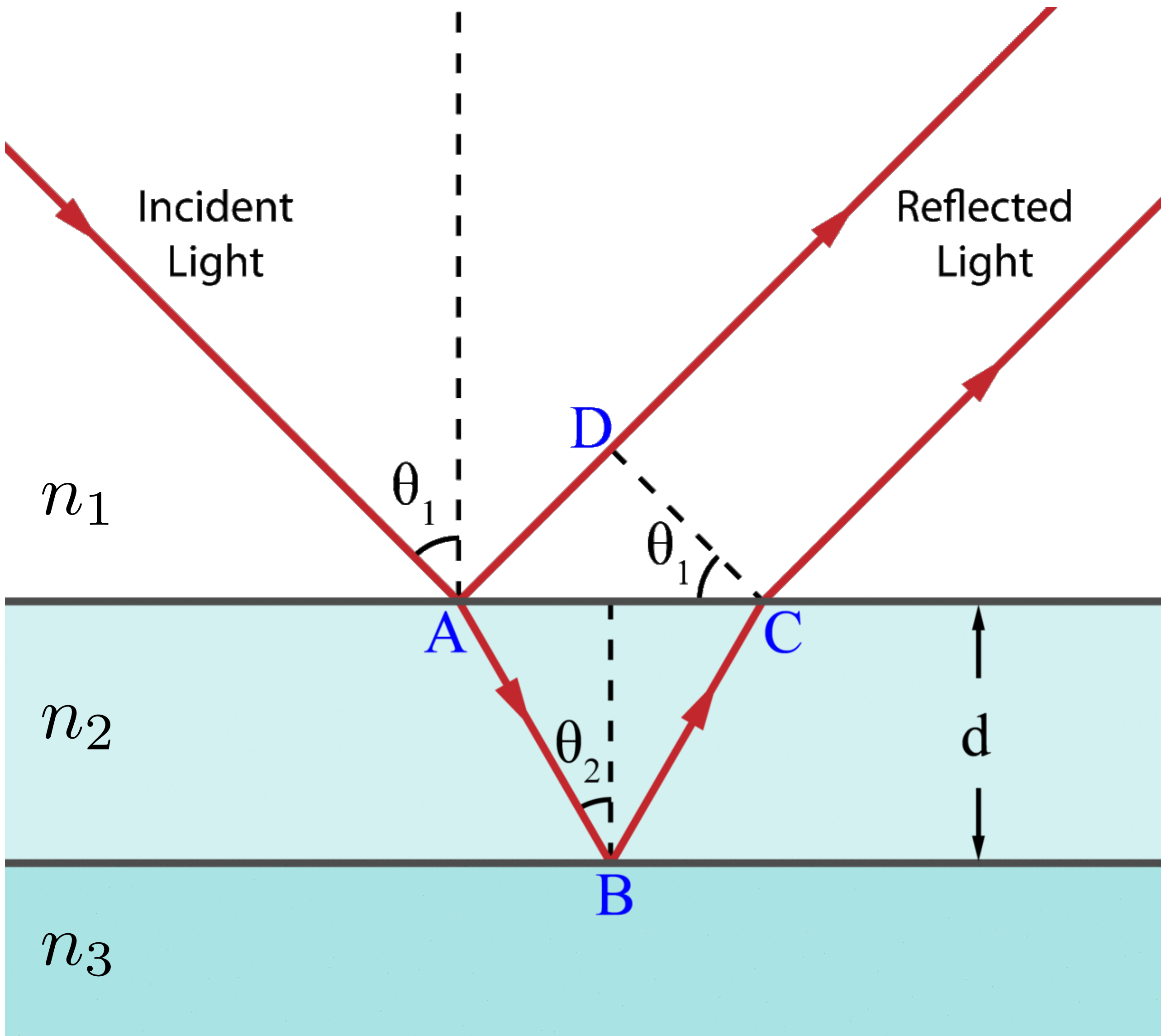
MAE 545: Lecture 4 (2/15)

Structural Color and Wrinkled Surfaces

Interference on thin films



difference between optical path lengths of the two reflected rays

$$OPD = n_2\left(\overline{AB} + \overline{BC}\right) - n_1\overline{AD}$$

$$OPD = 2n_2 d\cos(\theta_2)$$

$n_1 < n_2 < n_3 \qquad n_1 > n_2 > n_3$

no additional phase difference due to reflections

constructive interference of reflected rays $\quad OPD = m\lambda$

destructive interference of reflected rays $\quad OPD = \left(m + \frac{1}{2}\right)\lambda$

$m = 0, \pm1, \pm2, \ldots$

$n_1 > n_2 < n_3 \qquad n_1 < n_2 > n_3$

additional π phase difference due to reflections

constructive interference of reflected rays $\quad OPD = \left(m + \frac{1}{2}\right)\lambda$

