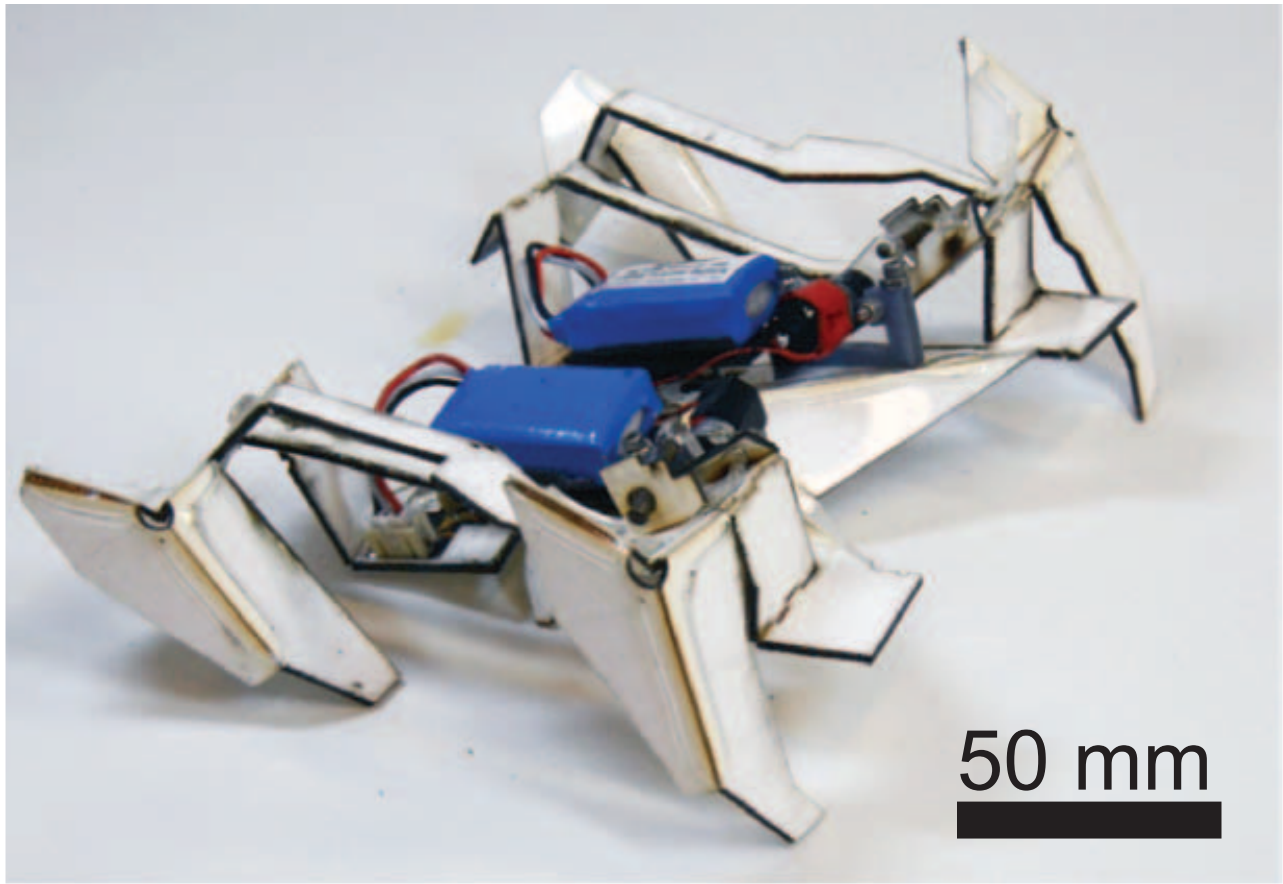
MAE 545: Lecture 11 (3/15)

Self-folding origami and robots

Helices



Shrinky-Dinks

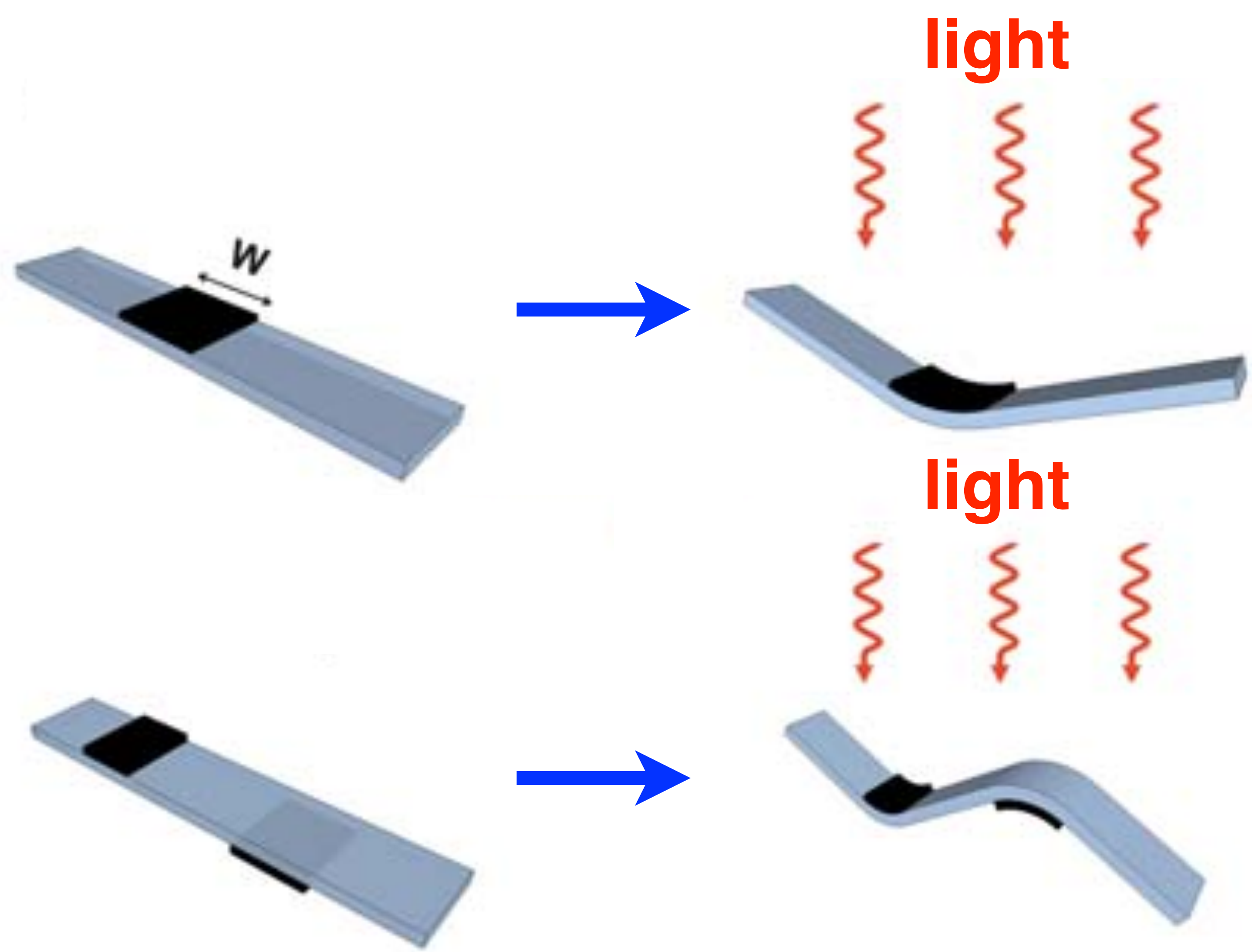
Shrinky-Dinks are sheets made of optically transparent, pre-strained polystyrene that shrink if heated to the glass transition temperature.

https://www.youtube.com/watch?v=m1mCoQFnOGU

Shrinky-Dinks

Shrinky-Dinks are sheets made of optically transparent, pre-strained polystyrene that shrink if heated to the glass transition temperature.

Localized heating and shrinking of Shrinky-Dinks can be achieved with patterning of black ink that absorb light.

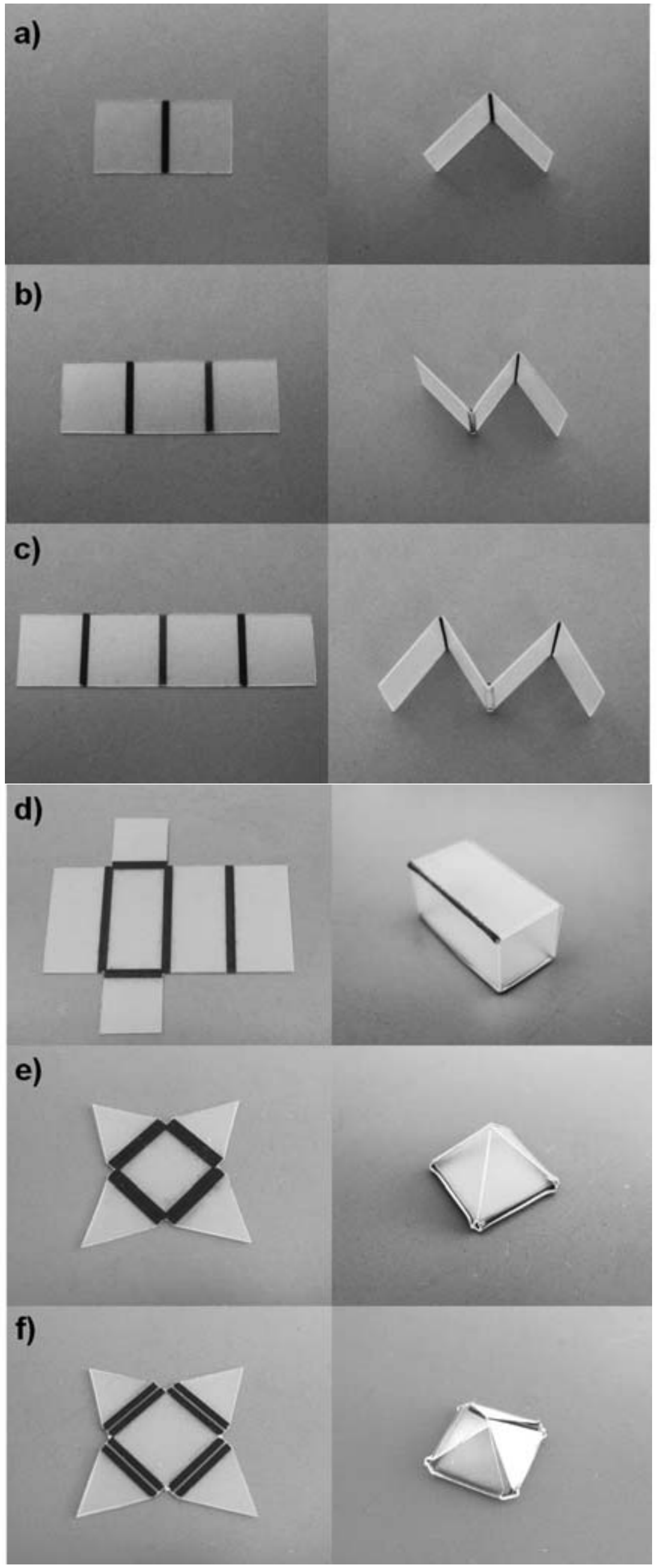


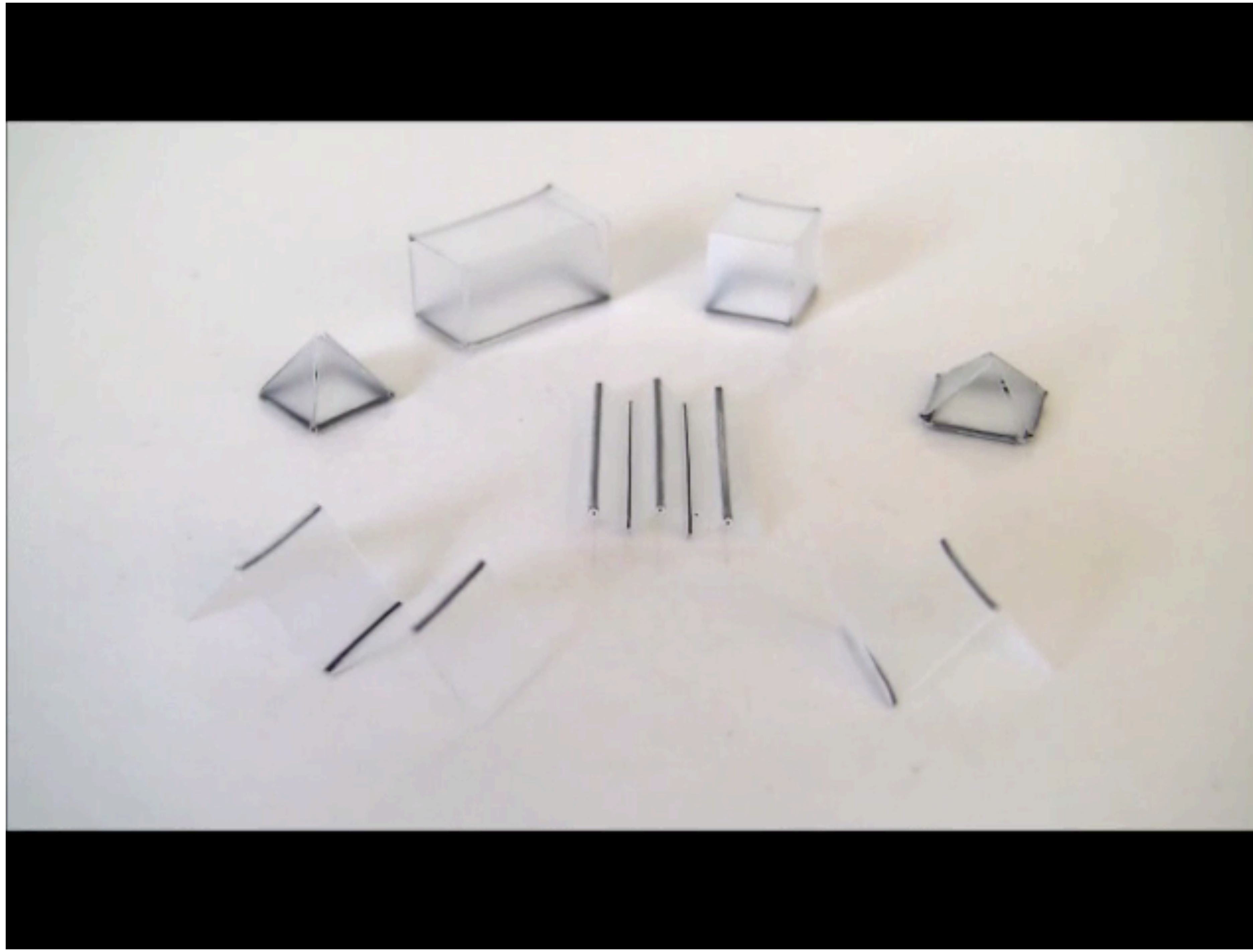
Folding angle can be controlled with the width of ink and with the exposure time of light.

