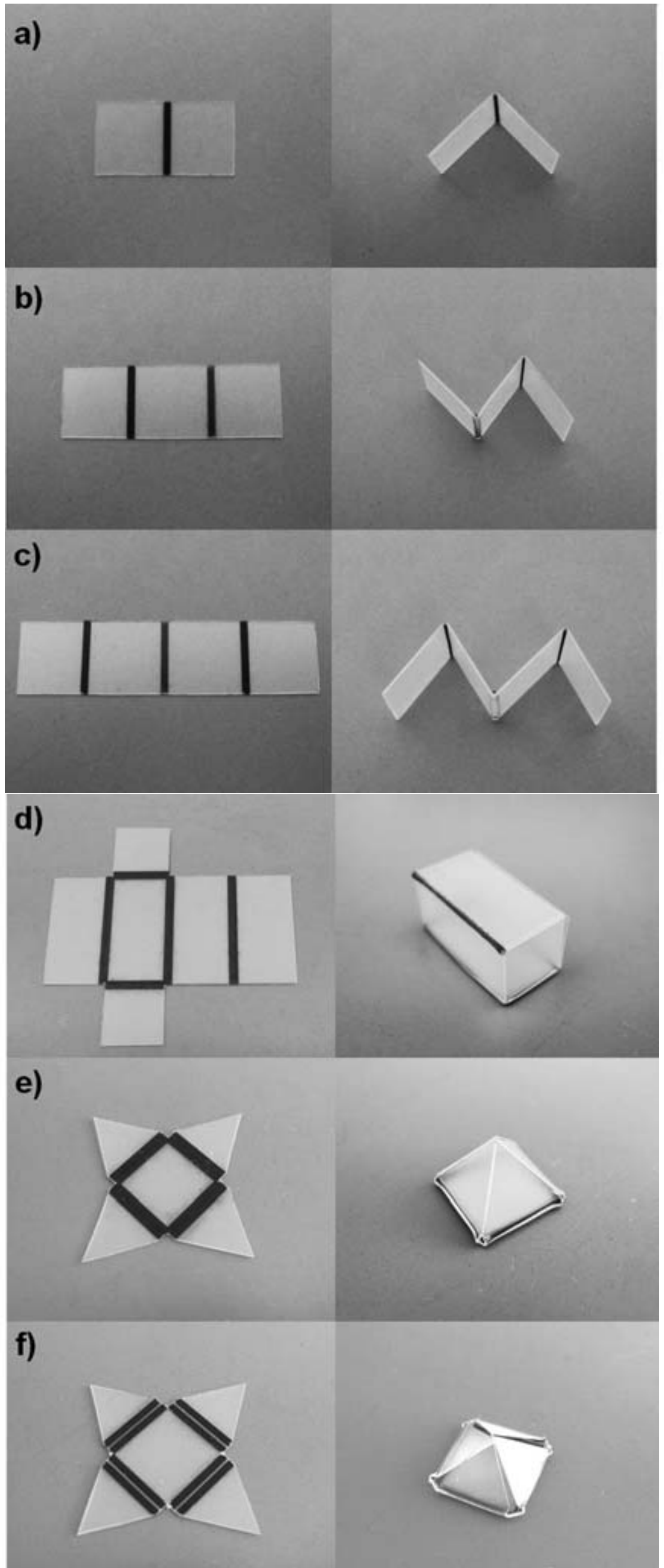


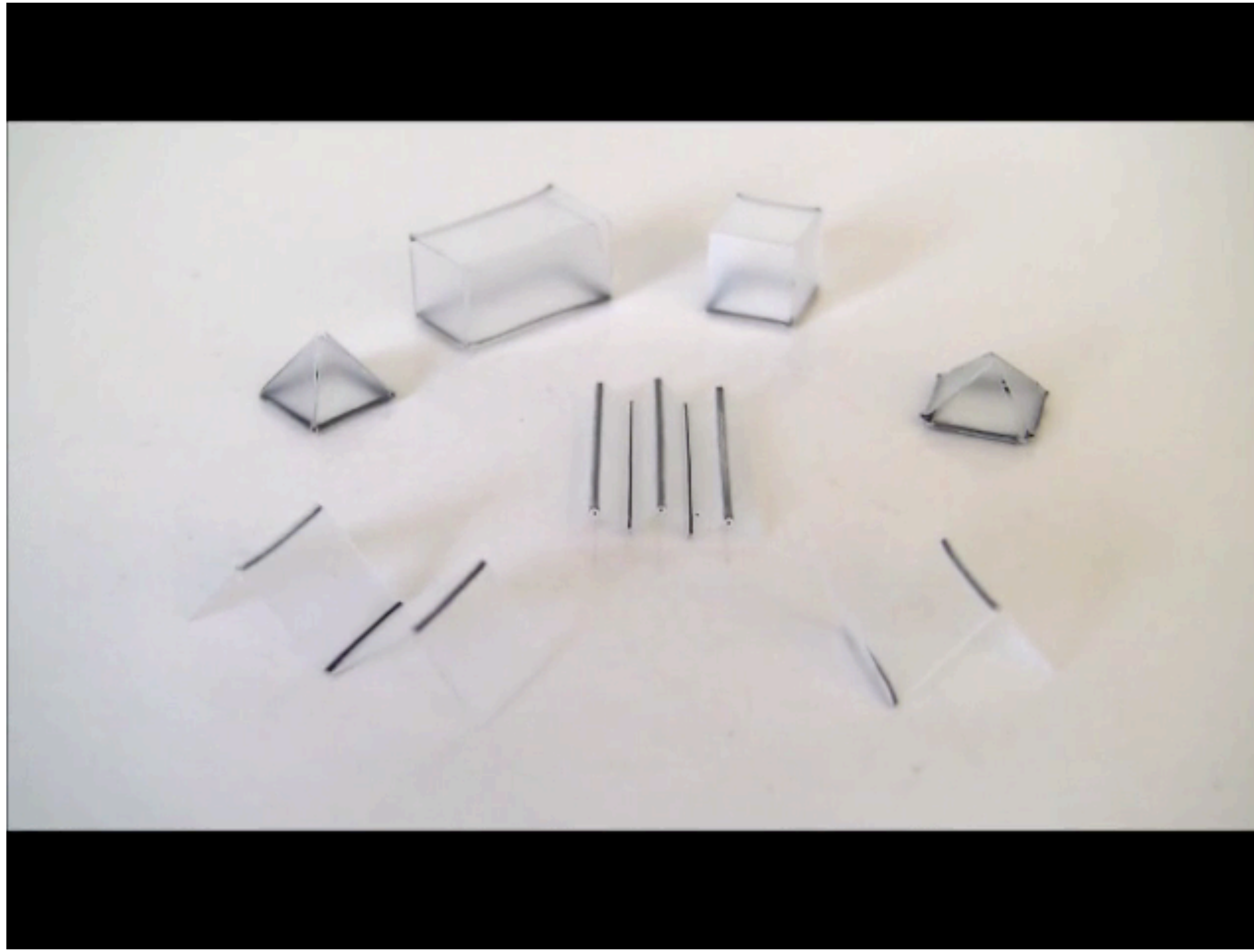
Folding angle can be controlled with the width of ink and with the exposure time of light.