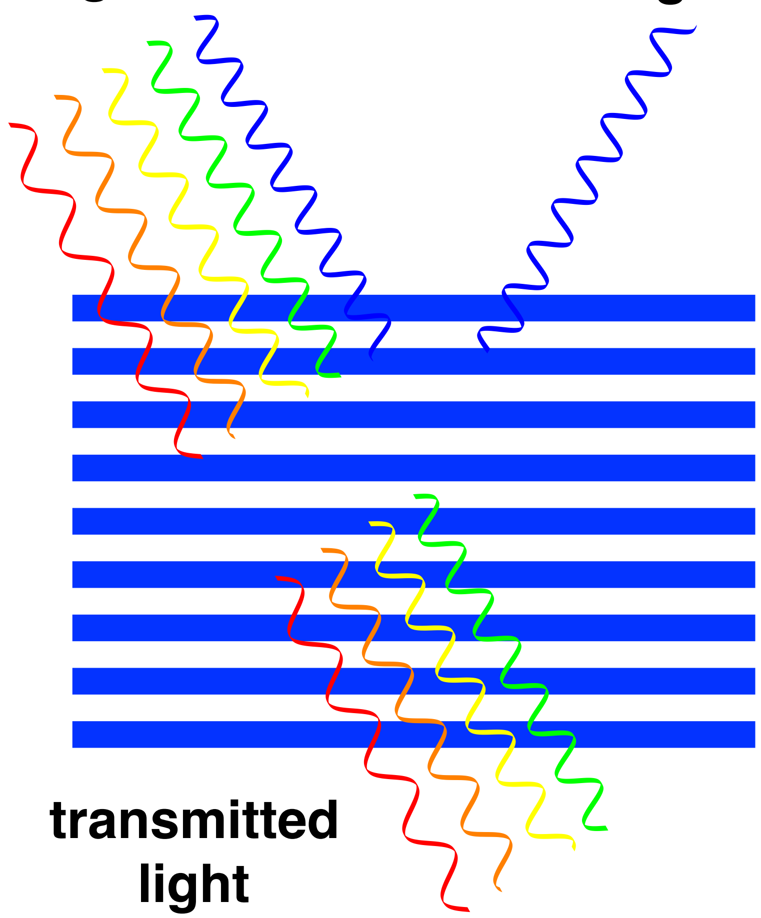
destructive interference of reflected rays $\quad OPD = m\lambda$

Structural colors on periodic structures

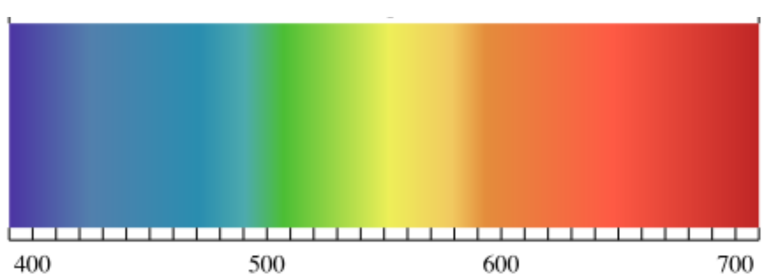
Single reflected color on structures with uniform spacing