J. Liu et al., Soft Mater **8**, 1764 (2012)

Shrinky-Dinks origami

size ~ cm

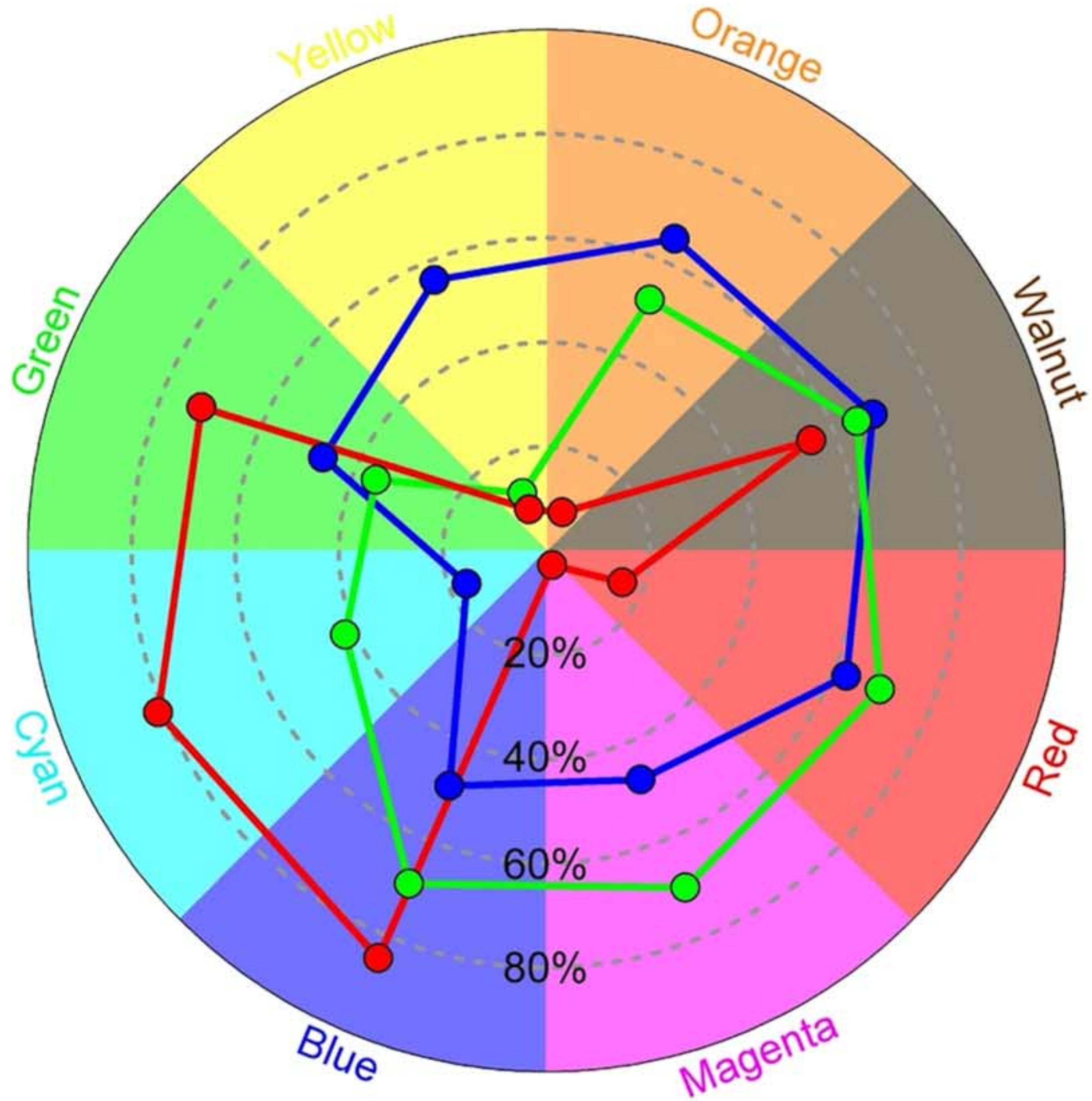


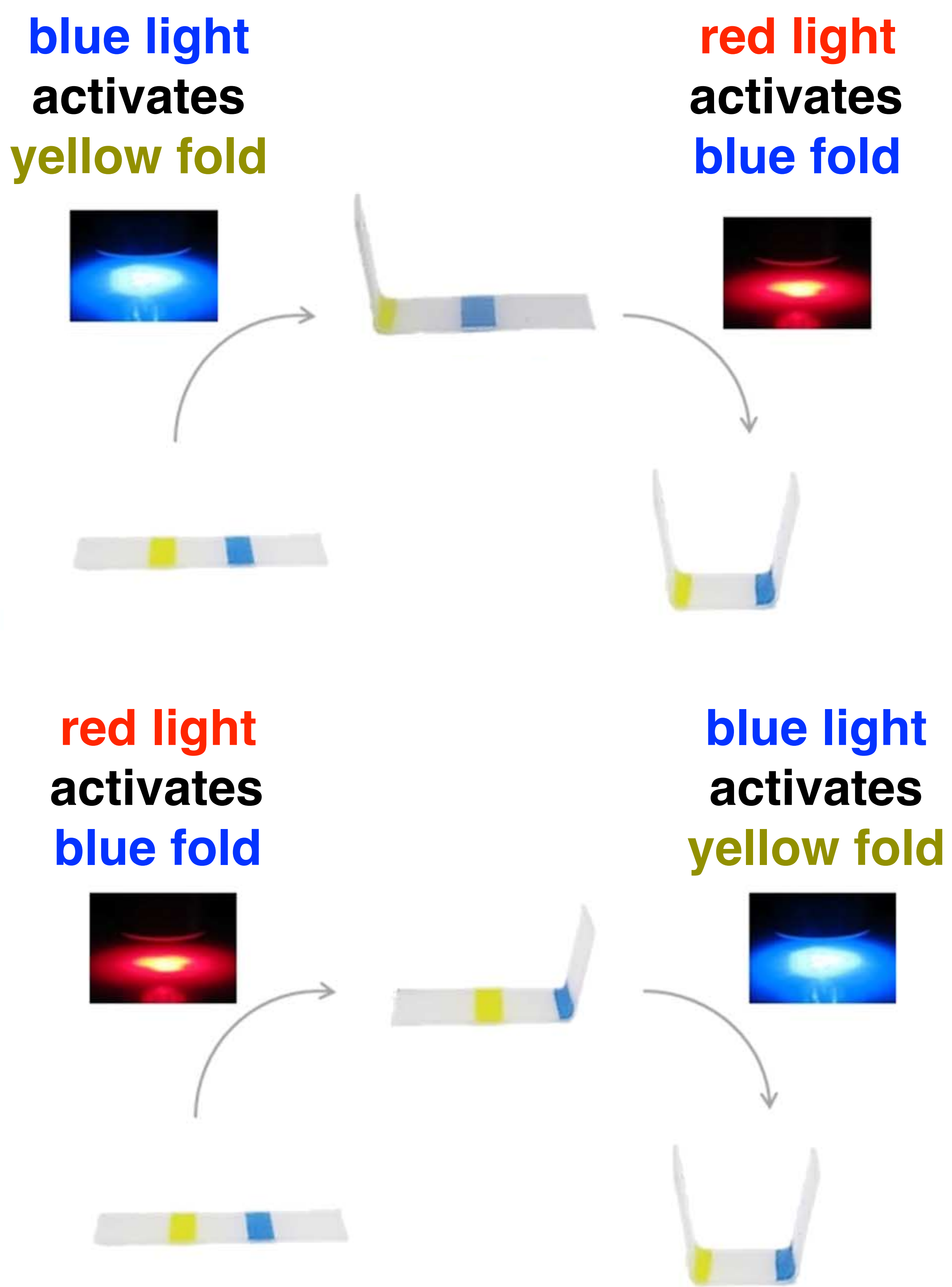


J. Liu et al., Soft Mater **8**, 1764 (2012)

Sequential folding of Shrinky-Dinks origami

Different ink colors have different absorption spectra for red, green and blue light.

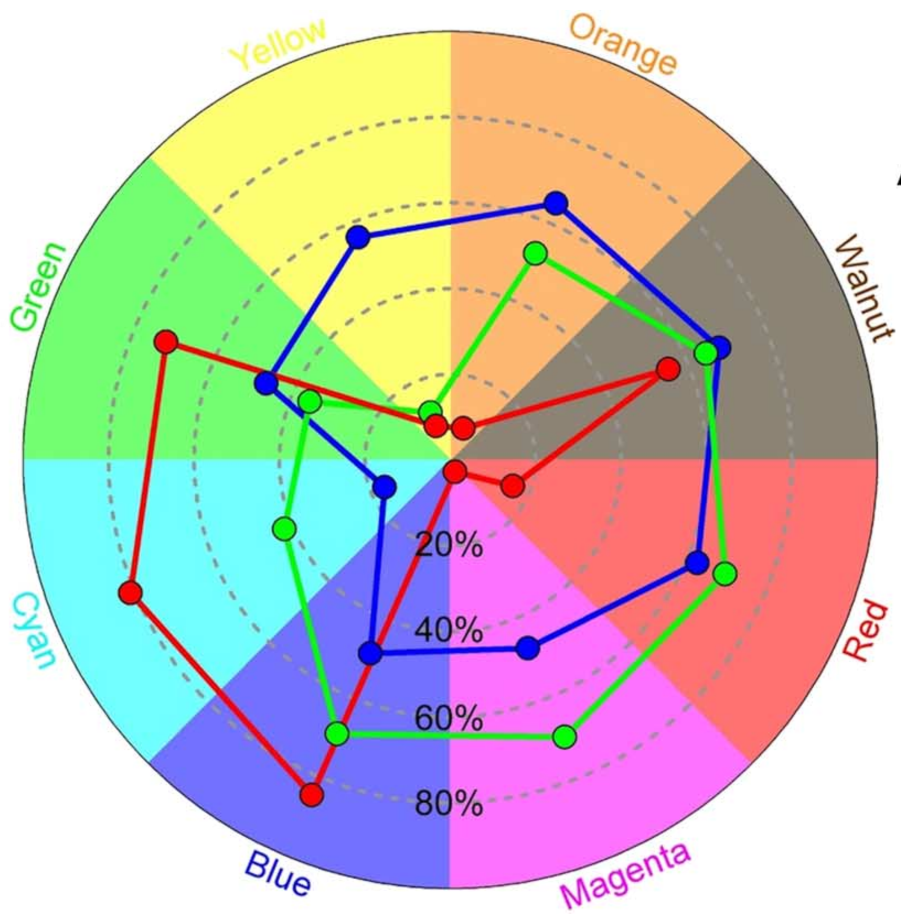


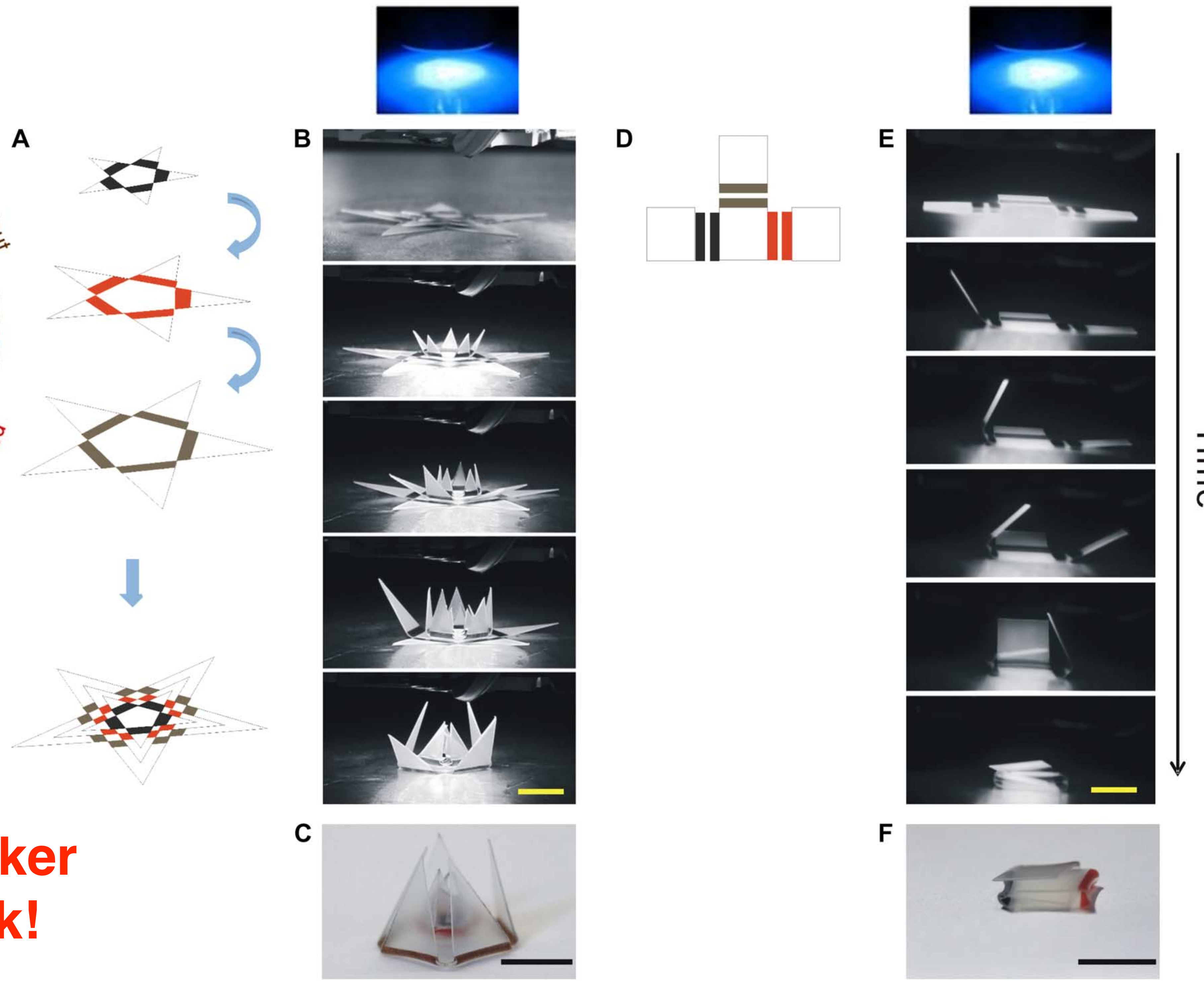


J. Liu et al., Sci. Adv. **3**, e1602417 (2017)

Sequential folding of Shrinky-Dinks origami

The order of folding corresponds to the amount of absorbed blue light (black > red > walnut)



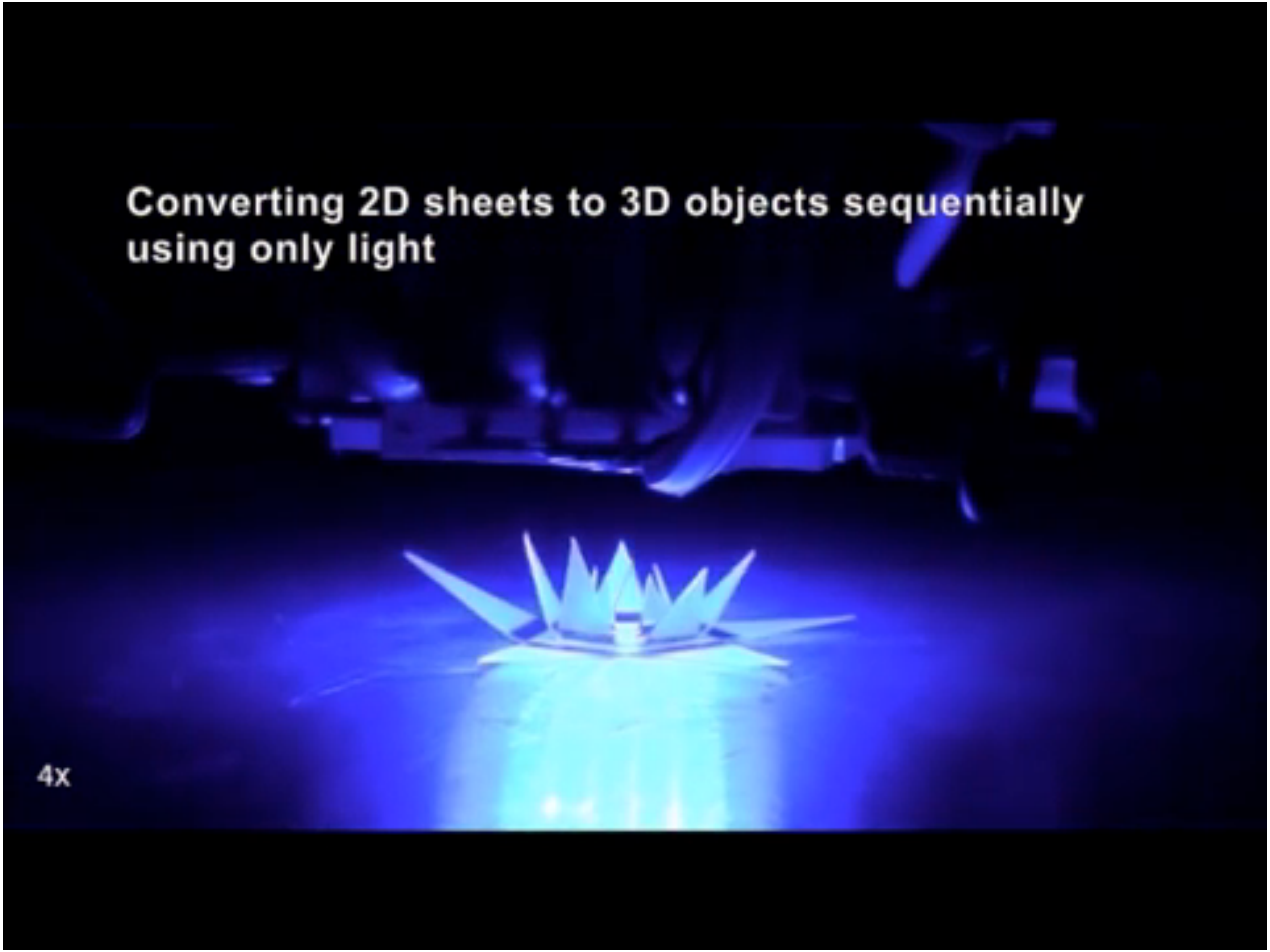


1 cm 1 cm

Note: red ink is thicker than the walnut ink!