J. Liu et al., Soft Mater **8**, 1764 (2012)

Shrinky-Dinks origami

size ~ cm

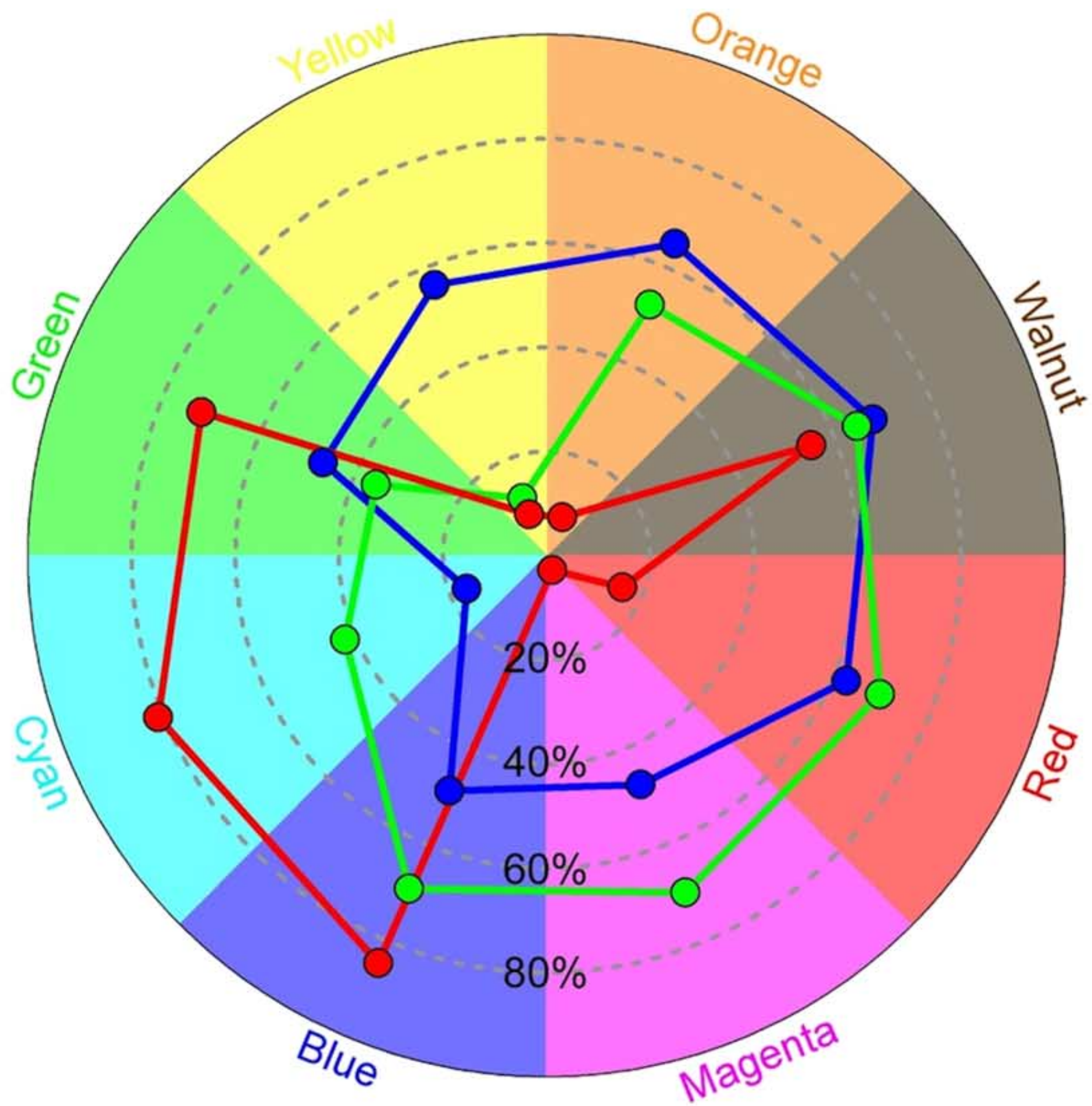


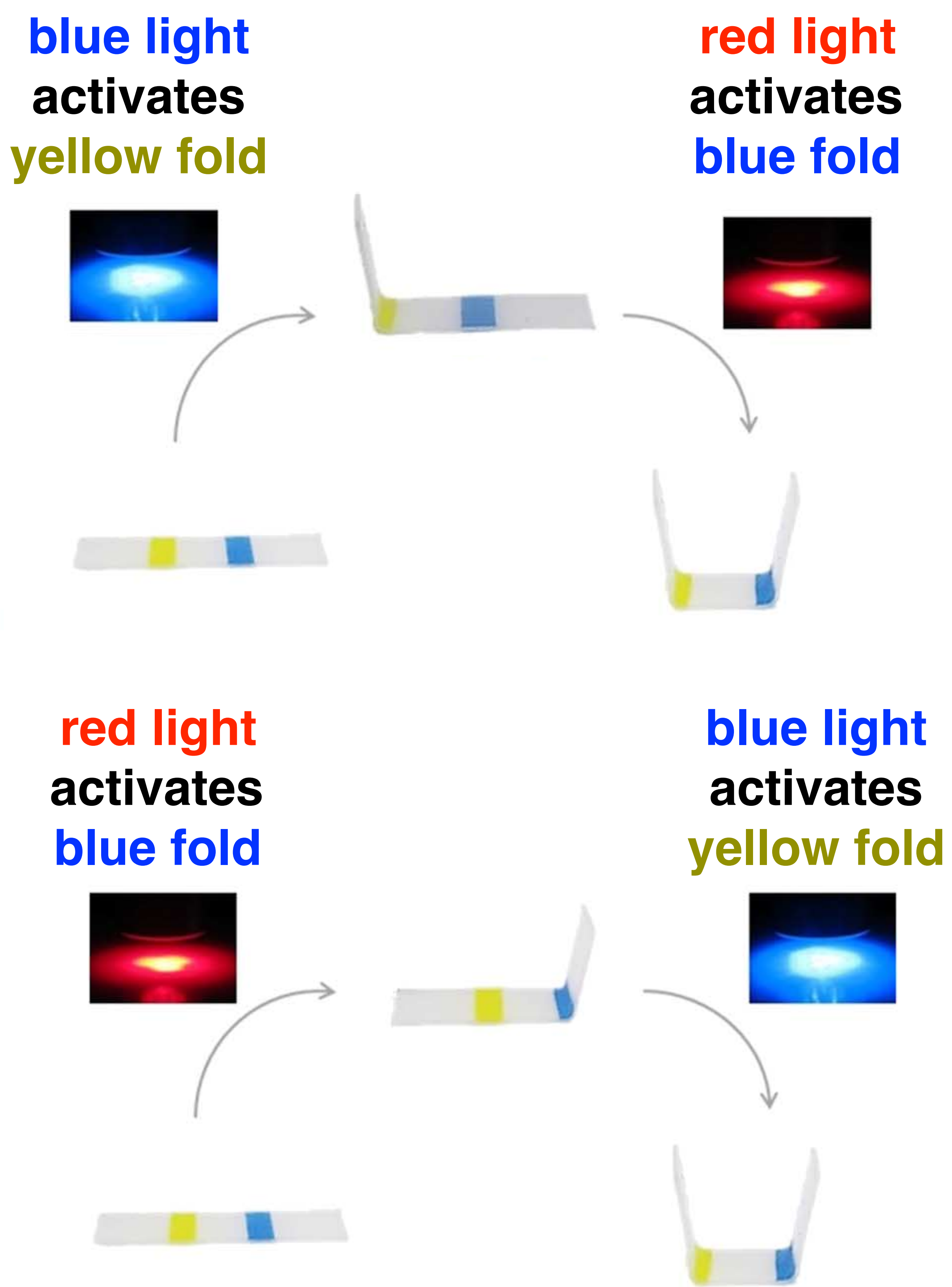


J. Liu et al., Soft Mater **8**, 1764 (2012)

Sequential folding of Shrinky-Dinks origami

Different ink colors have different absorption spectra for red, green and blue light.