incoming light

reflected light

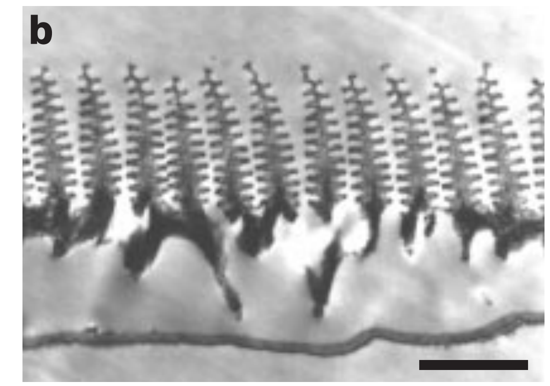


transmitted light



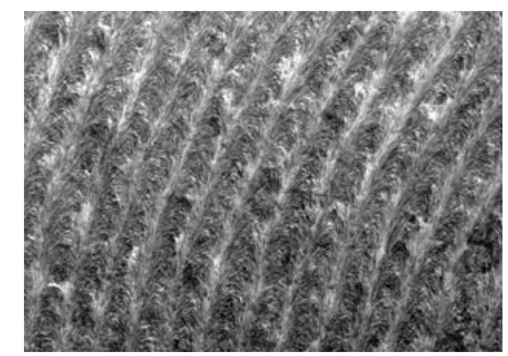
wavelength $\lambda[\mathrm{nm}]$

Morpho butterfly



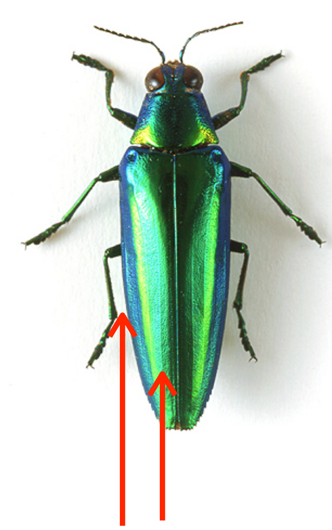
$1.7\mu\mathrm{m}$

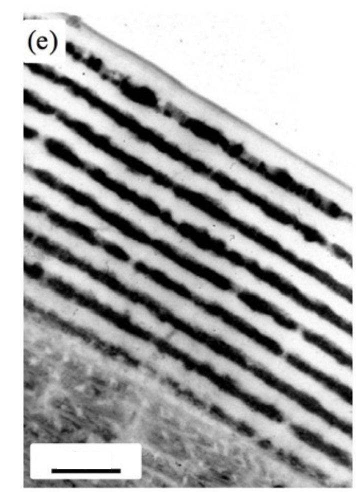
Marble berry



$250\mathrm{nm}$

Chrysochroa raja beetle





$1\mu\mathrm{m}$

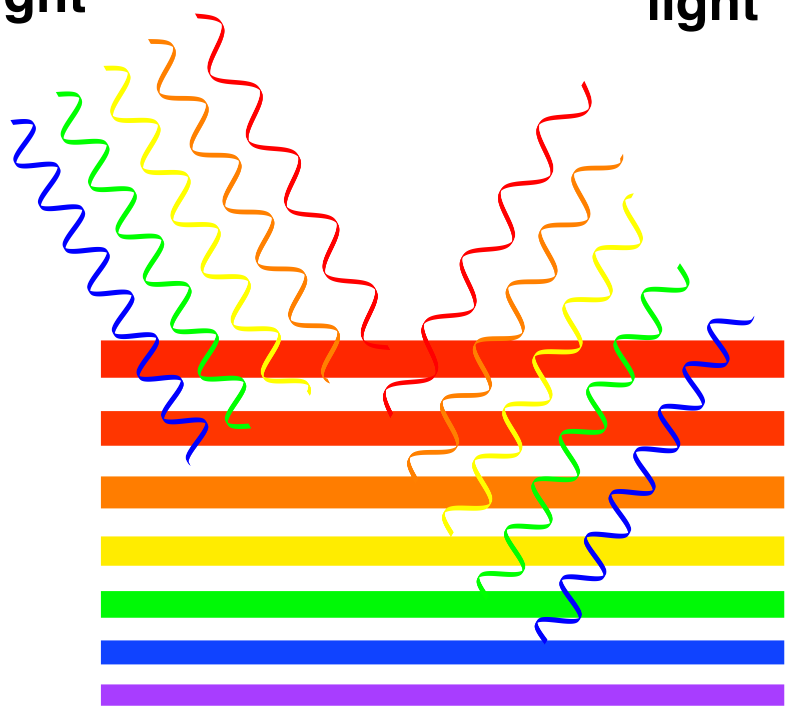
reflected color depends on the viewing angle!