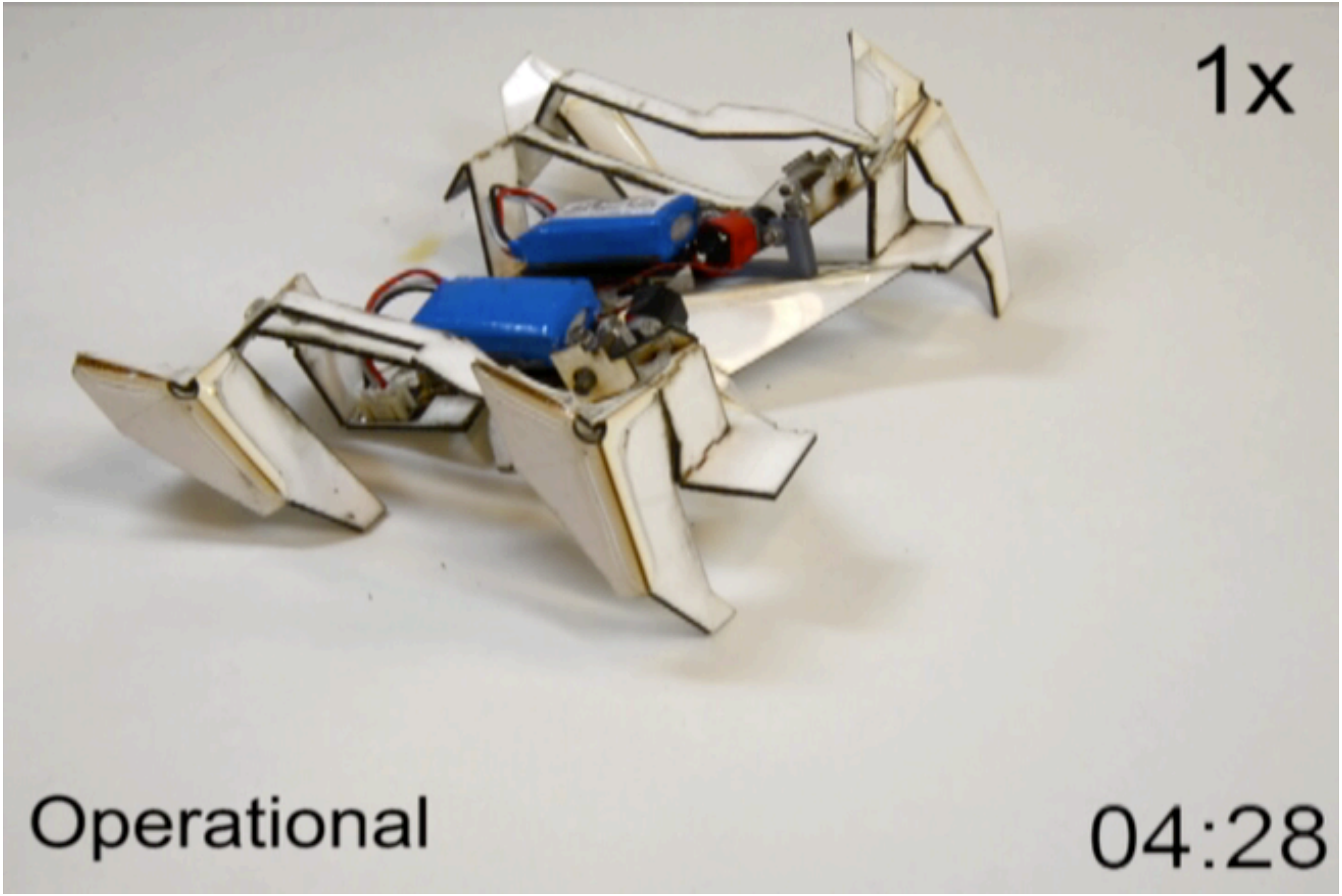
J. Liu et al., Sci. Adv. **3**, e1602417 (2017)

Sequential folding of Shrinky-Dinks origami



J. Liu et al., Sci. Adv. **3**, e1602417 (2017)

Self-folding robots (in 4 min)



S. Felton et al., Science **345**, 644 (2014)

Robot assembly

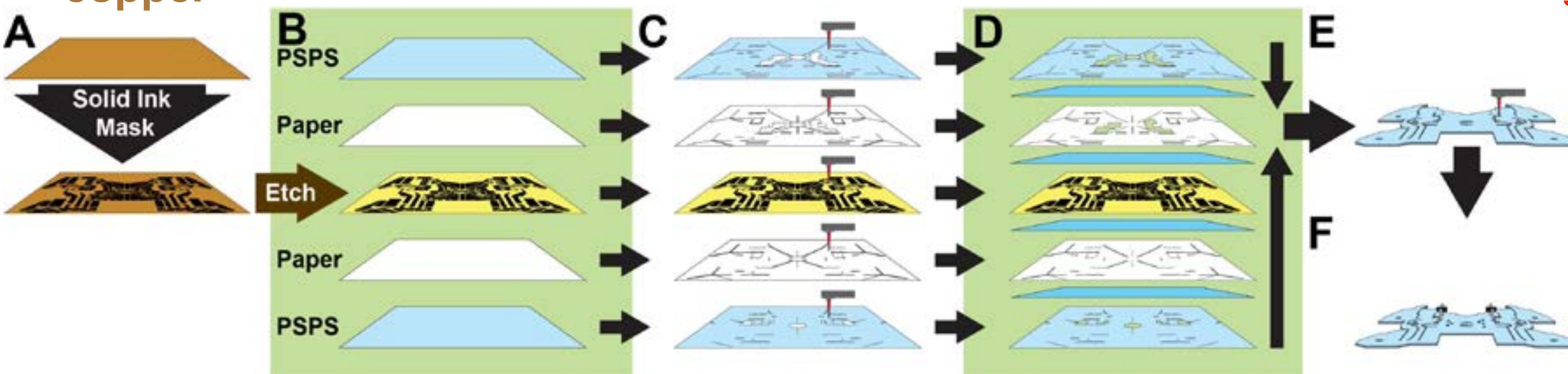
Chemical etching of copper outside the ink mask

Laser cutting of layers

Gluing of layers

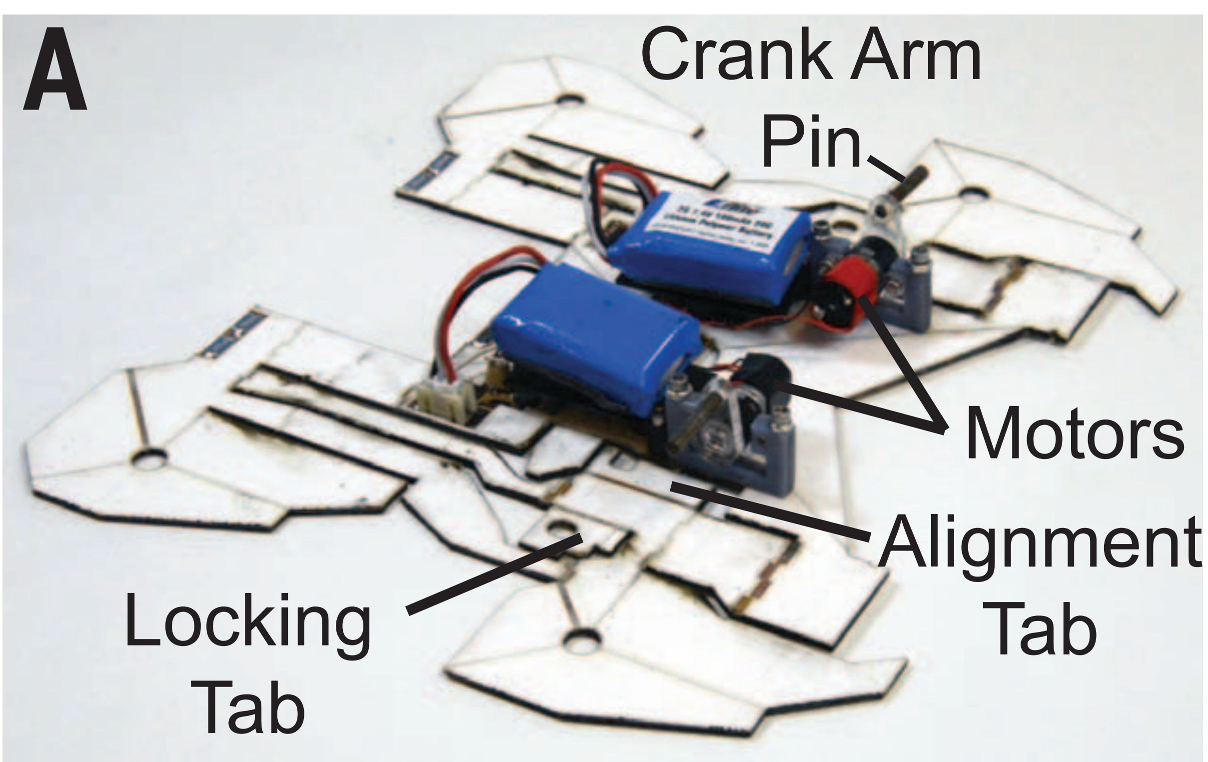
copper

Laser cutting



"Shrinky-Dinks"

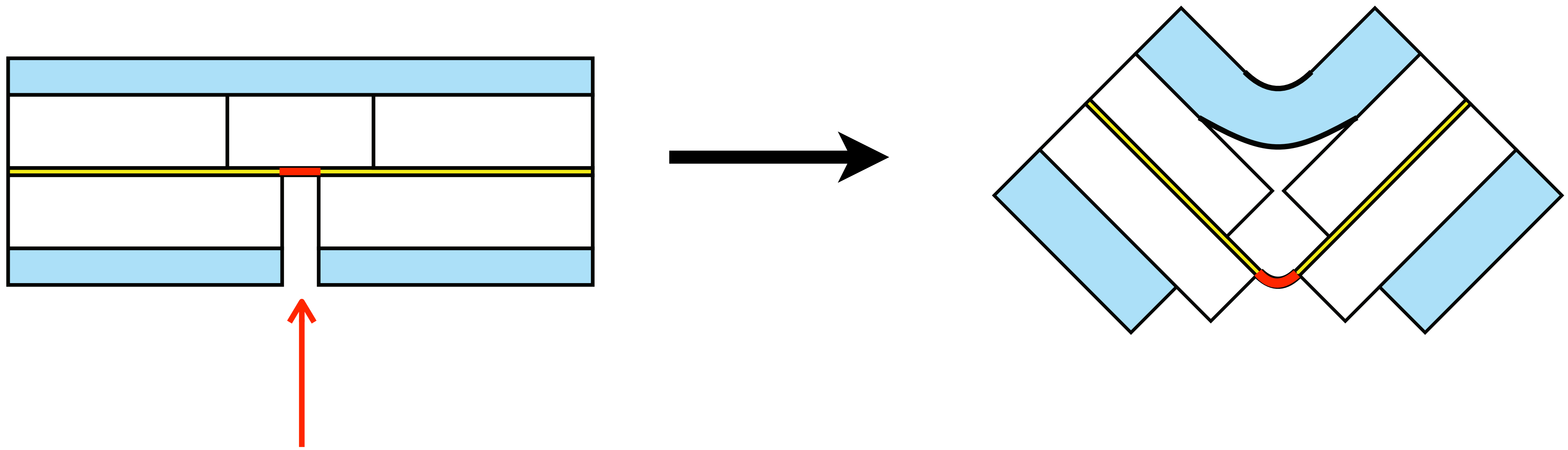
installment of electrical components, motors, and batteries



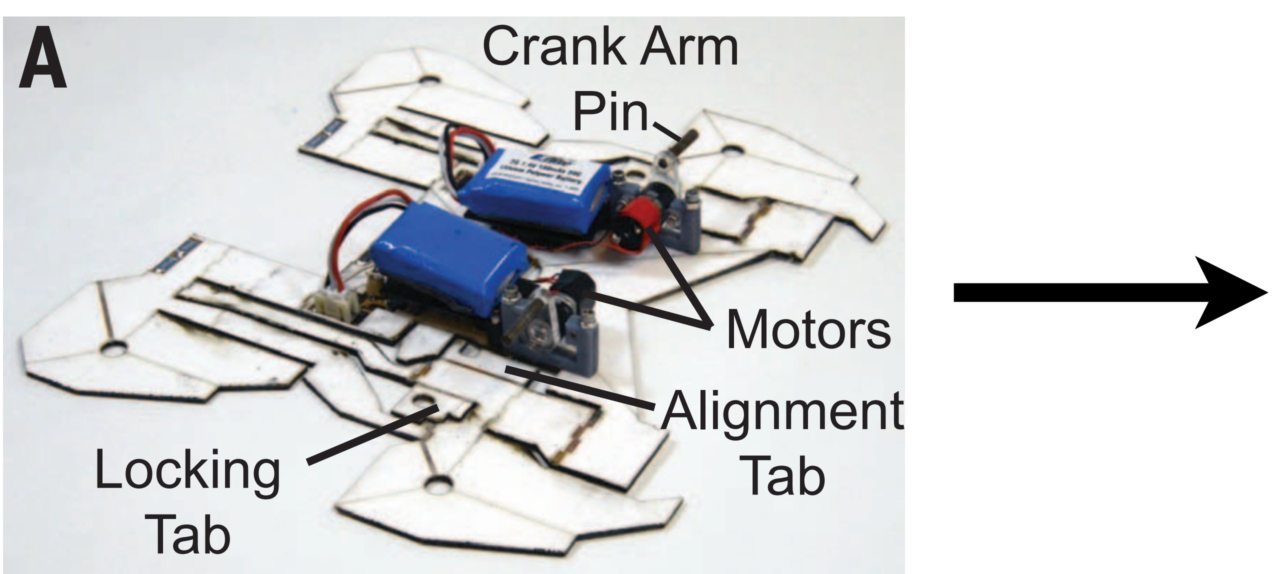
S. Felton et al., Science **345**, 644 (2014)

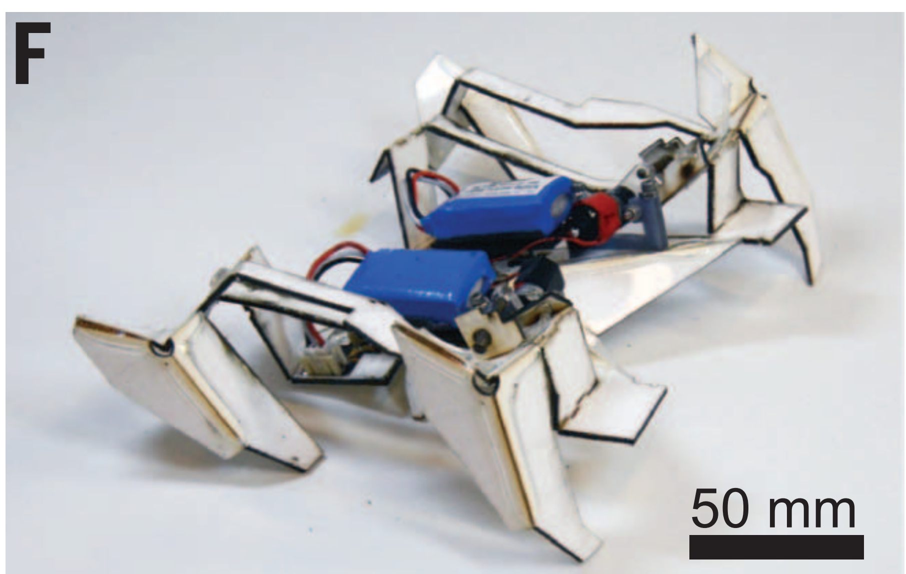
Folding of robot

electric current through patterned copper network locally heats up and shrinks the “Shrinky-Dinks” layer



copper





How can we actuate the assembled robot?