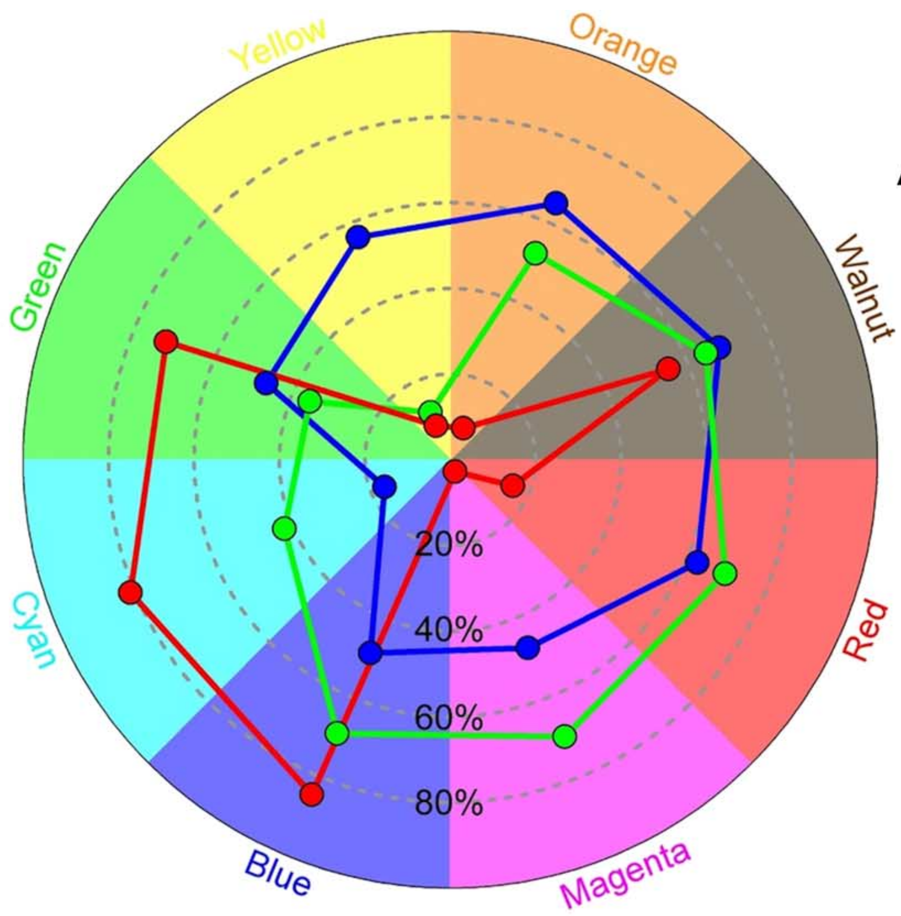


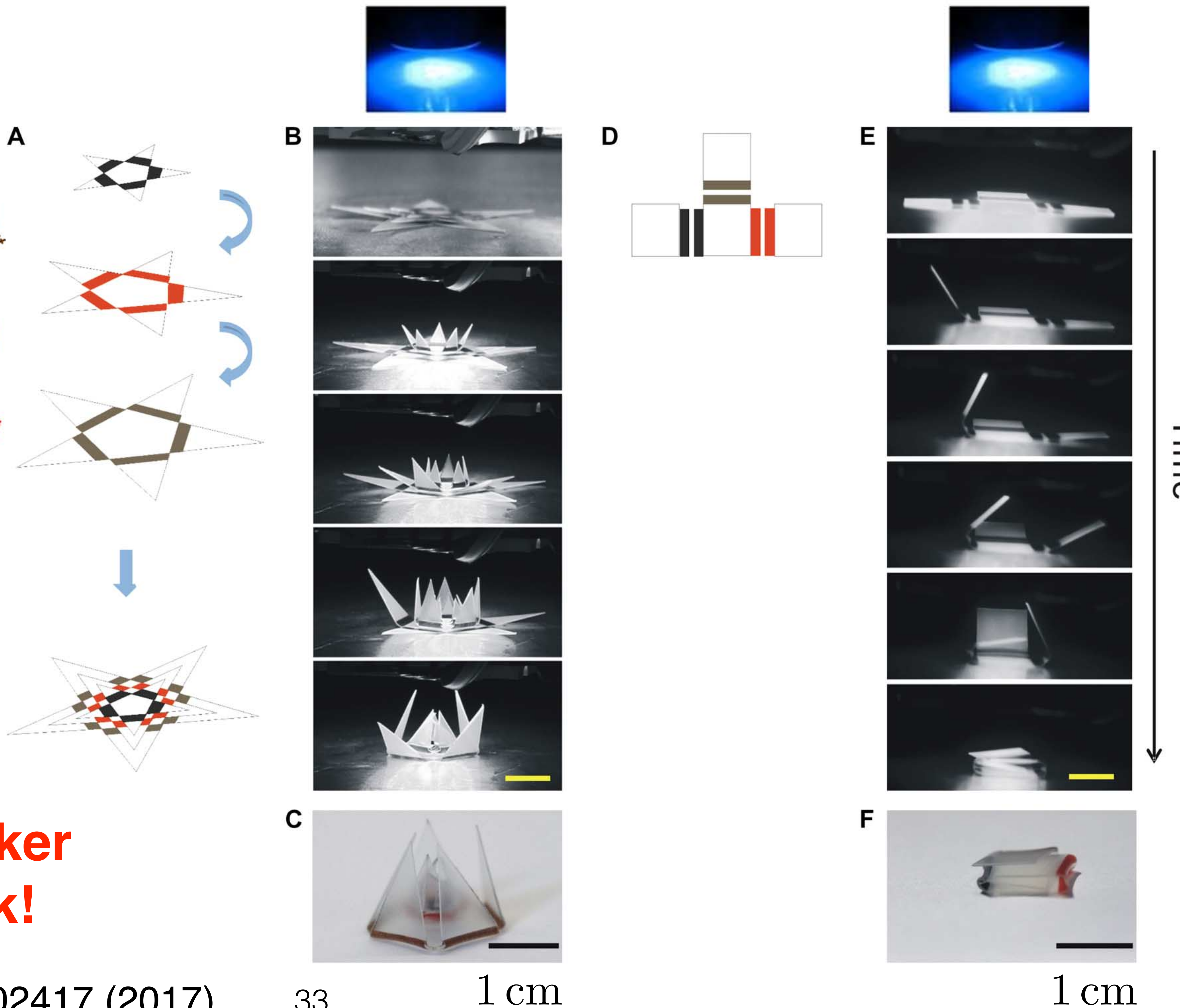


J. Liu et al., Sci. Adv. **3**, e1602417 (2017)

Sequential folding of Shrinky-Dinks origami

The order of folding corresponds to the amount of absorbed blue light (black > red > walnut)