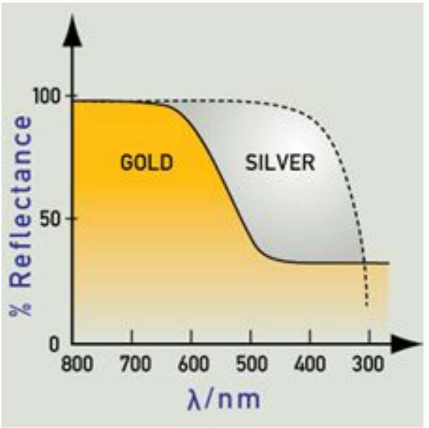
Silver and gold structural colors

Many colors reflected on structures with varying spacing

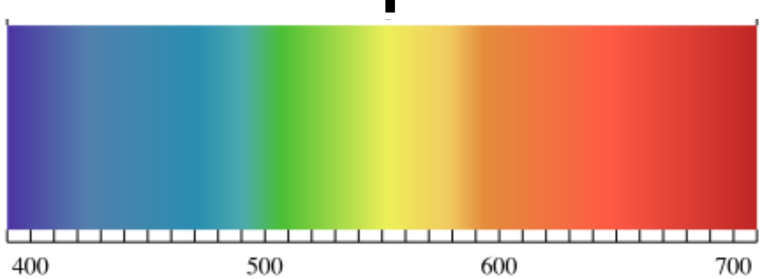
incoming light

reflected light





visible spectrum

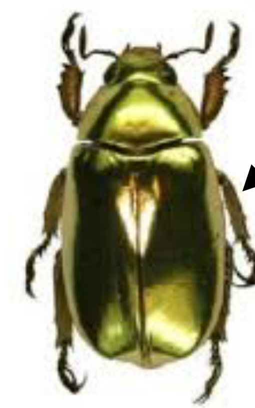


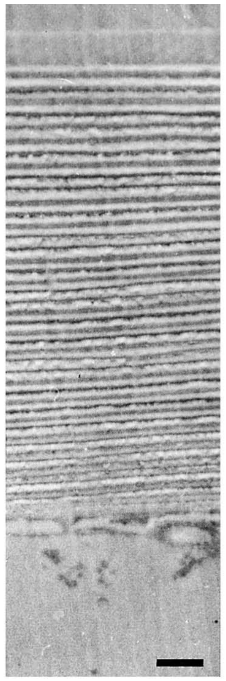
wavelength $\lambda[\mathrm{nm}]$

chirped structure

Chrysina limbata beetle

Chrysina aurigans beetle





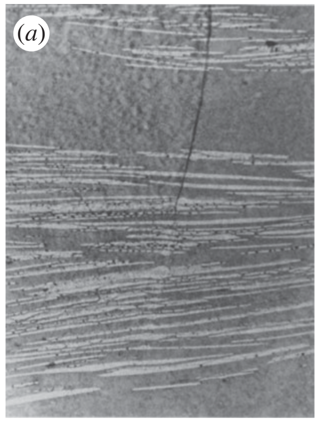
$1\mu\mathrm{m}$

thicker

thinner

disordered layer spacing

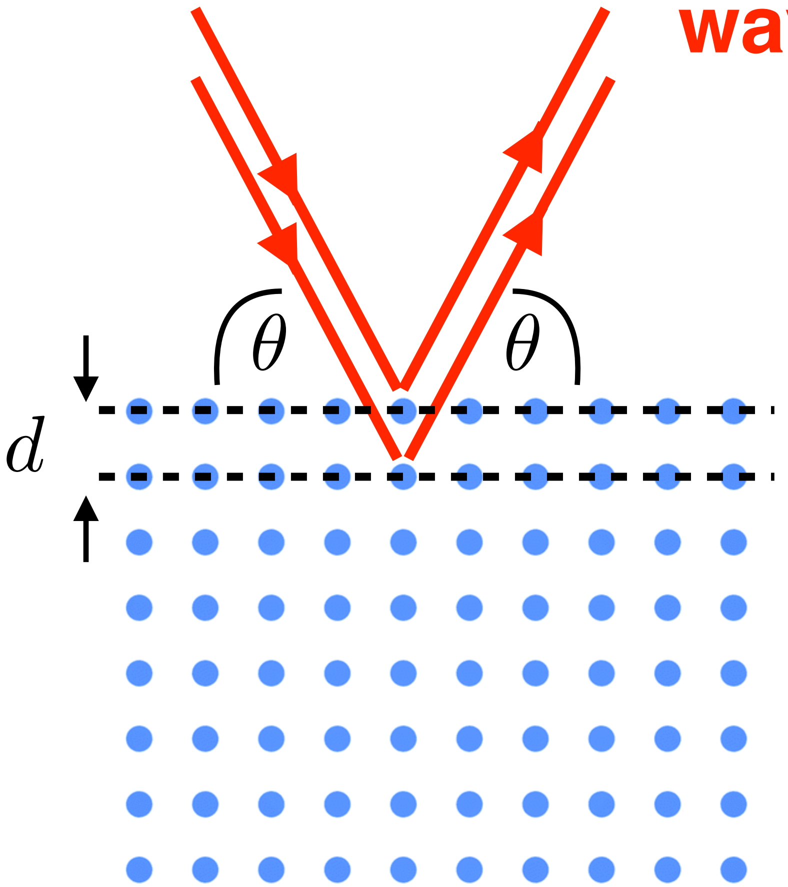
bleak fish

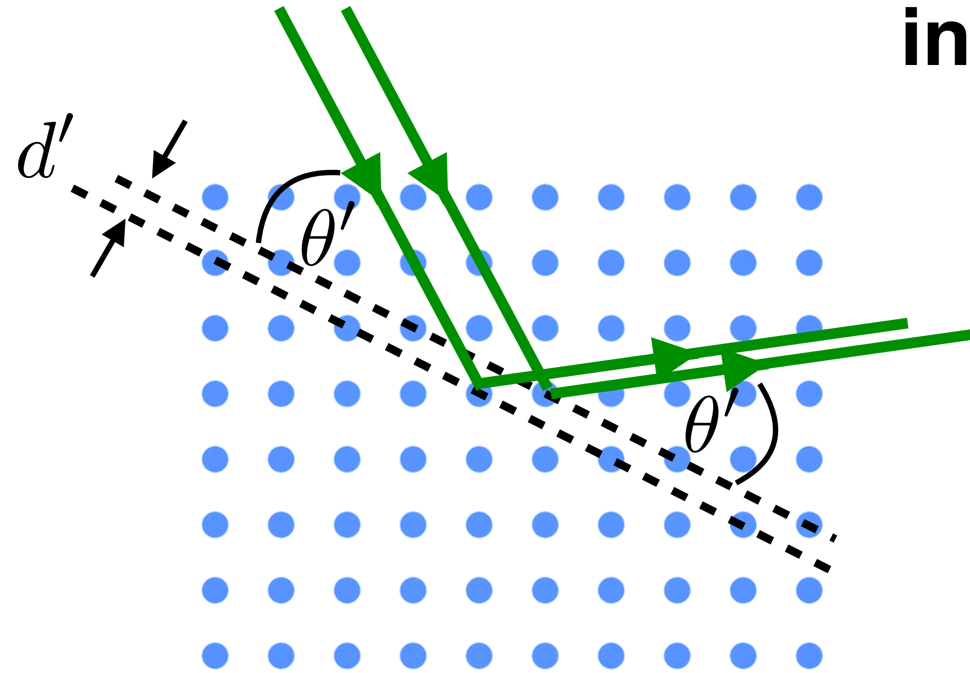


$1\mu\mathrm{m}$

Bragg scattering on crystal layers

Constructive interference for waves with different wavelengths occurs in different crystal planes!