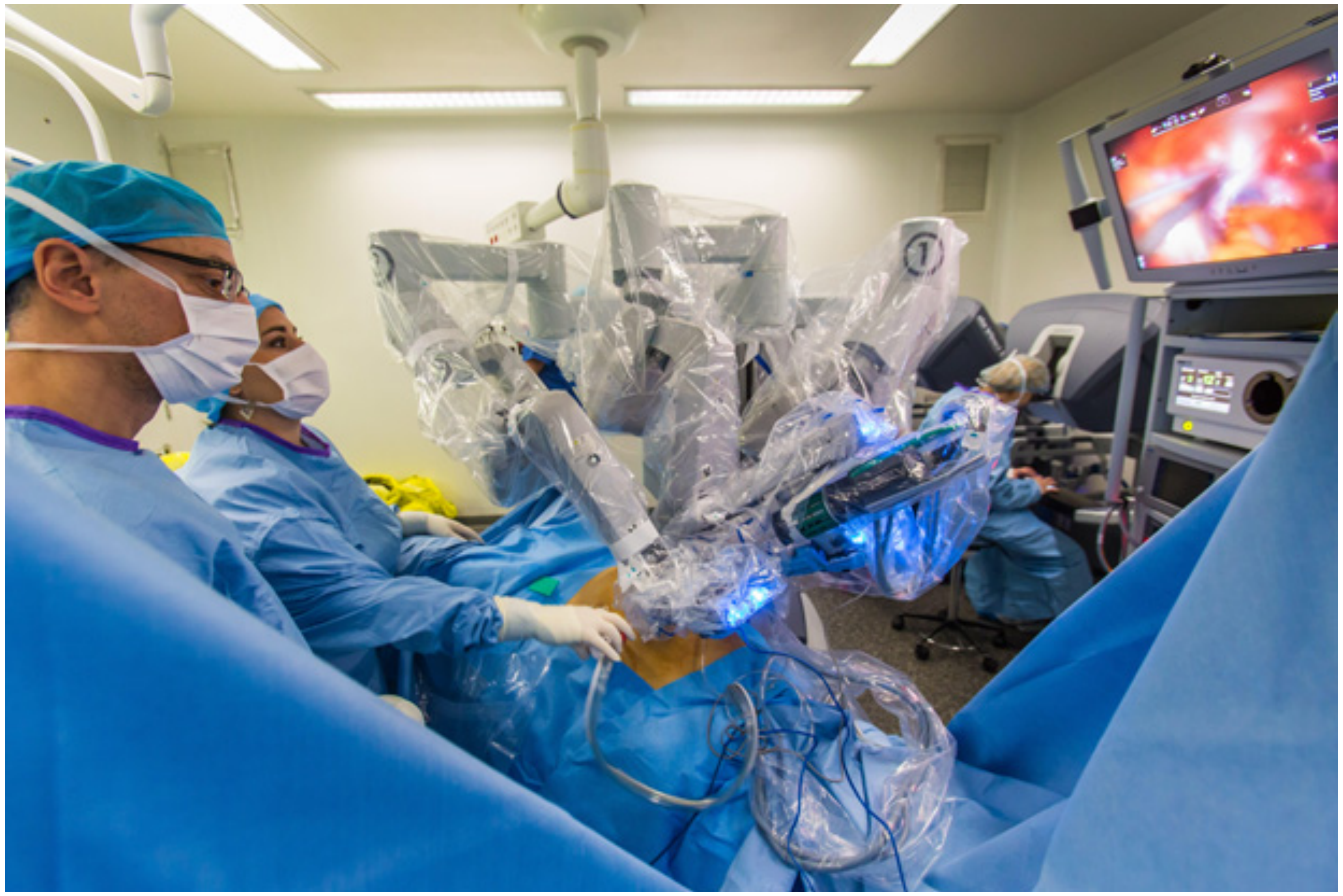
S. Felton et al., Science **345**, 644 (2014)

Structures with mechanisms

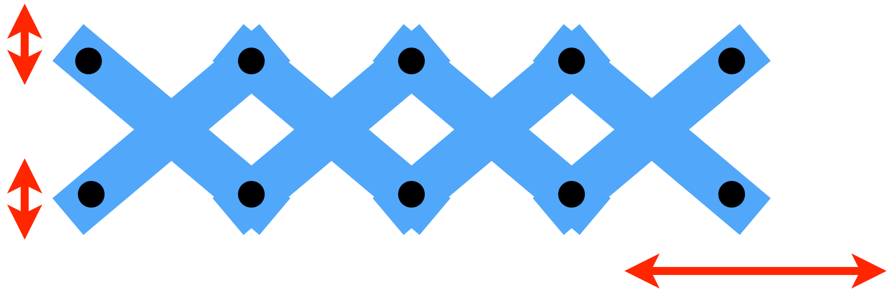
Structures composed of bars and hinges, which have fewer constraints than degrees of freedom, have specific mechanisms (=modes of deformations)

scissor lift

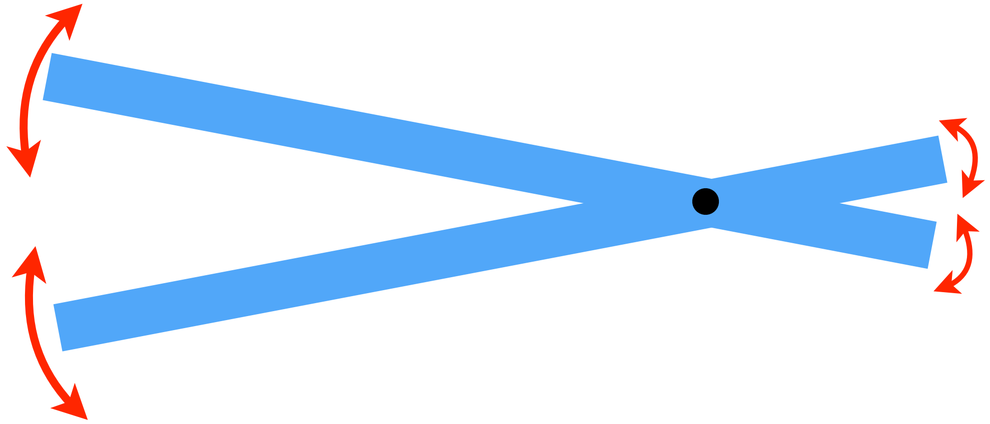
precise robotic surgeries



changing direction of motion

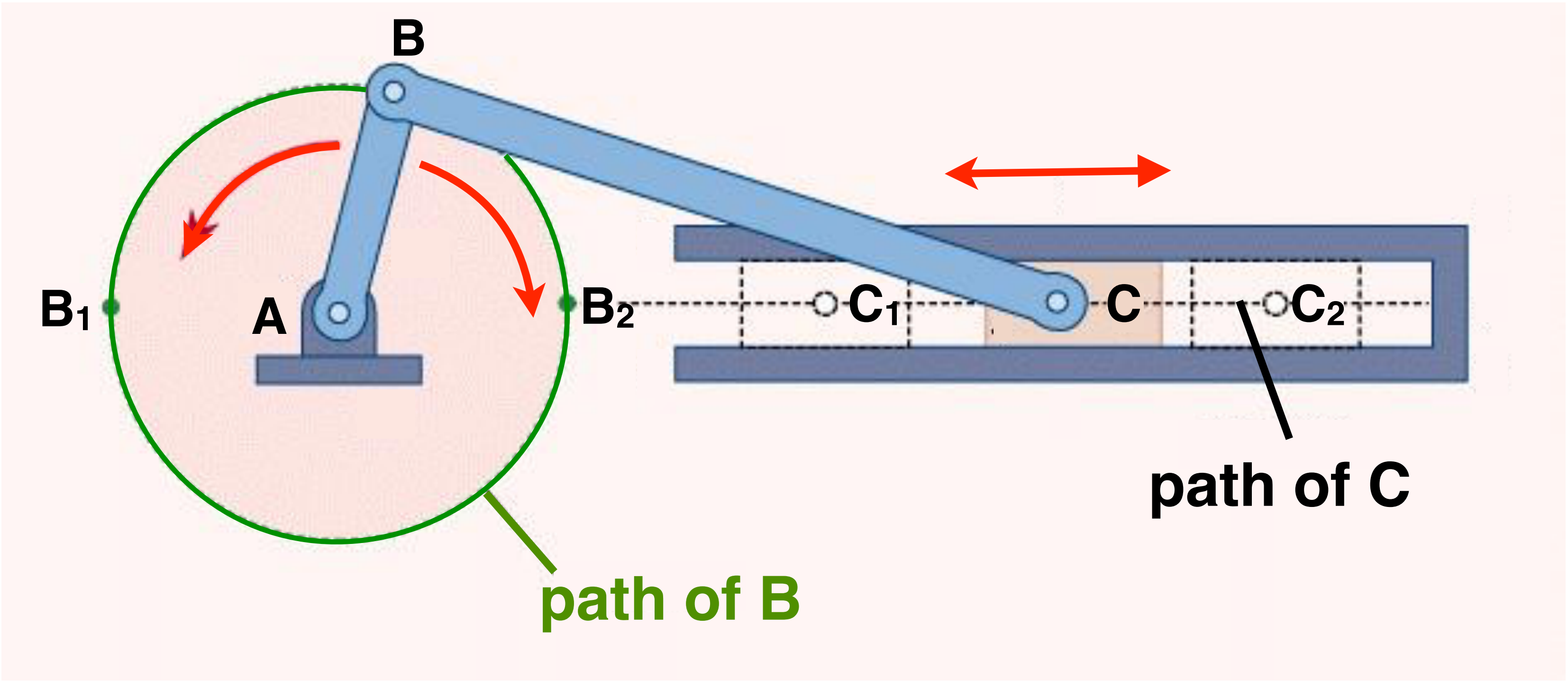


amplifying/reducing amplitude of motion

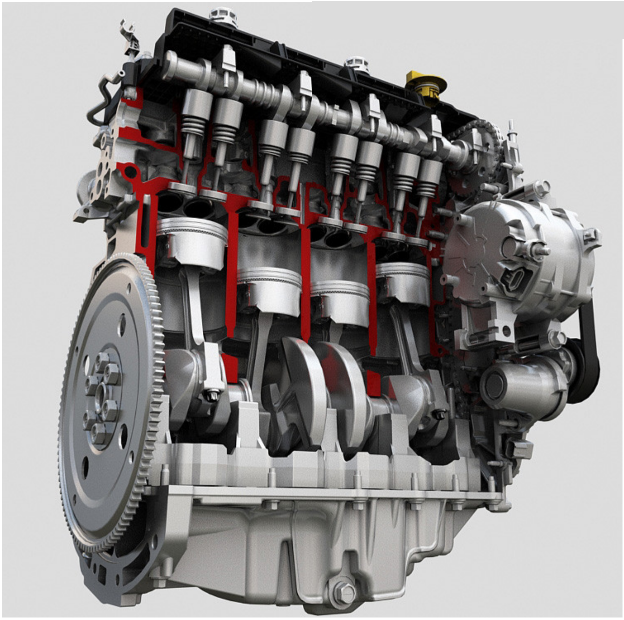


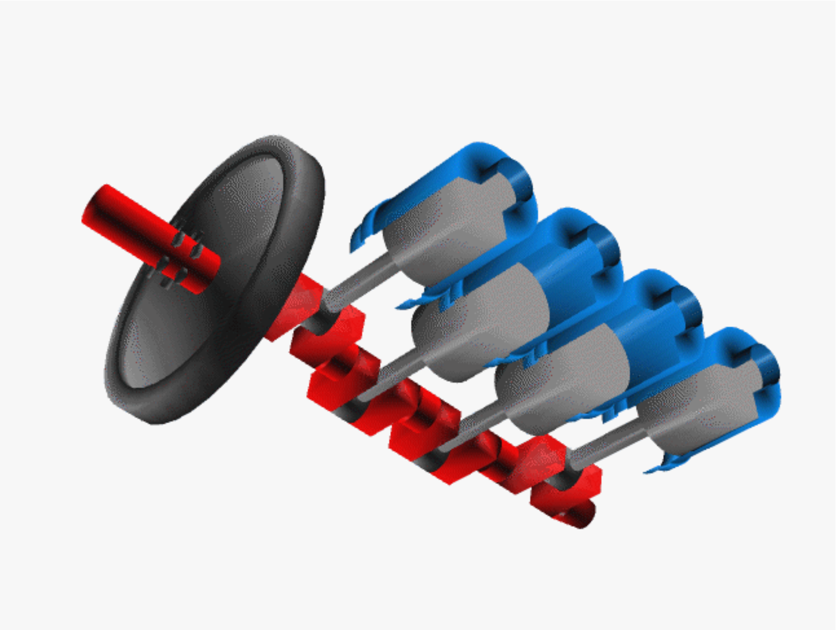
Crank slider mechanism

Crank slider mechanism converts linear to rotary motion!



Crank slider mechanism in car engines

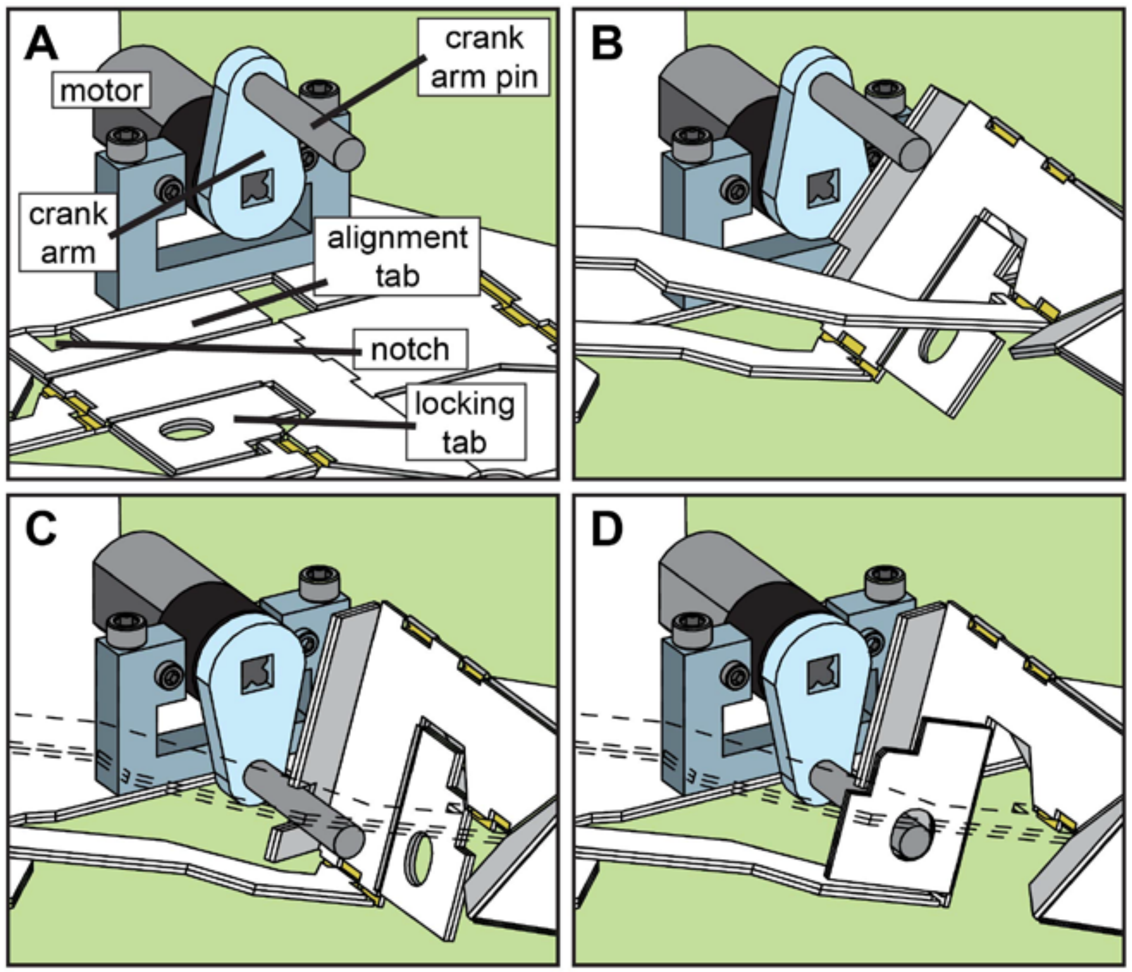




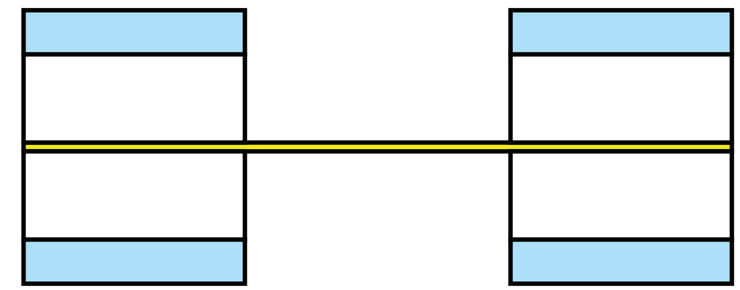
https://en.wikipedia.org/wiki/Crank_(mechanism)

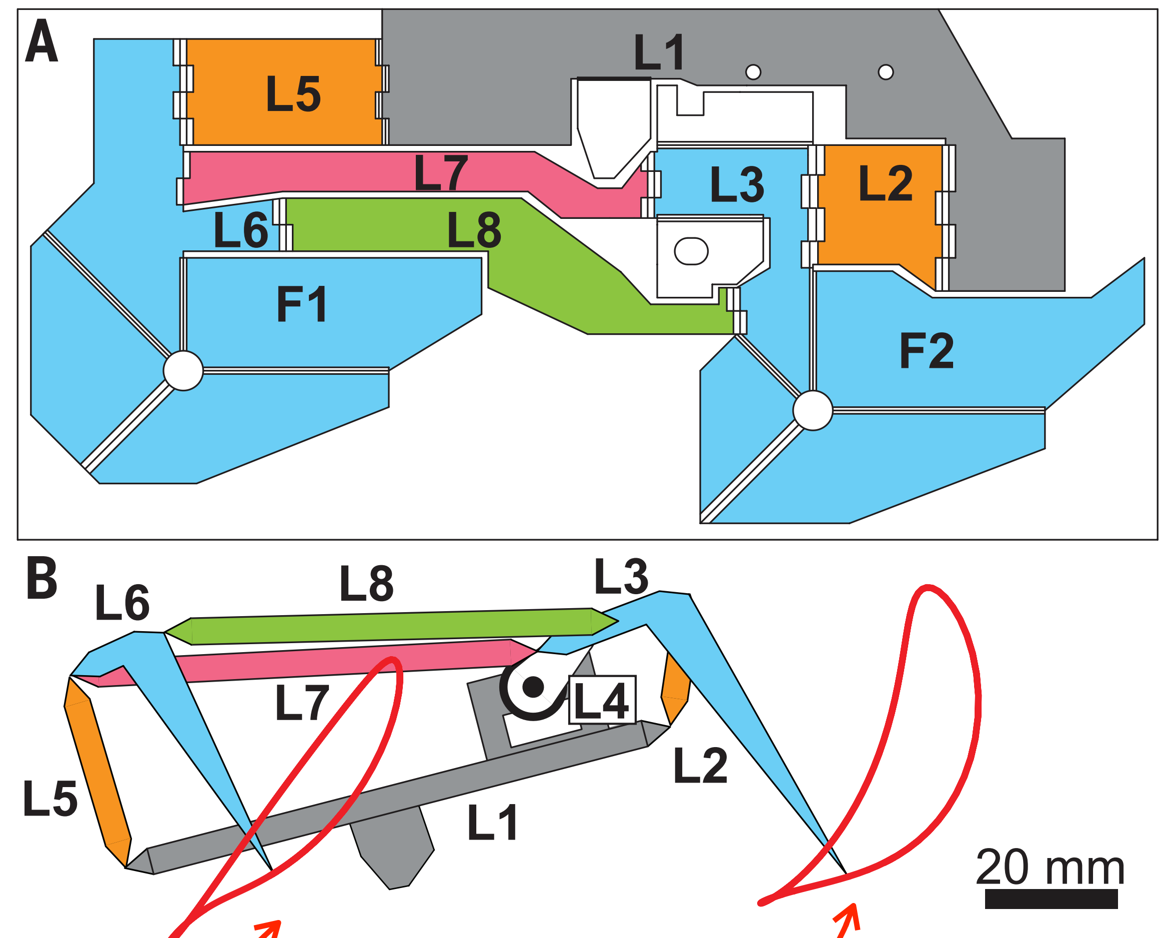
Robot actuation

sequential folding enables locking of the crank arm to the robot structure



hinge





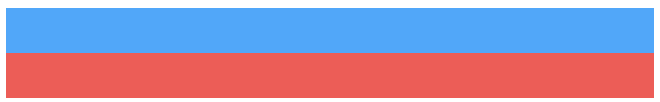
rotary motor moves the crank arm, which controls the movement of robot legs via a specific structure mechanism

S. Felton et al., Science **345**, 644 (2014)

Helices in plants

How are helices formed?