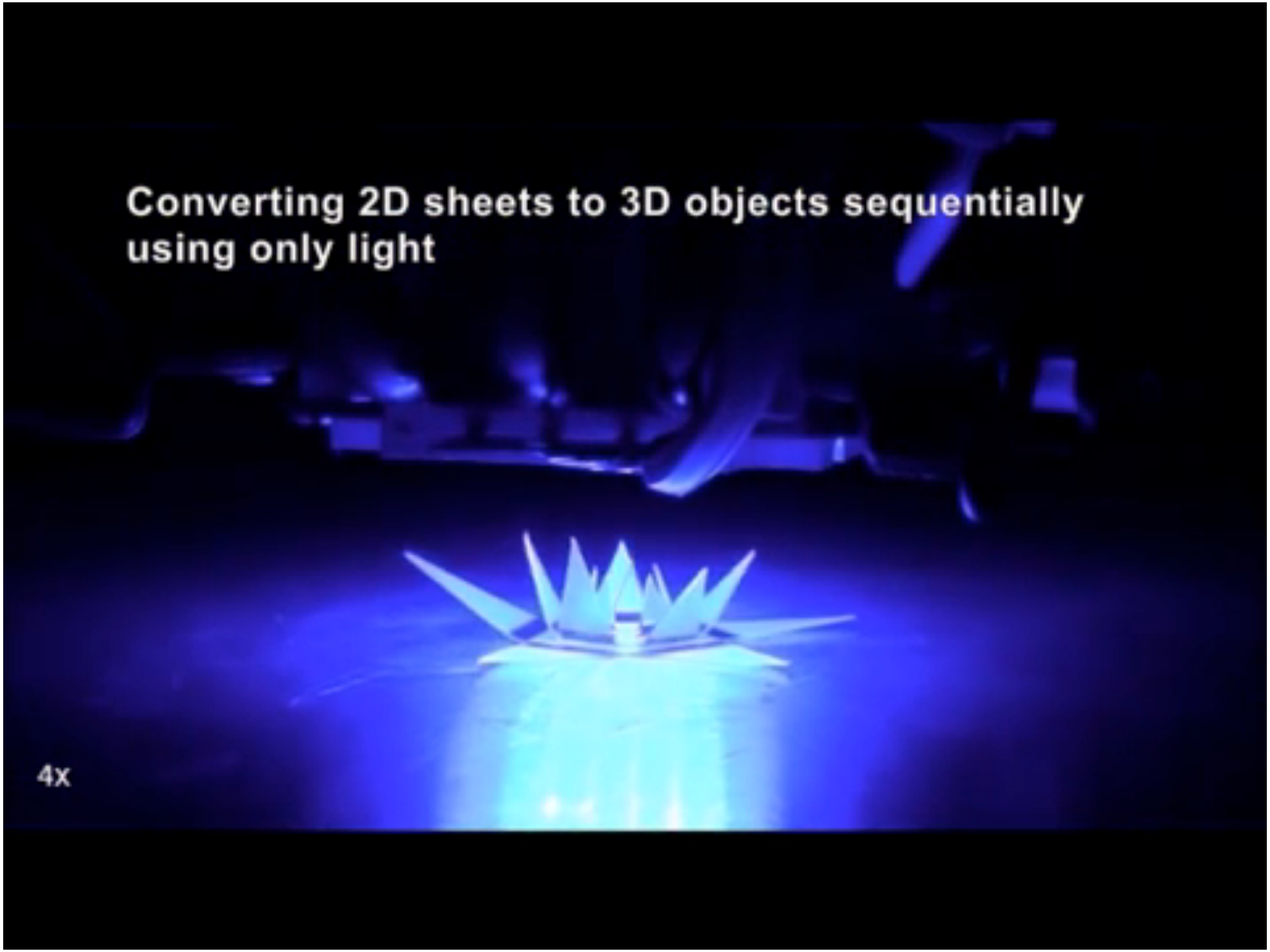




Note: red ink is thicker than the walnut ink!

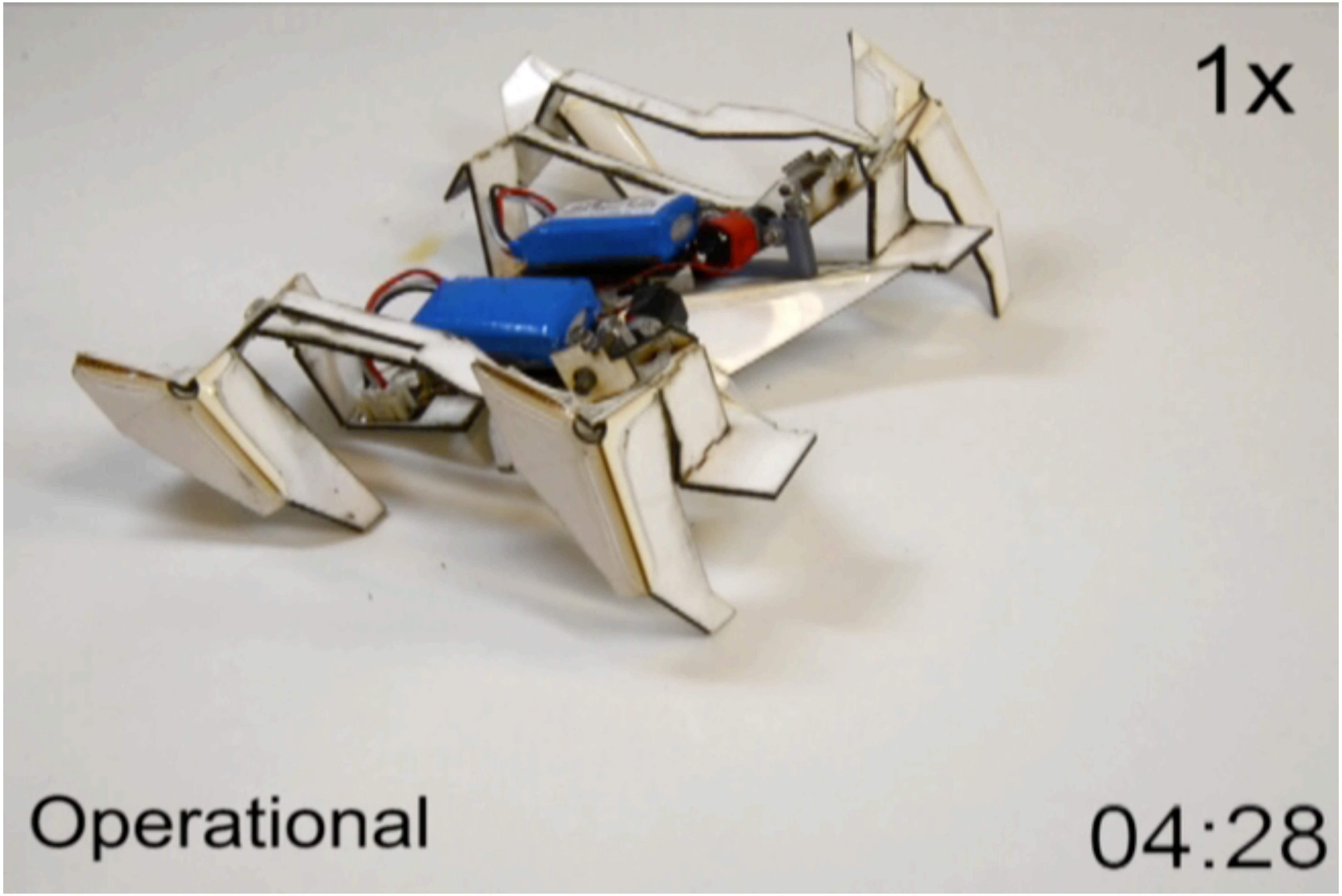
J. Liu et al., Sci. Adv. **3**, e1602417 (2017)