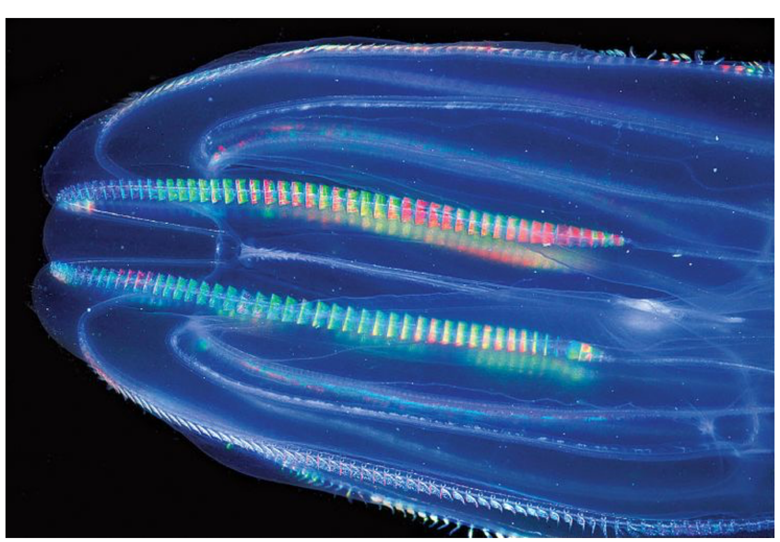
constructive interference condition

$$2d \sin\theta = m\lambda$$

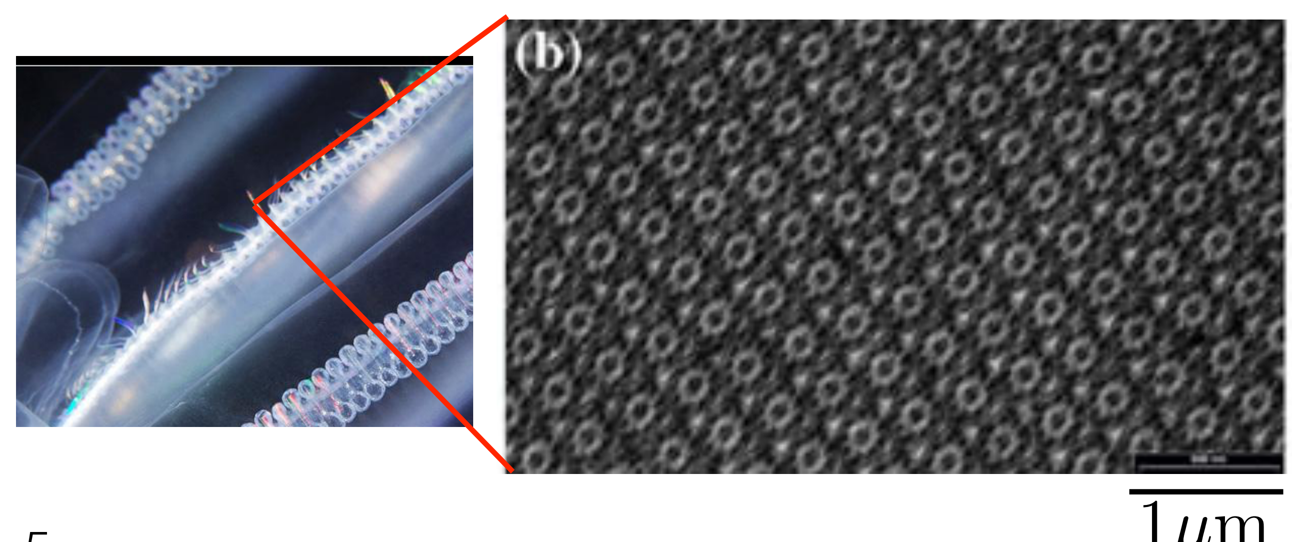
$$2d' \sin\theta' = m\lambda'$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Comb jelly