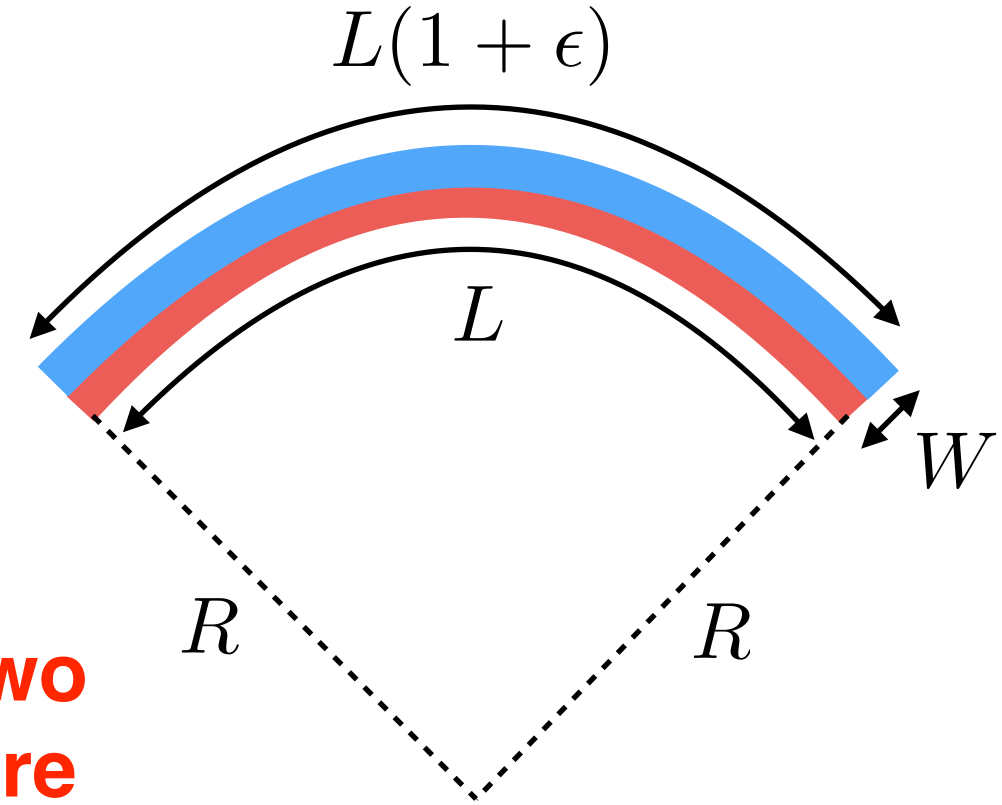
Differential growth or differential shrinking produces spontaneous curvature



faster growth of the top layer

→

more shrinking of the bottom layer



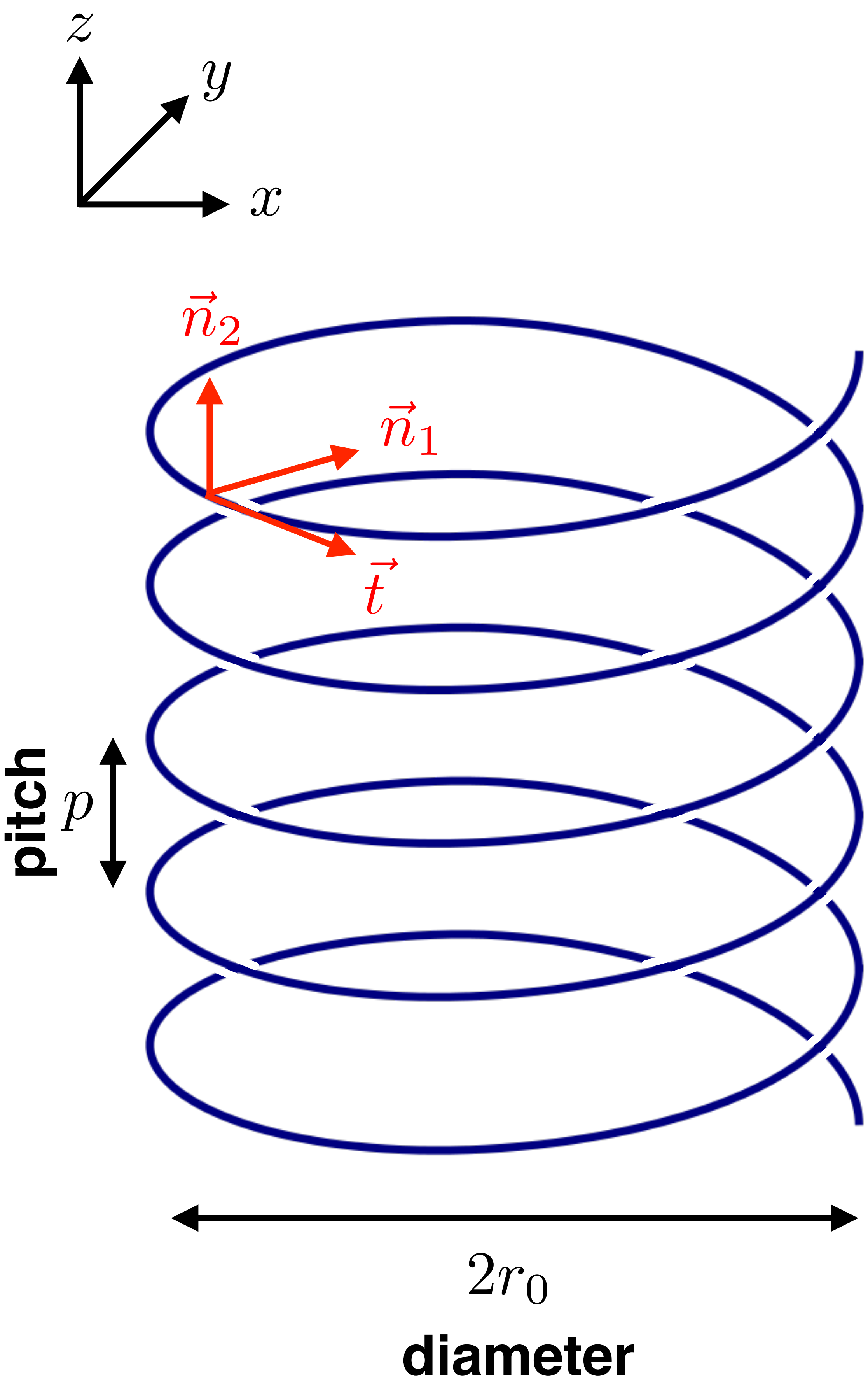
Differential growth (shrinking) of the two layers produces spontaneous curvature

$$\boxed{K = \frac{1}{R} = \frac{\epsilon}{W}}$$

$$\frac{L(1+\epsilon)}{L} = \frac{R+W}{R}$$

Filaments that are longer than $L > 2\pi R$ form helices to avoid steric interactions.

Helix



Mathematical description

$$\vec{r}(s) = \left(r_0 \cos(s/\lambda), r_0 \sin(s/\lambda), \frac{p}{2\pi\lambda} s \right)$$

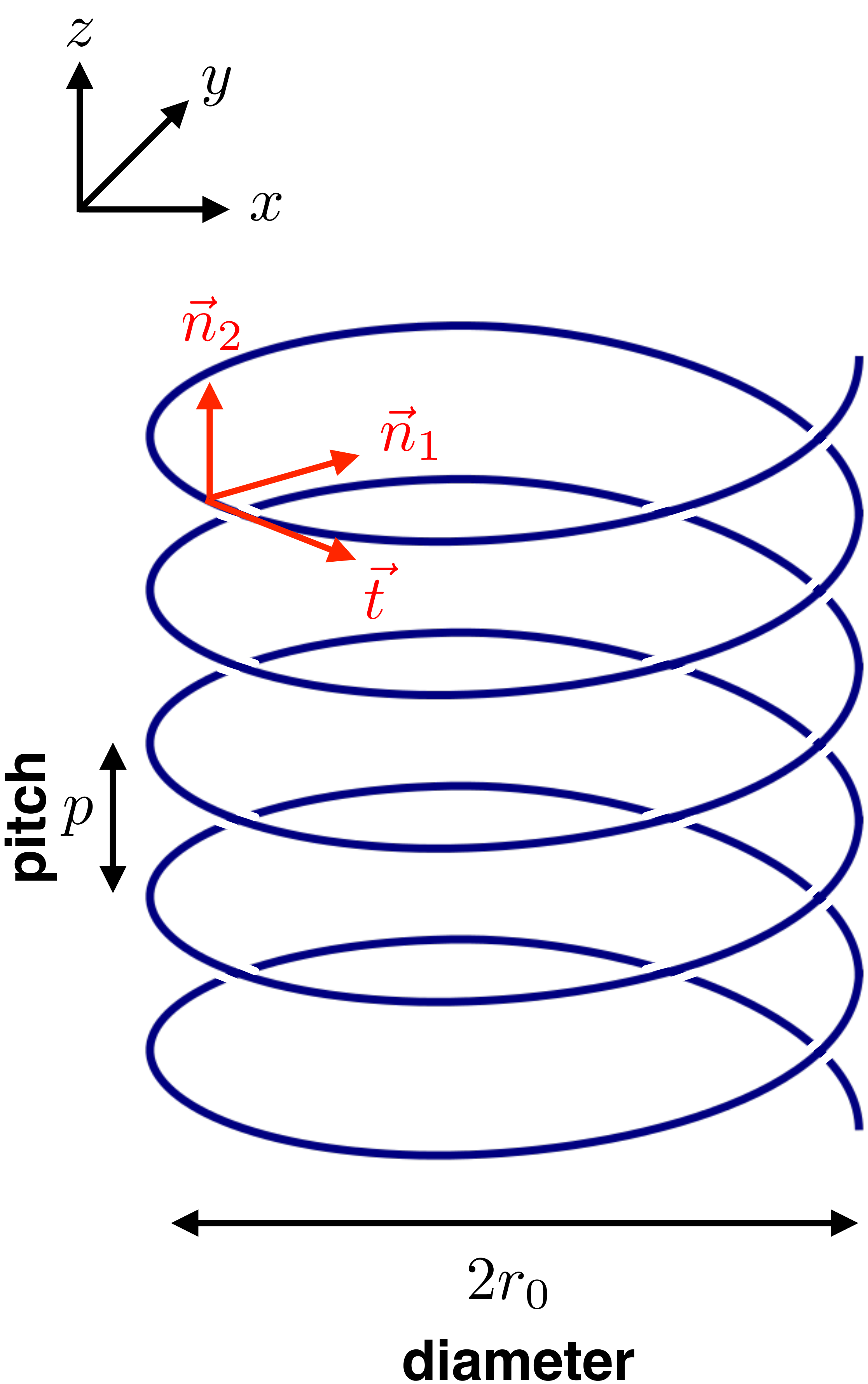
Set λ to fix the metric in natural parametrization:

$$\vec{t}(s) = \frac{d\vec{r}}{ds} = \left(-\frac{r_0}{\lambda} \sin(s/\lambda), \frac{r_0}{\lambda} \cos(s/\lambda), \frac{p}{2\pi\lambda} \right)$$

$$g = \vec{t} \cdot \vec{t} = \frac{r_0^2}{\lambda^2} + \frac{p^2}{4\pi^2\lambda^2} = 1$$

$$\lambda = \sqrt{r_0^2 + (p/2\pi)^2}$$

Helix



Mathematical description

$$\vec{r}(s) = \left(r_0 \cos(s/\lambda), r_0 \sin(s/\lambda), \frac{p}{2\pi\lambda} s \right)$$

$$\lambda = \sqrt{r_0^2 + (p/2\pi)^2}$$

Tangent and normal vectors

$$\vec{t}(s) = \frac{d\vec{r}}{ds} = \left(-\frac{r_0}{\lambda} \sin(s/\lambda), \frac{r_0}{\lambda} \cos(s/\lambda), \frac{p}{2\pi\lambda} \right)$$

$$\vec{n}_1(s) = \left(-\cos(s/\lambda), -\sin(s/\lambda), 0 \right)$$

$$\vec{n}_2(s) = \left(\frac{p}{2\pi\lambda} \sin(s/\lambda), -\frac{p}{2\pi\lambda} \cos(s/\lambda), \frac{r_0}{\lambda} \right)$$

Helix curvatures

$$\vec{n}_1 \cdot \frac{d^2\vec{r}}{ds^2} = \frac{r_0}{\lambda^2} = \frac{r_0}{r_0^2 + (p/2\pi)^2} = K$$

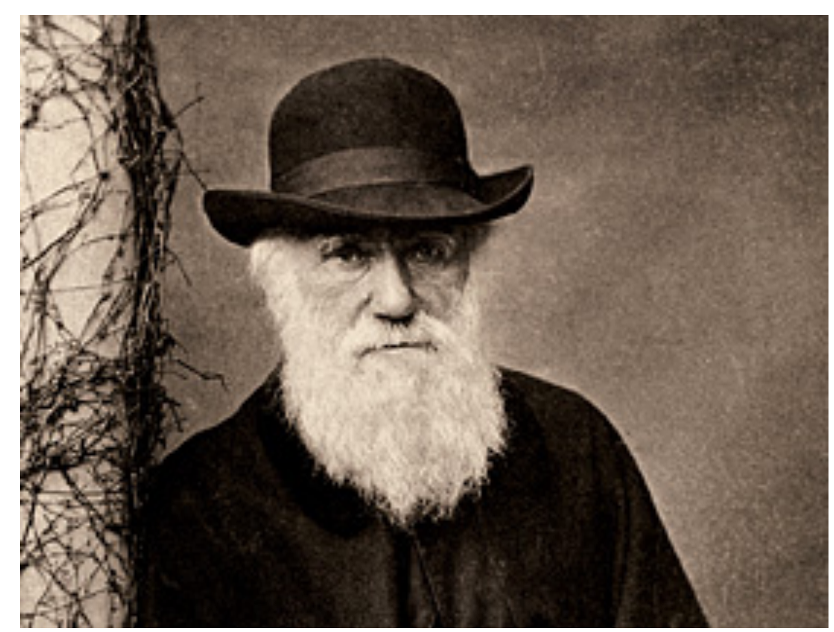
$$\vec{n}_2 \cdot \frac{d^2\vec{r}}{ds^2} = 0$$