Sequential folding of Shrinky-Dinks origami



J. Liu et al., Sci. Adv. **3**, e1602417 (2017)

Self-folding robots (in 4 min)



S. Felton et al., Science **345**, 644 (2014)

Robot assembly

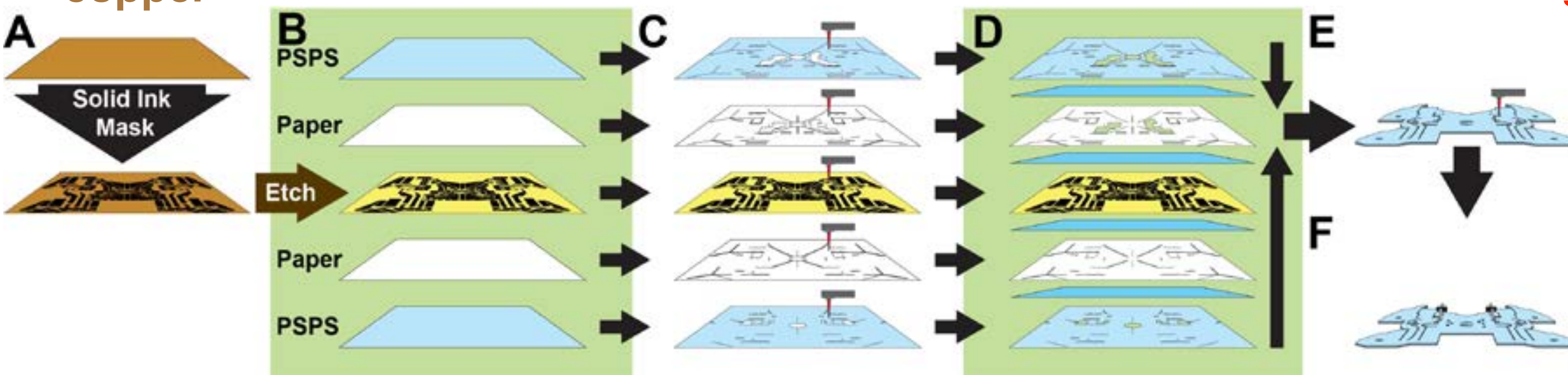
copper

Chemical etching of copper outside the ink mask

Laser cutting of layers

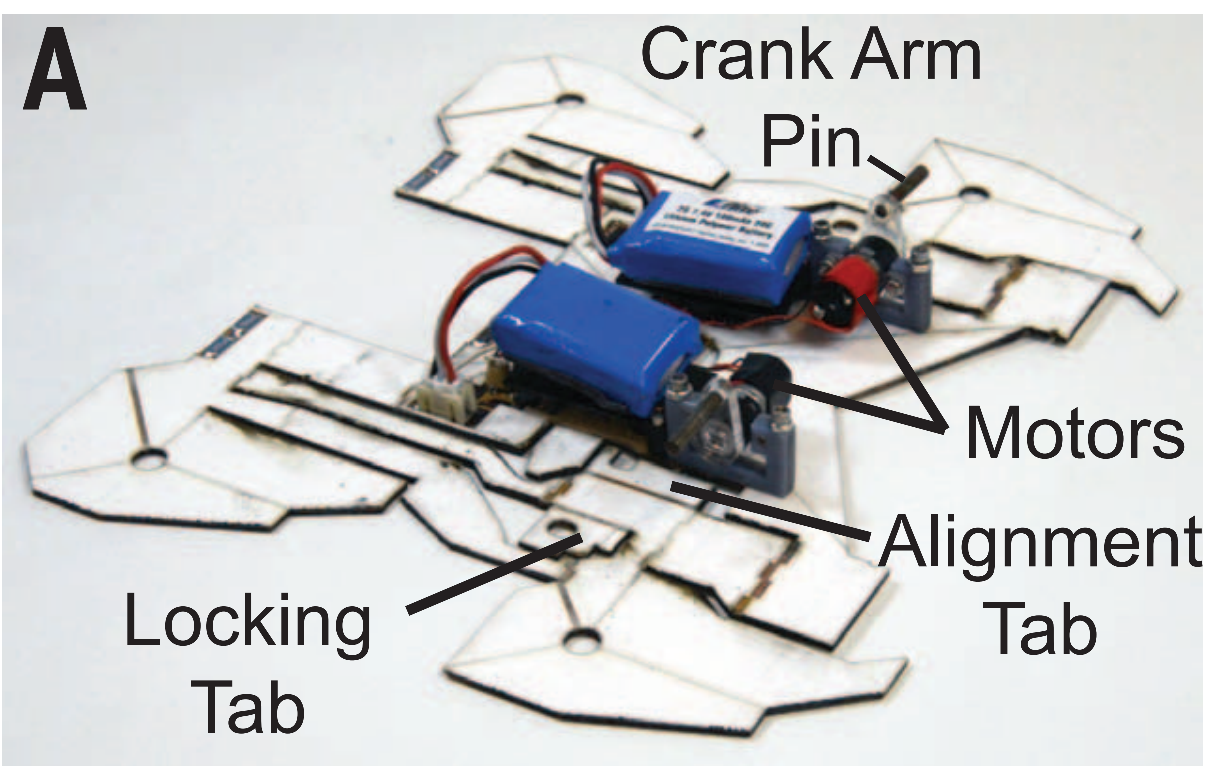
Gluing of layers

Laser cutting



"Shrinky-Dinks"

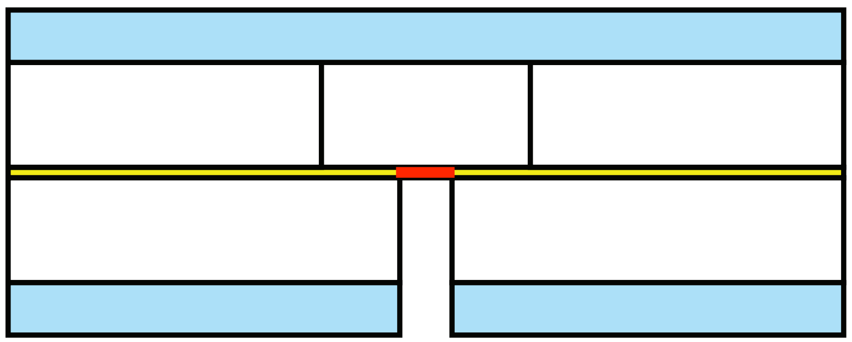
installment of electrical components, motors, and batteries