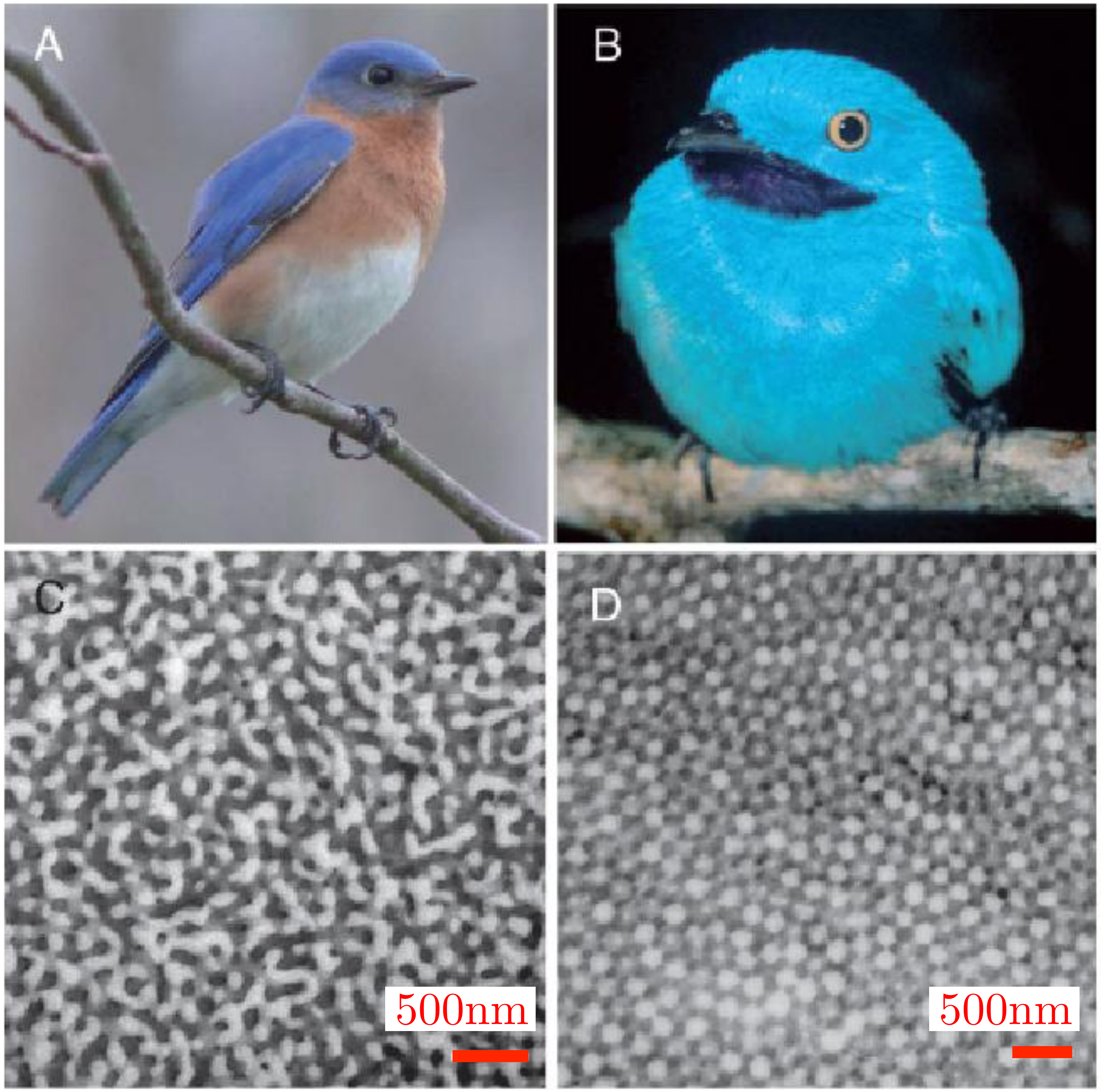


Beating cilia are changing crystal orientation



Scattering on disordered structures

Eastern bluebird | **Plum-throated Cotinga**



V. Saranathan et al.,
J. R. Soc. Interface 9, 2563 (2012)

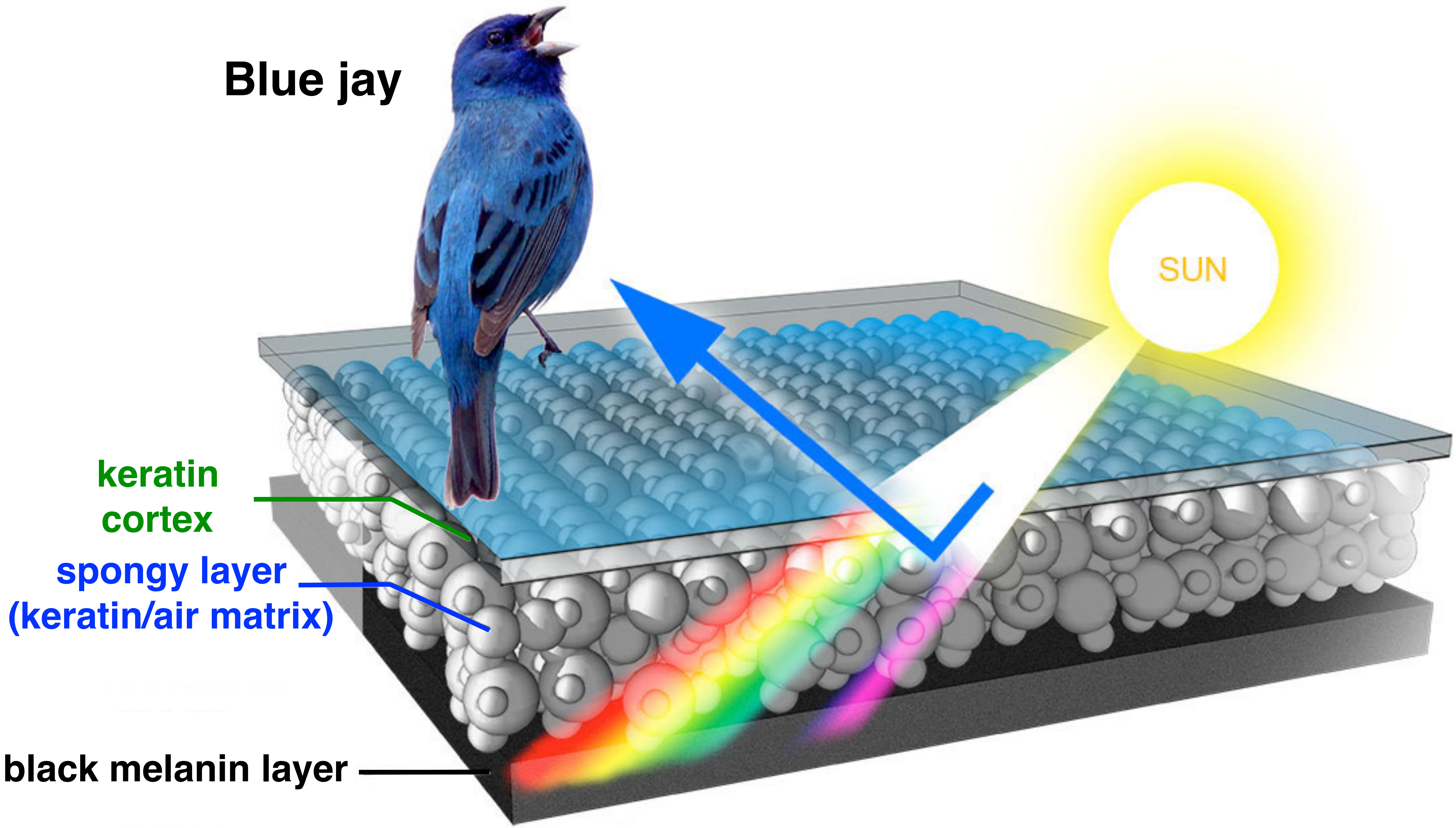
Disordered structures with a characteristic length scale.

This length scale determines what light wavelengths are preferentially scattered.

The selectively reflected wavelengths are the same in all directions!

This gives rise to blue colors in these birds.