Cucumber tendril climbing via helical coiling

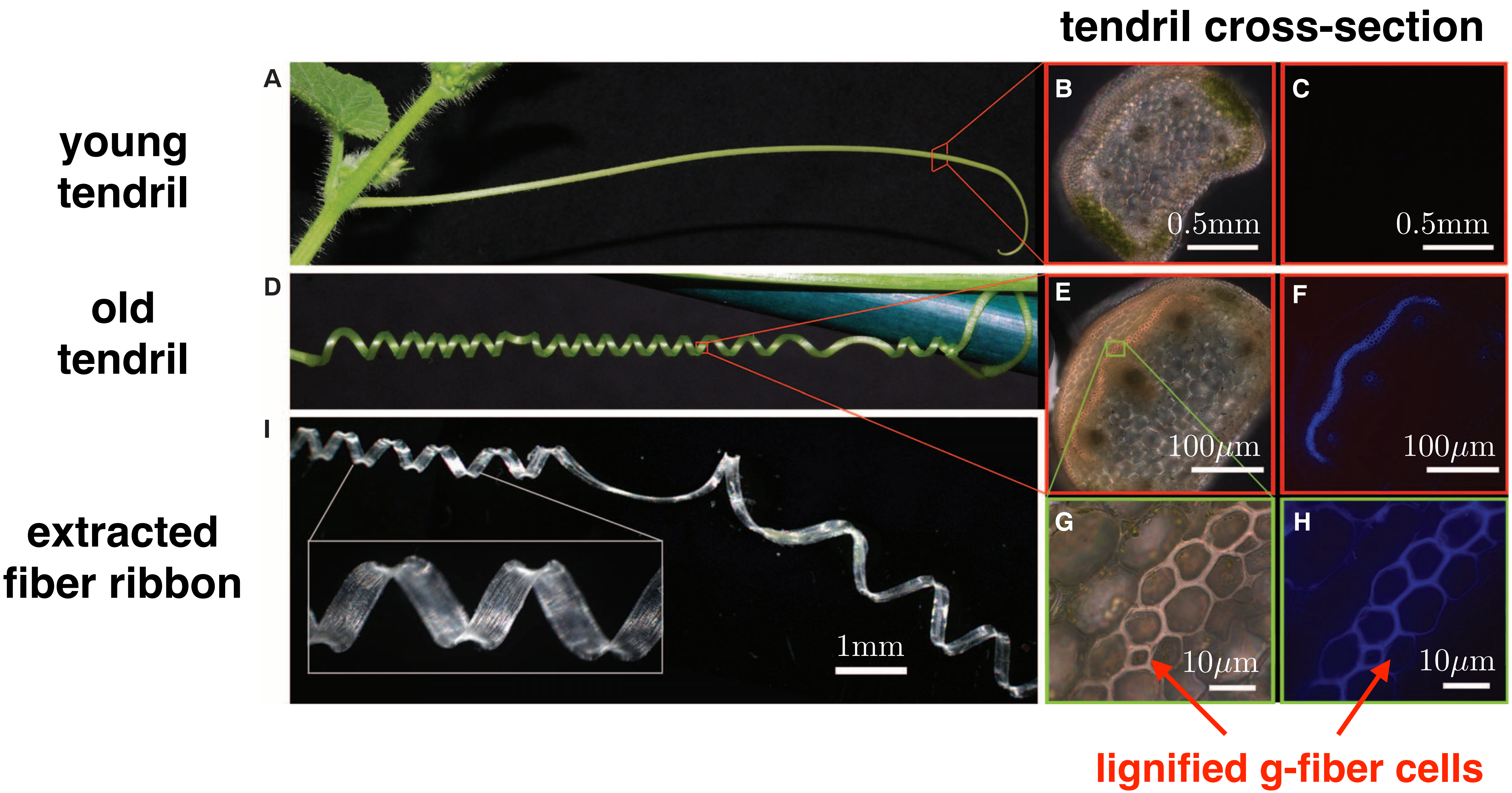
S. J. Gerbode et al., Science 337, 1087 (2012)

Cucumber tendrils want to pull themselves up above other plants in order to get more sunlight.

Already studied by Charles Darwin in 1865:



Helical coiling of cucumber tendril



Coiling in older tendrils is due to a thin layer of stiff, lignified gelatinous fiber cells, which are also found in wood.

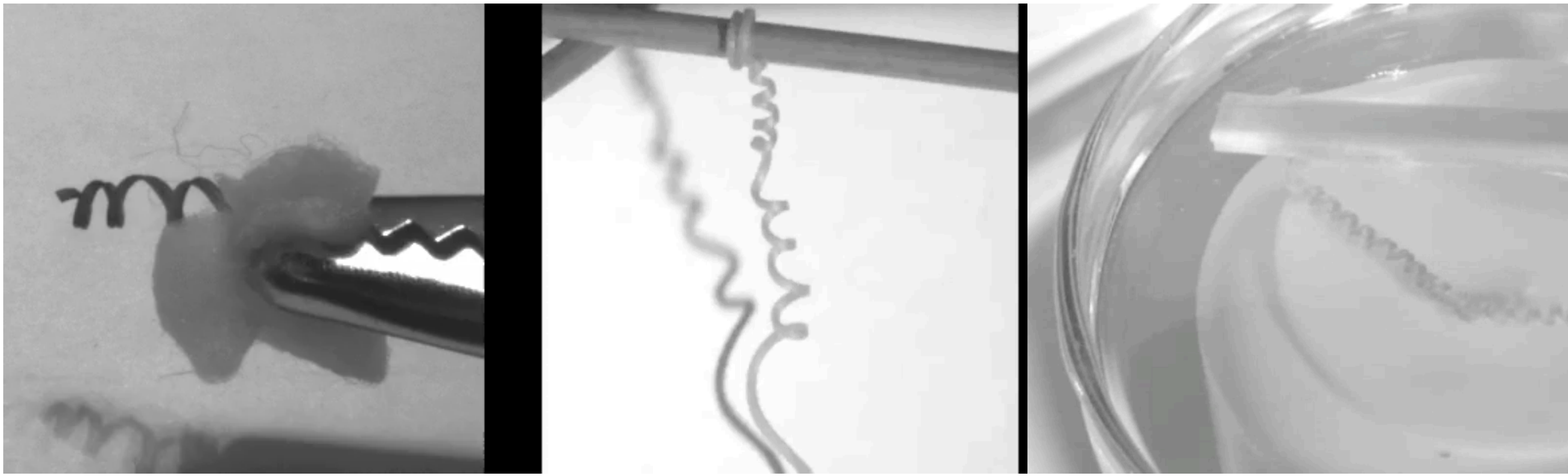
S. J. Gerbode et al., Science 337, 1087 (2012)

Helical coiling of cucumber tendril

Drying of fibber ribbon increases coiling

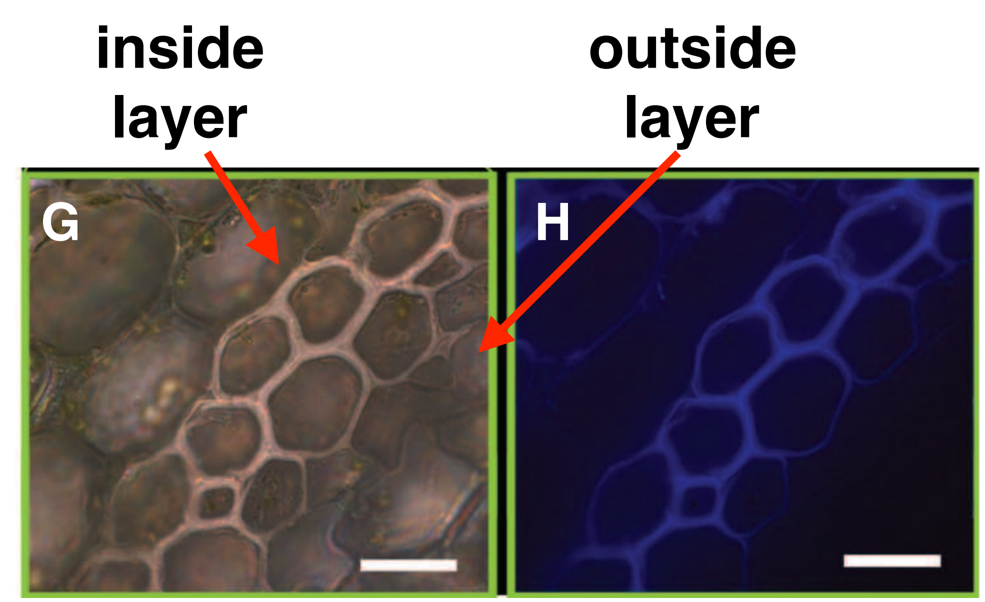
Drying of tendril increases coiling

Rehydrating of tendril reduces coiling



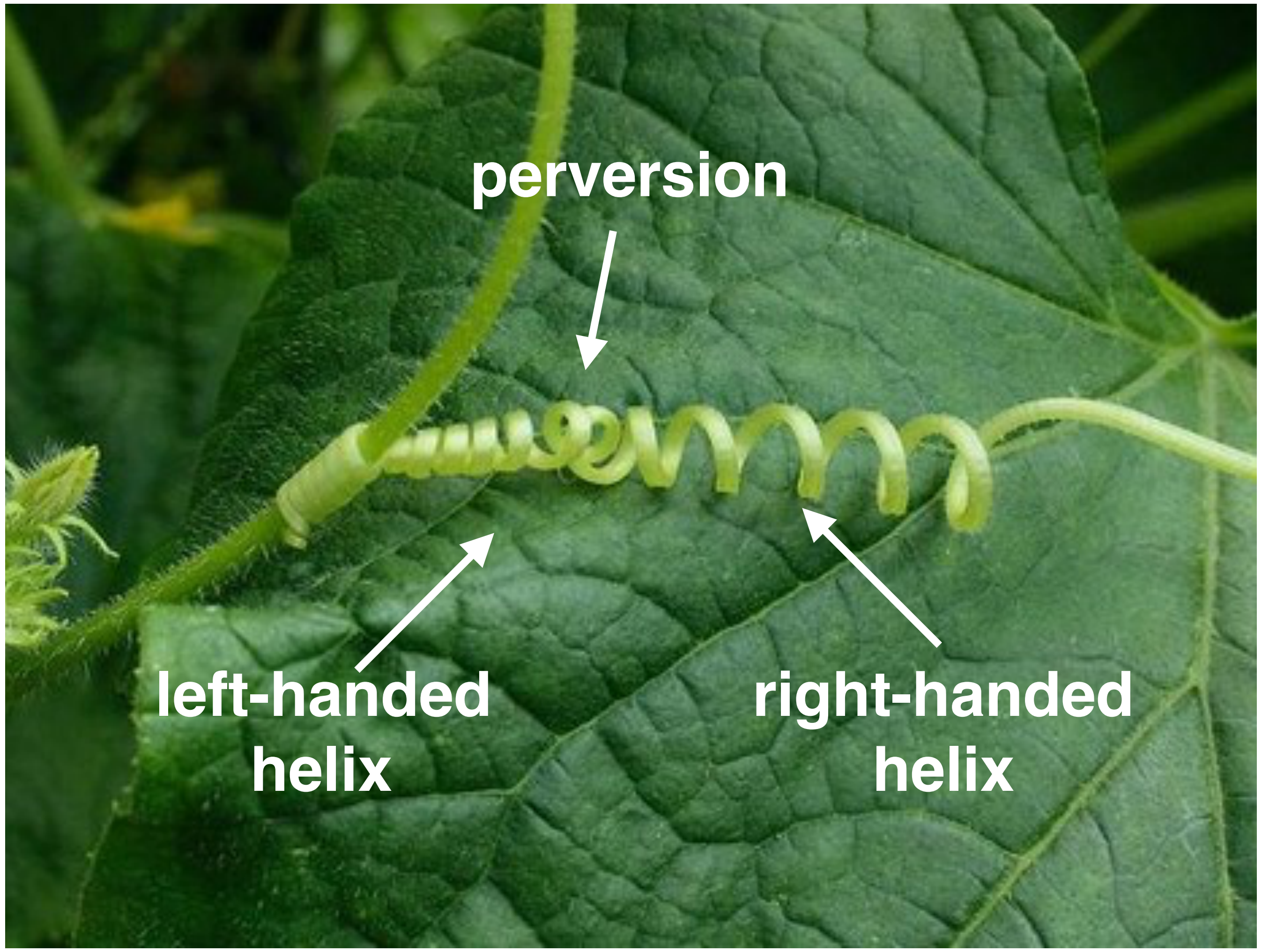
During the lignification of g-fiber cells water is expelled, which causes shrinking.

The inside layer is more lignified and therefore shrinks more and is also stiffer than the outside layer.



S. J. Gerbode et al., Science 337, 1087 (2012)

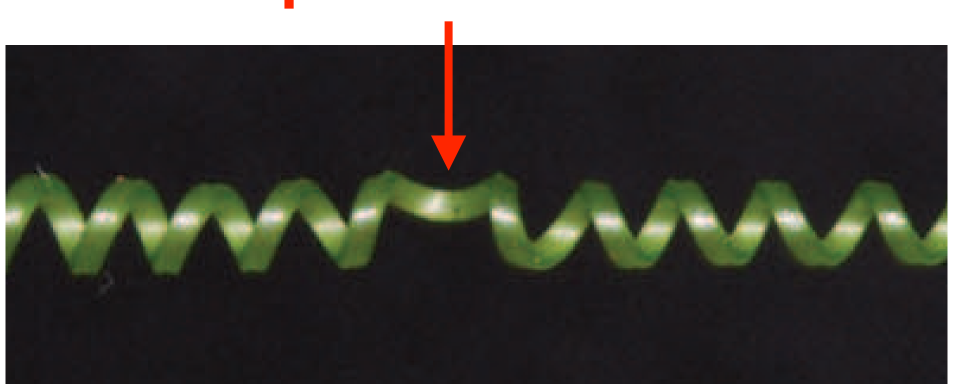
Coiling of tendrils in opposite directions



Ends of the tendril are fixed and cannot rotate. This constraints the linking number.