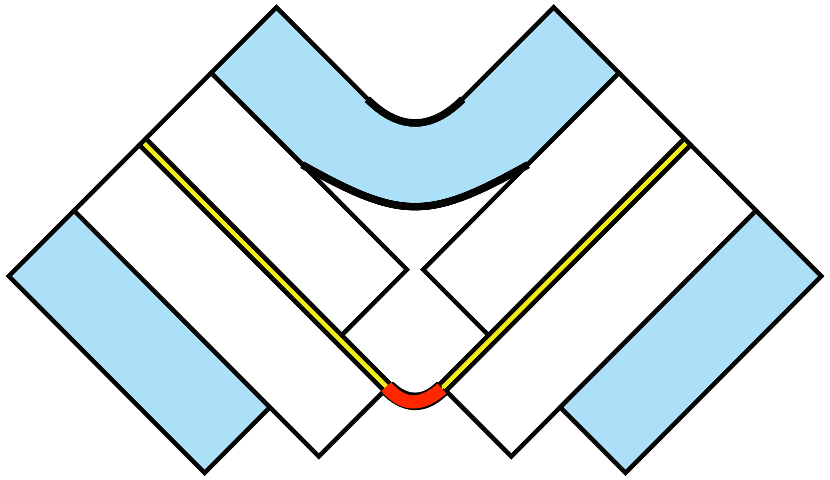
S. Felton et al., Science **345**, 644 (2014)

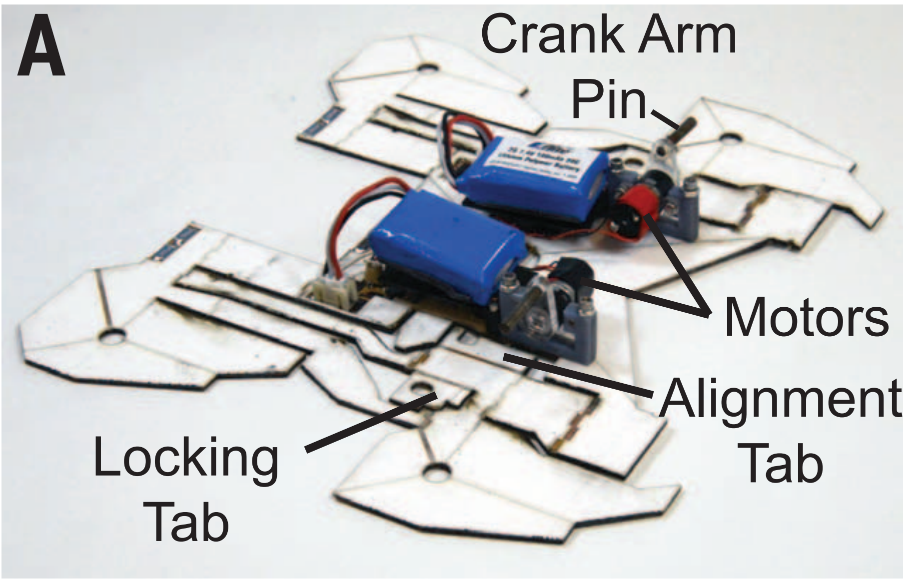
Folding of robot

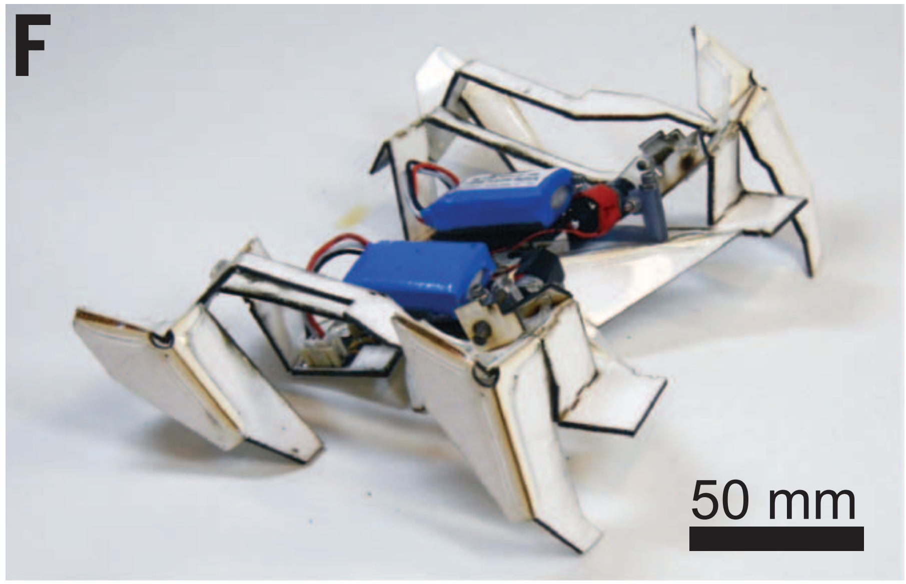
electric current through patterned copper network locally heats up and shrinks the "Shrinky-Dinks" layer



copper







How can we actuate the assembled robot?

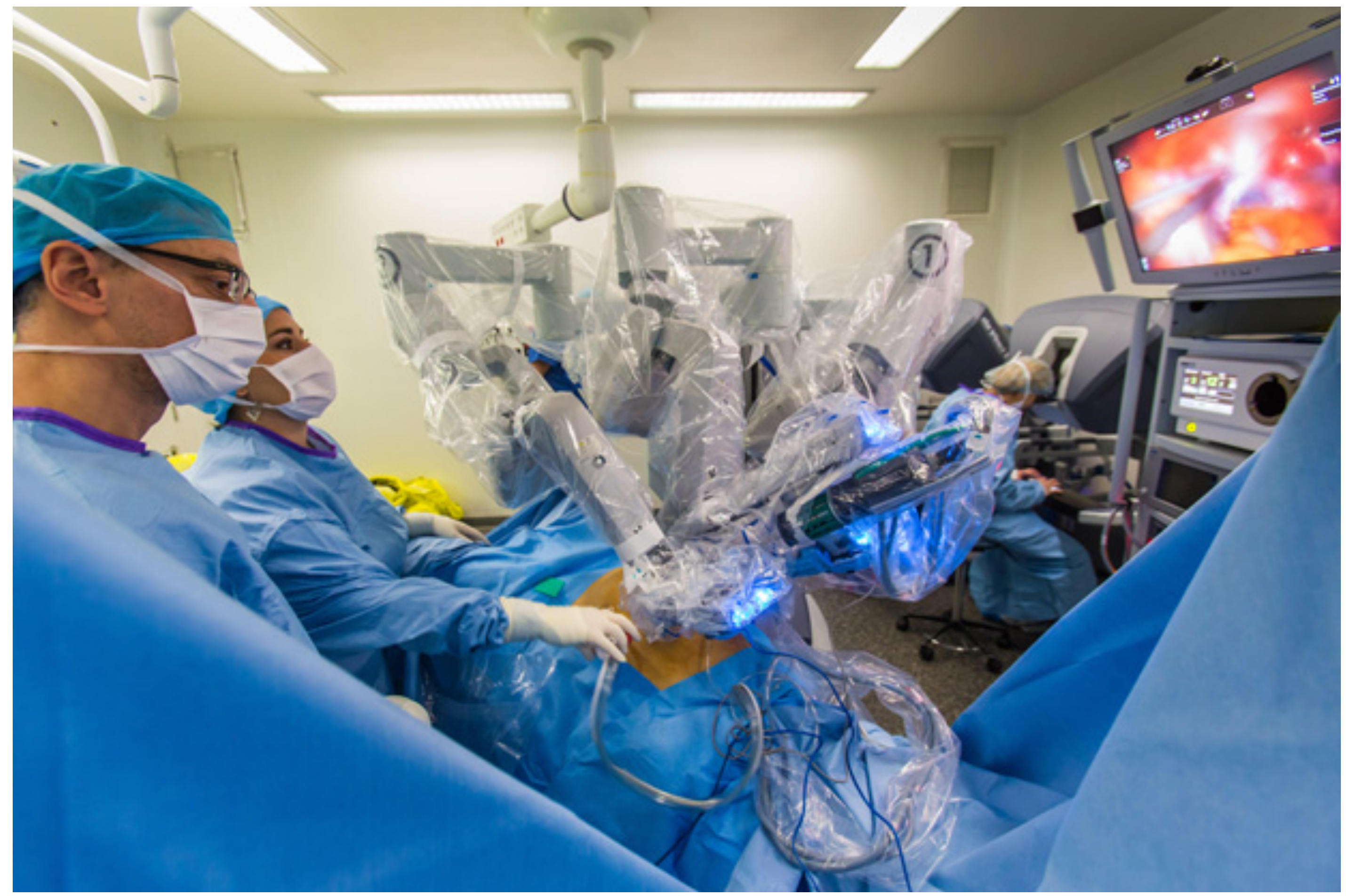
S. Felton et al., <u>Science</u> **345**, 644 (2014)