Scattering on disordered structures



https://academy.allaboutbirds.org/how-birds-make-colorful-feathers/

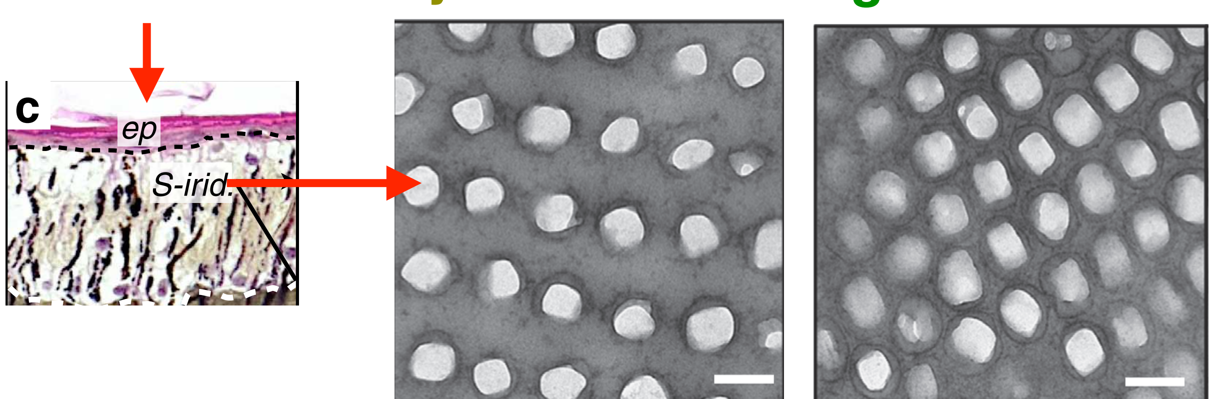
Dynamic structural colors

Chameleon (speed 8x)

J. Teyssier et al., Nat. Comm. 6, 6368 (2015)

Changes in osmotic concentration lead to the swelling of cells in excited chameleon. This changes the spacing of periodic structure from which the ambient light is reflected.

skin | **yellow color** | **green color**

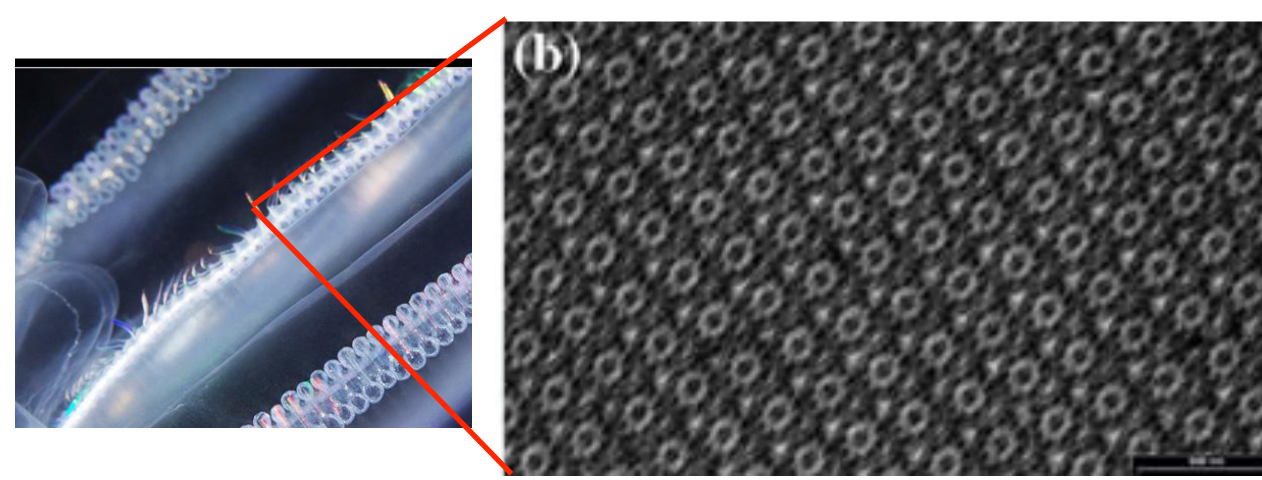


200nm 200nm

Comb Jelly (real time)