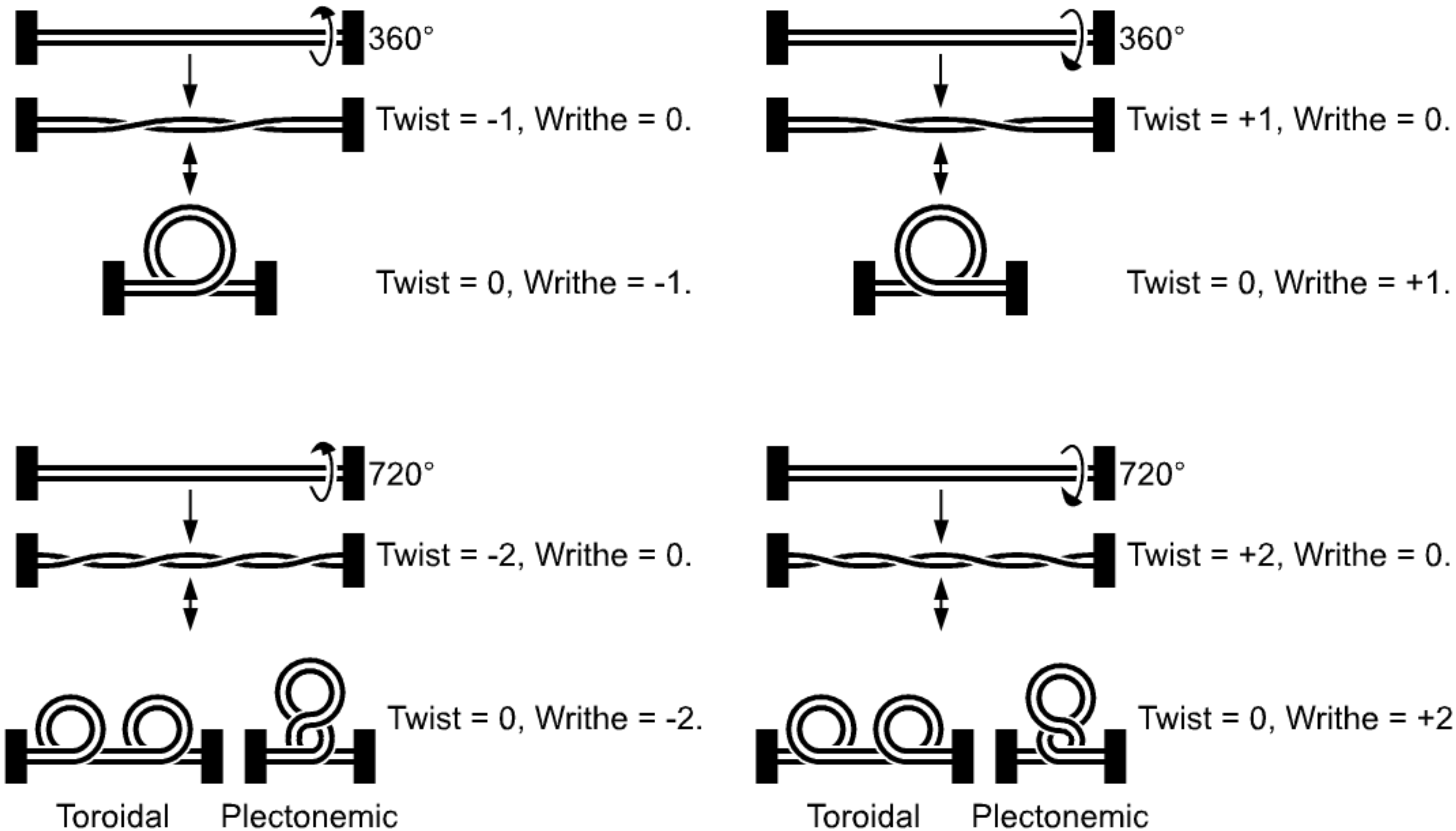
Link = Twist + Writhe

perversion

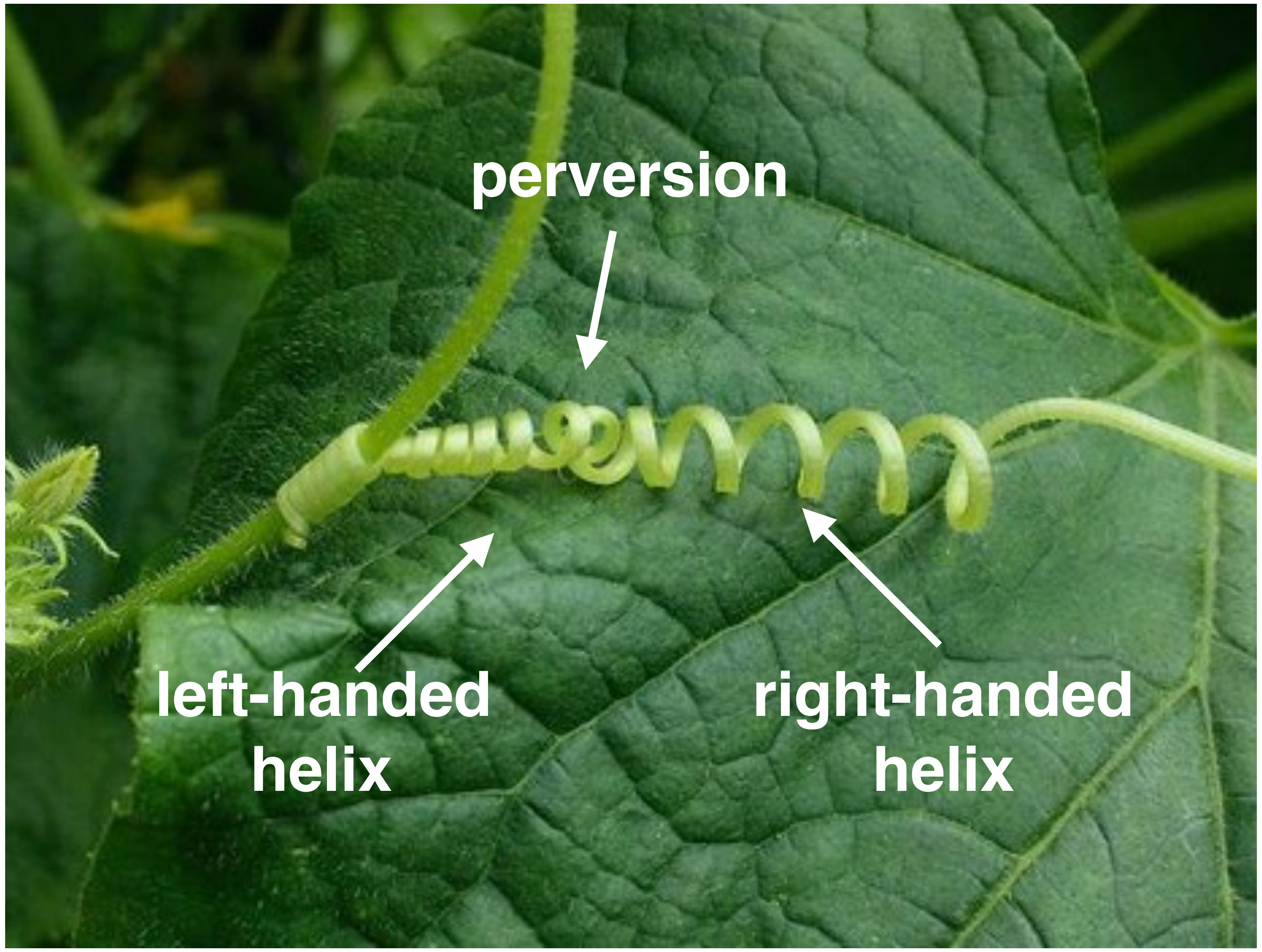


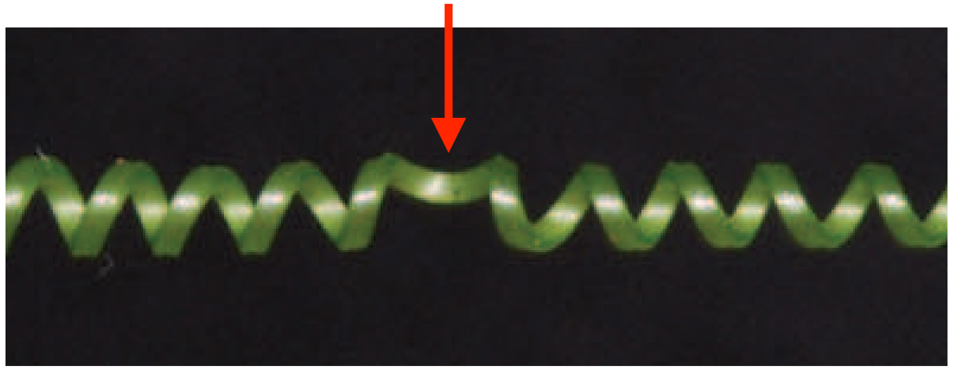
Twist, Writhe and Linking numbers

Ln=Tw+Wr	linking number: total number of turns of a particular end
Tw	twist: number of turns due to twisting the beam
Wr	writhe: number of crossings when curve is projected on a plane



Coiling of tendrils in opposite directions





Ends of the tendril are fixed and cannot rotate. This constraints the linking number.

Link = Twist + Writhe

Coiling in the same direction increases Writhe, which needs to be compensated by the twist.

In order to minimize the twisting energy tendrils combine two helical coils of opposite handedness (=opposite Writhe).

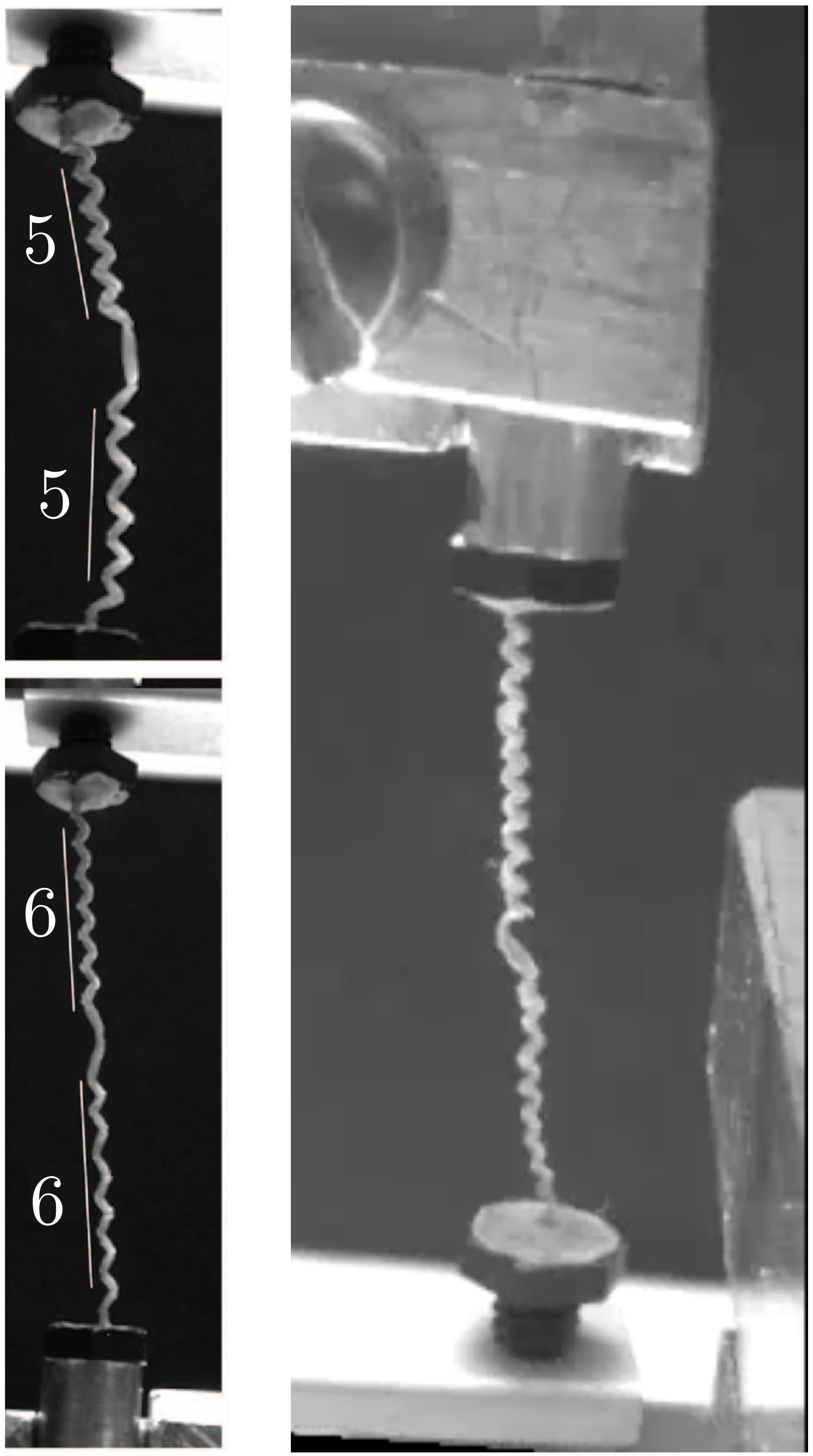
Note: there is no bending energy when the curvature of two helices correspond to the spontaneous curvature due to the differential shrinking of fiber.

Overwinding of tendril coils

Old tendrils overwind when stretched.

relaxed

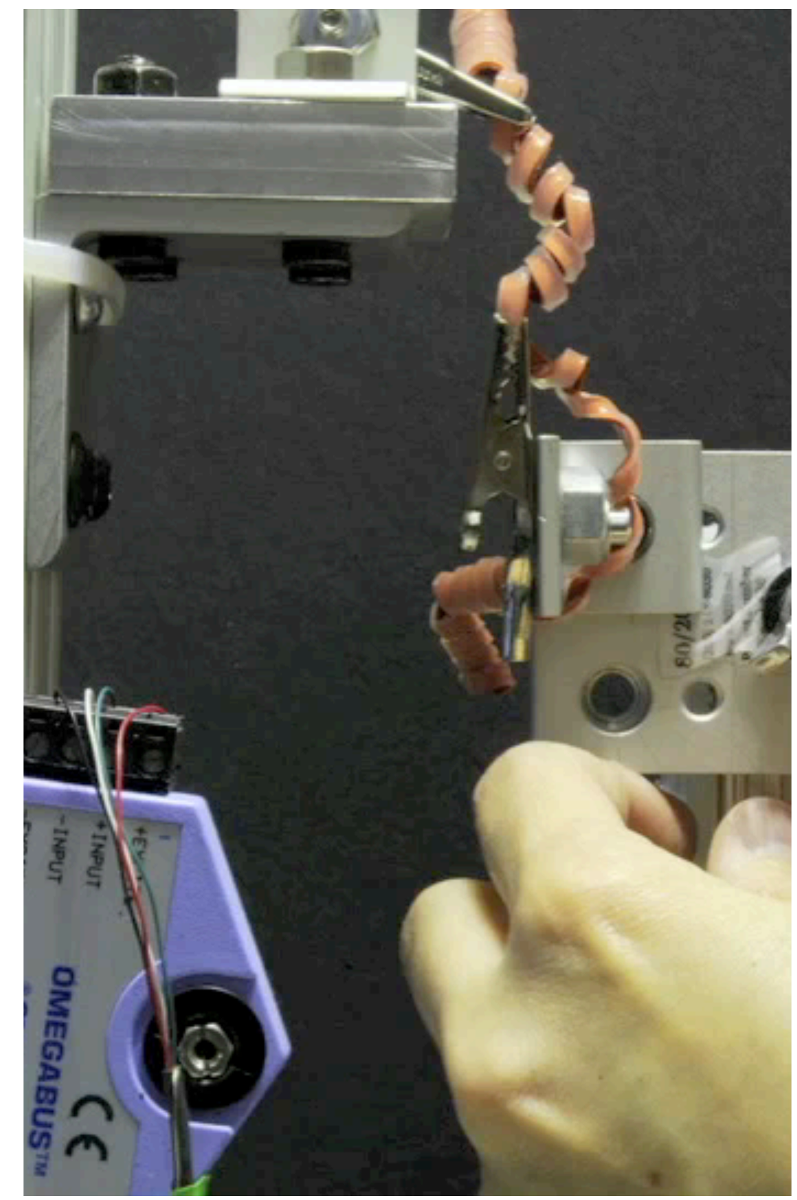
stretched



Rubber model unwinds when stretched.

relaxed

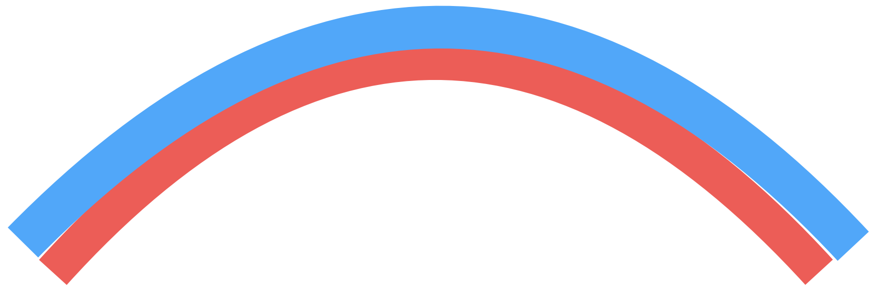
stretched



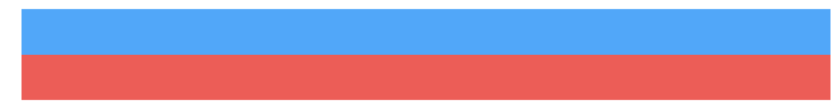
S. J. Gerbode et al., Science 337, 1087 (2012)

Overwinding of tendril coils

Preferred curved state



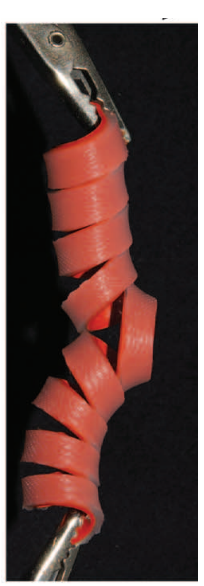
Flattened state



In tendrils the red inner layer is much stiffer then the outside blue layer.

High bending energy cost associated with stretching of the stiff inner layer!

Tendrils try to keep the preferred curvature when stretched!

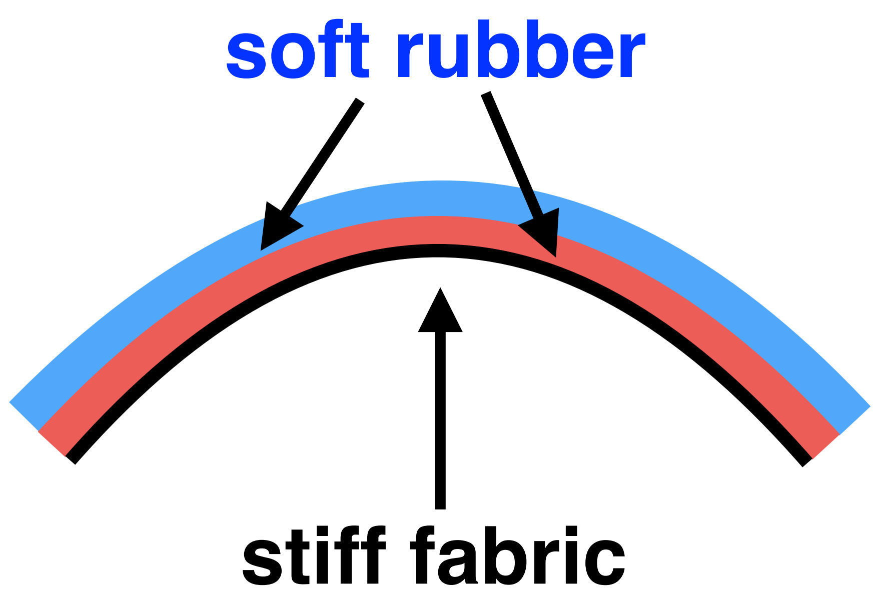


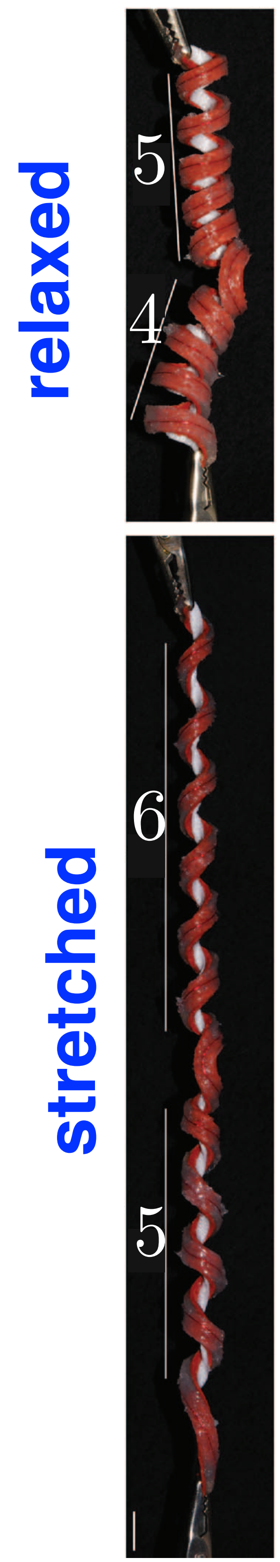
In rubber models both layers have similar stiffness.