Structures with mechanisms

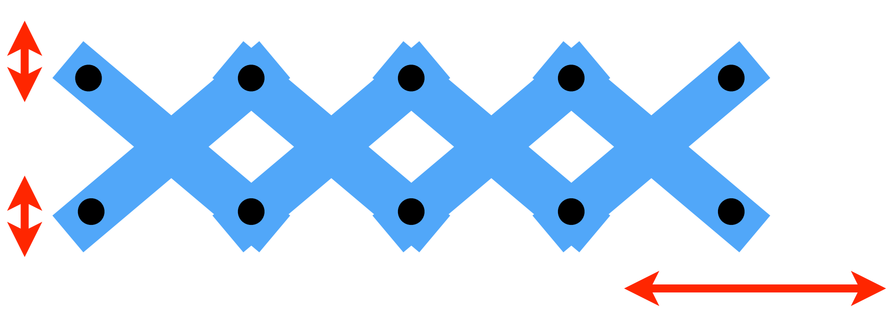
Structures composed of bars and hinges, which have fewer constraints than degrees of freedom, have specific mechanisms (=modes of deformations)

scissor lift

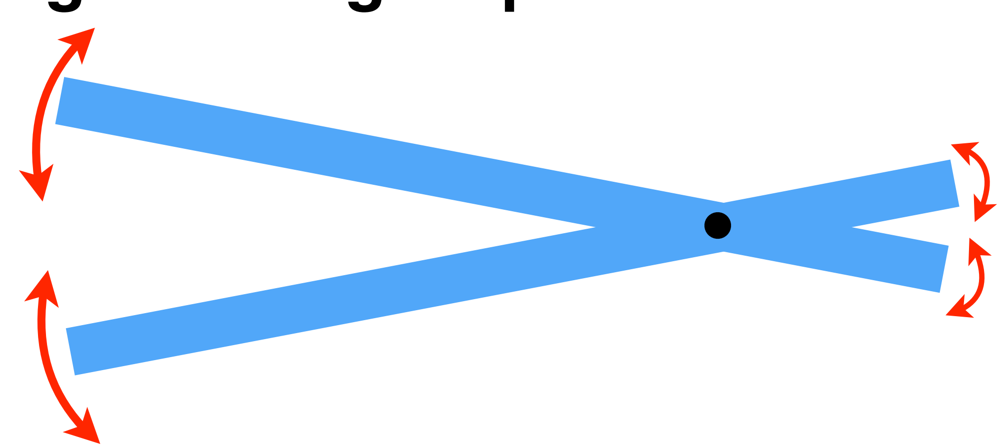
precise robotic surgeries



changing direction of motion

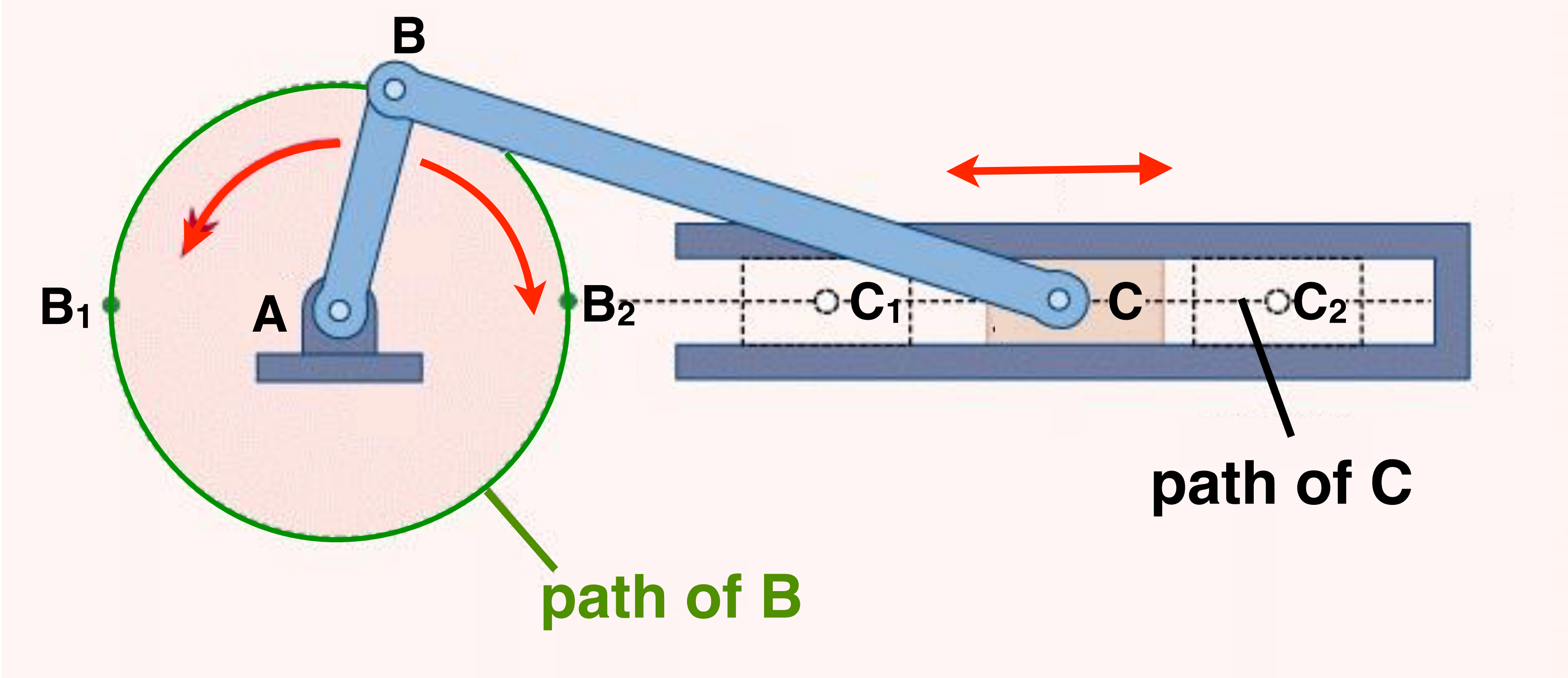


amplifying/reducing amplitude of motion

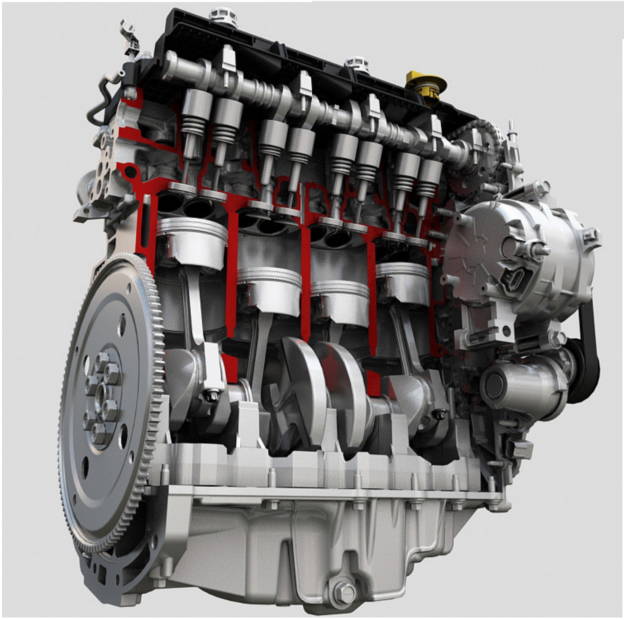


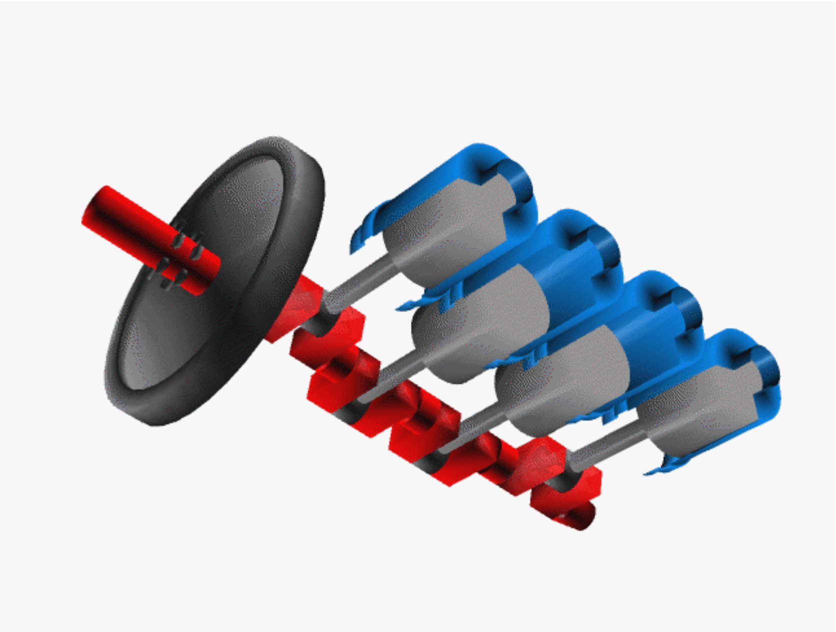
Crank slider mechanism

Crank slider mechanism converts linear to rotary motion!



Crank slider mechanism in car engines

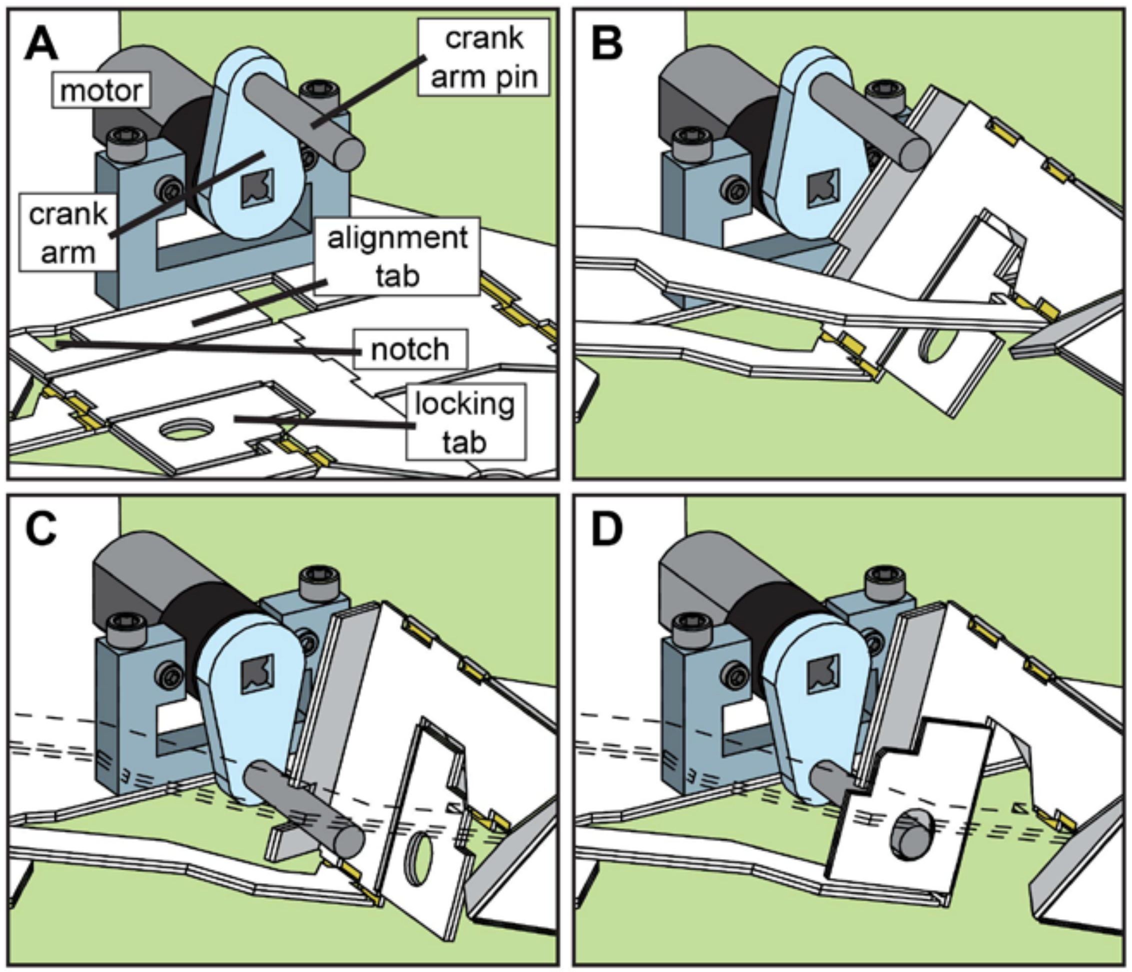




https://en.wikipedia.org/wiki/Crank_(mechanism)

Robot actuation

sequential folding enables locking of the crank arm to the robot structure



rotary motor moves the crank arm, which controls the movement of robot legs via a specific structure mechanism

S. Felton et al., Science **345**, 644 (2014)

hinge