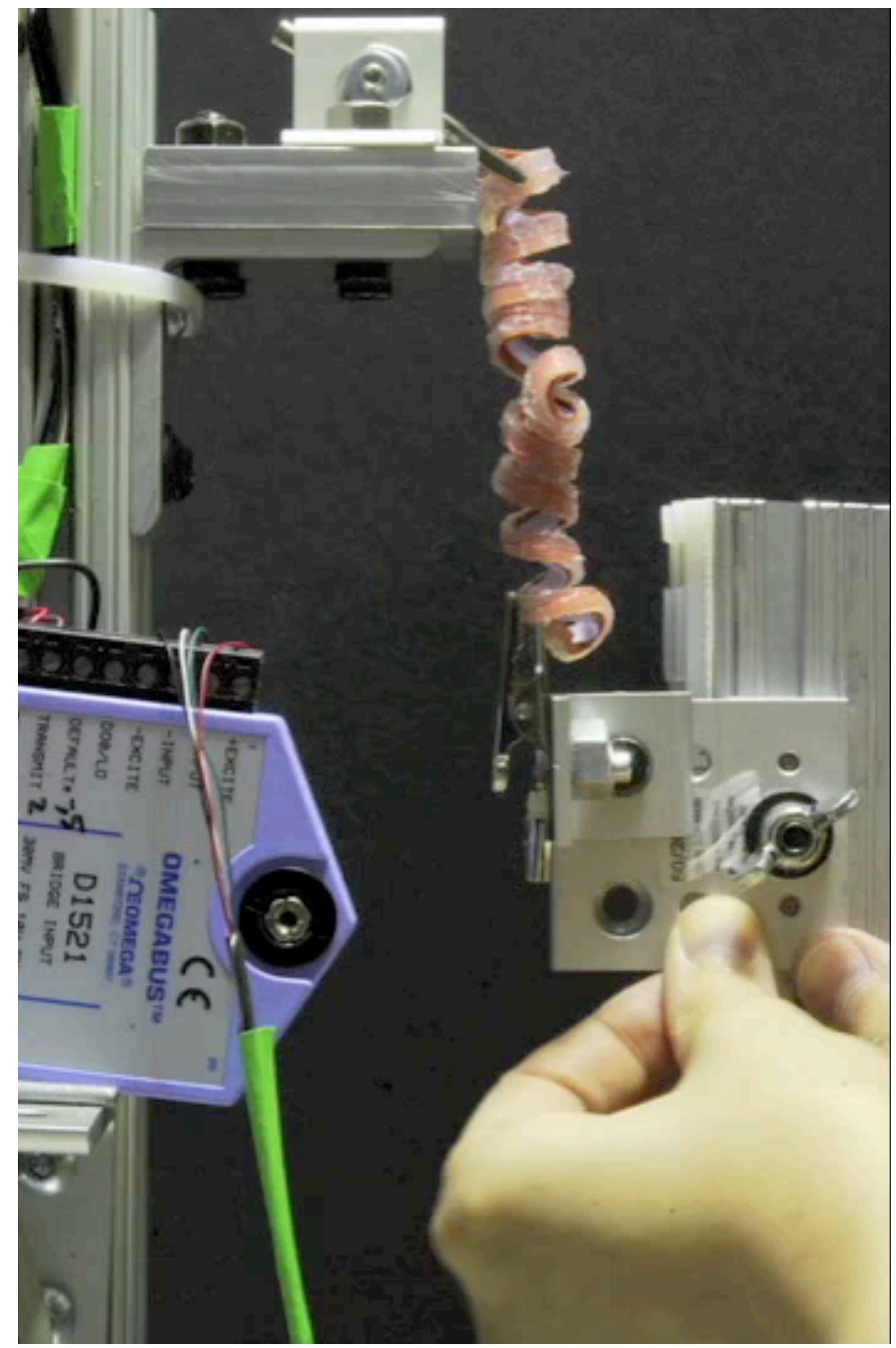
Small bending energy.

S. J. Gerbode et al., Science 337, 1087 (2012)

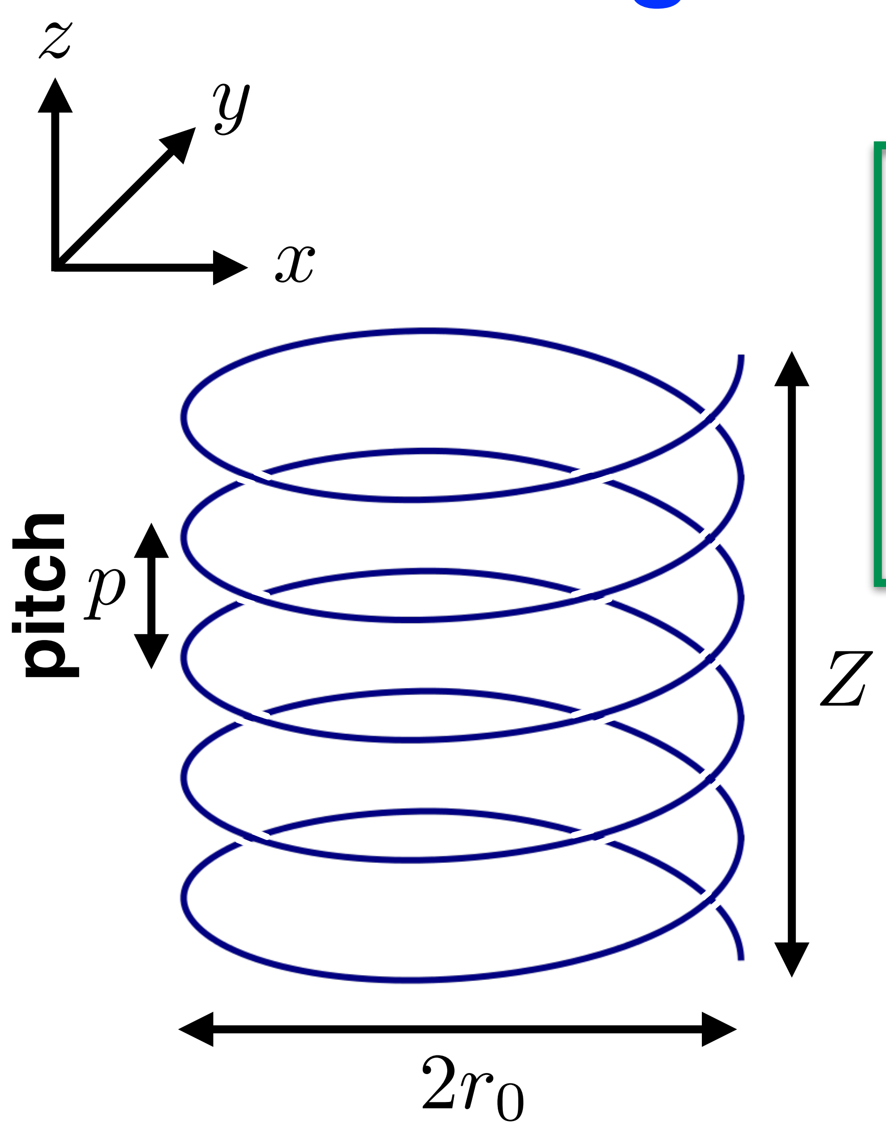
Overwinding of rubber models with an additional stiff fabric on the inside layers







Overwinding of helix with infinite bending modulus



Mathematical description

$$\vec{r}(s) = \left(r_0 \cos(s/\lambda), r_0 \sin(s/\lambda), \frac{p}{2\pi\lambda} s \right)$$

$$\lambda = \sqrt{r_0^2 + (p/2\pi)^2} \qquad Z = pN = p(L/2\pi\lambda)$$

Infinite bending modulus fixes the helix curvature during stretching

$$K = \frac{r_0}{r_0^2 + (p/2\pi)^2}$$

Helix pitch and radius

$$r_0 = \frac{1}{K}\left(1 - \frac{Z^2}{L^2}\right)$$

$$p = \frac{2\pi Z}{KL}\sqrt{1 - \frac{Z^2}{L^2}}$$

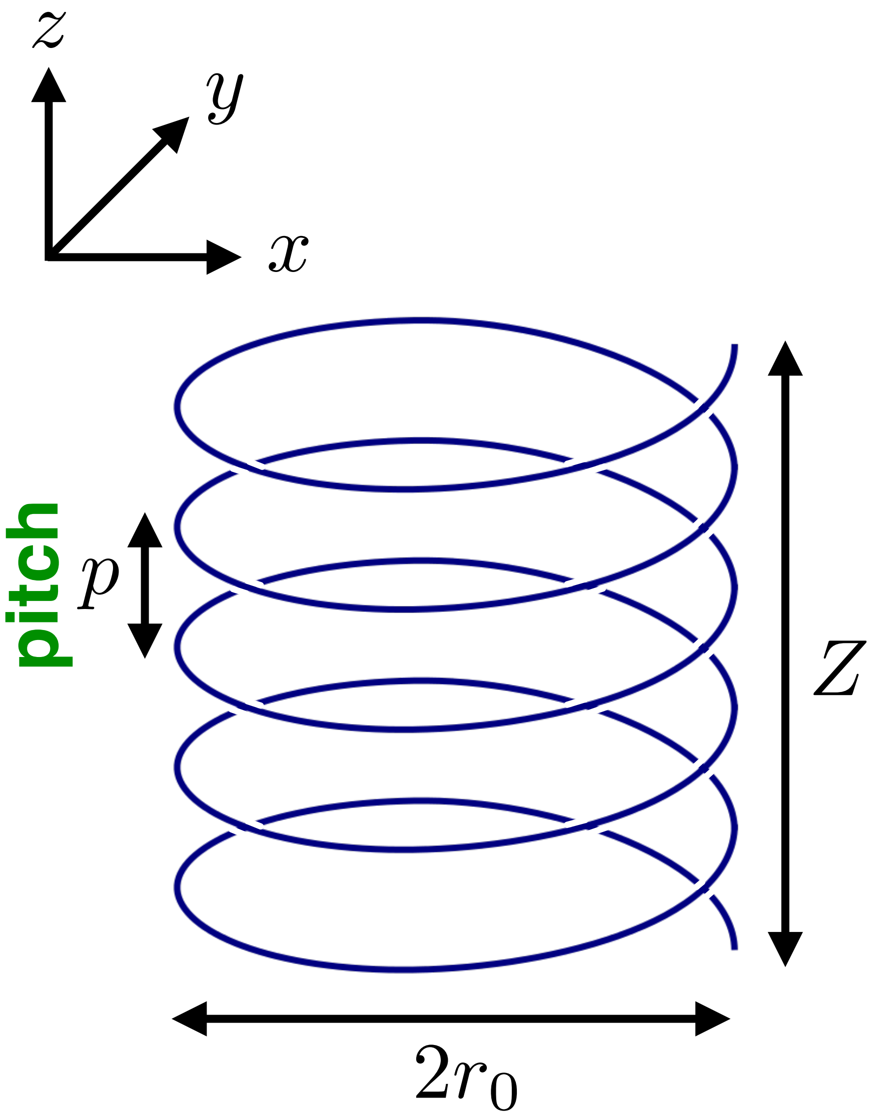
Number of loops

$$N = \frac{Z}{p} = \frac{KL}{2\pi\sqrt{1-(Z/L)^2}}$$

L **length of the helix backbone**

$N = \frac{Z}{p}$ **number of loops**

Overwinding of helix with infinite bending modulus



diameter

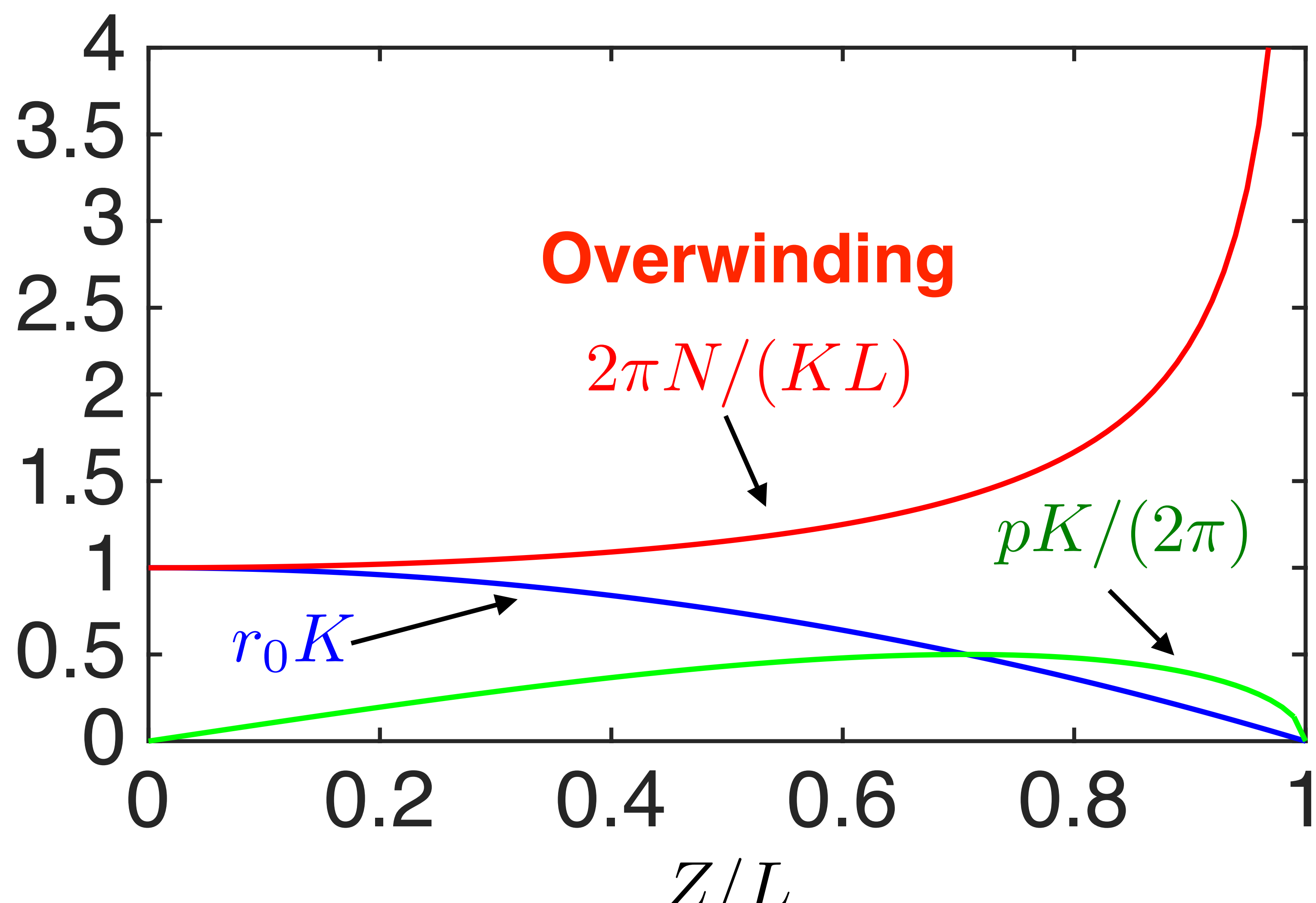
Helix pitch and radius

$$r_0 = \frac{1}{K}\left(1 - \frac{Z^2}{L^2}\right)$$

$$p = \frac{2\pi Z}{KL}\sqrt{1 - \frac{Z^2}{L^2}}$$

Number of loops

$$N = \frac{Z}{p} = \frac{KL}{2\pi\sqrt{1-(Z/L)^2}}$$



L **length of the helix backbone**

$N = \frac{Z}{p}$ **number of loops**

Further reading