https://www.youtube.com/watch?v=Qy90d0XvJIE

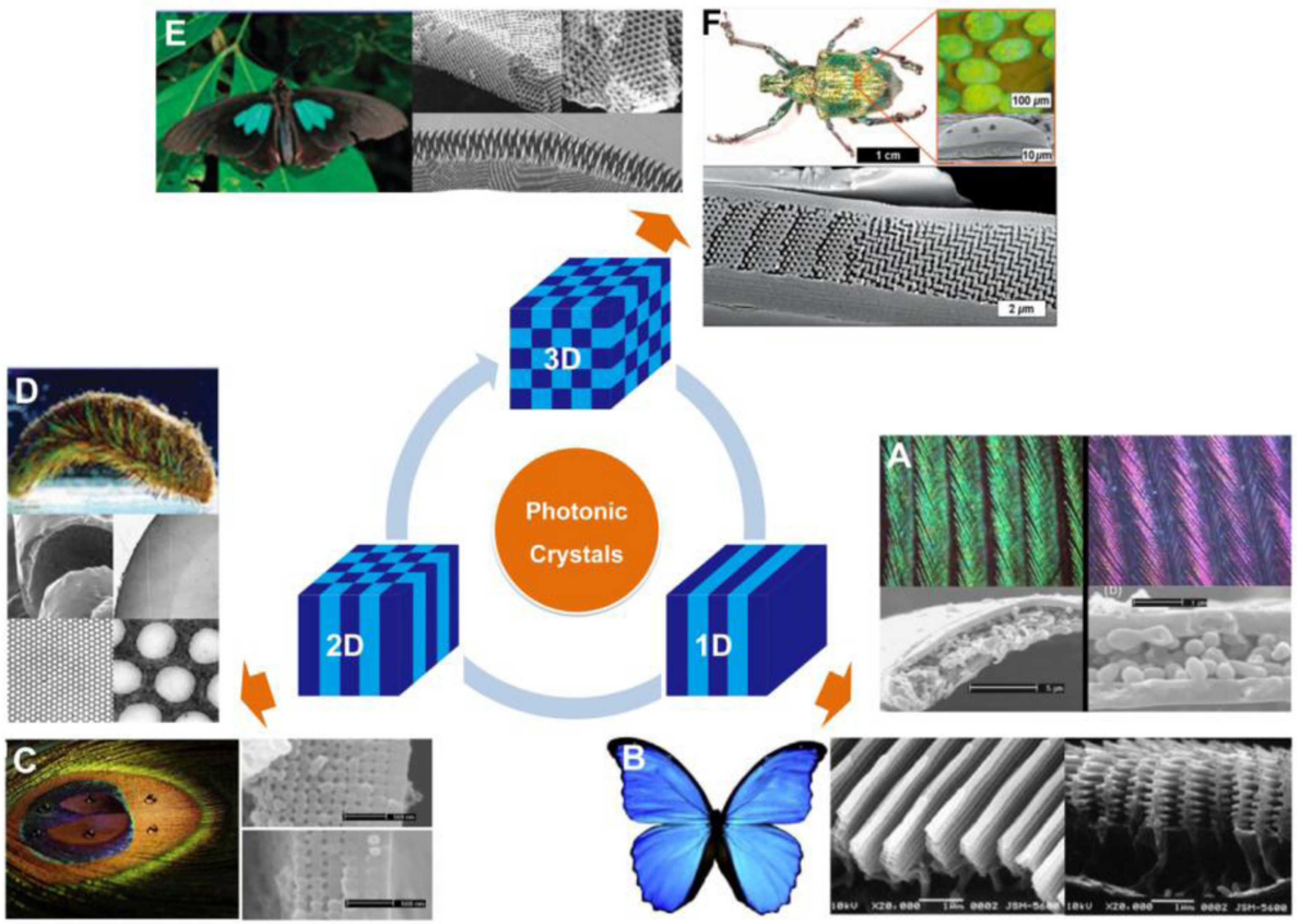
Rainbow color waves are produced by the beating of cilia, which change the orientation of periodic structure from which the ambient light is reflected.



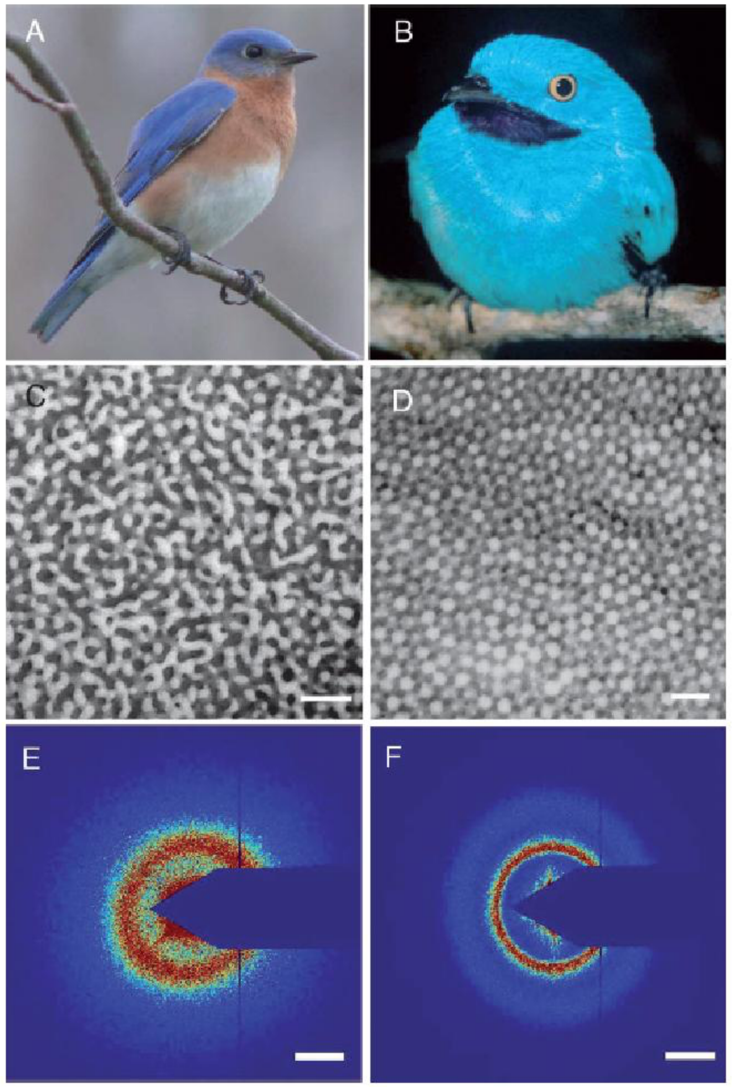
$1\mu m$

Structural colors

Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.

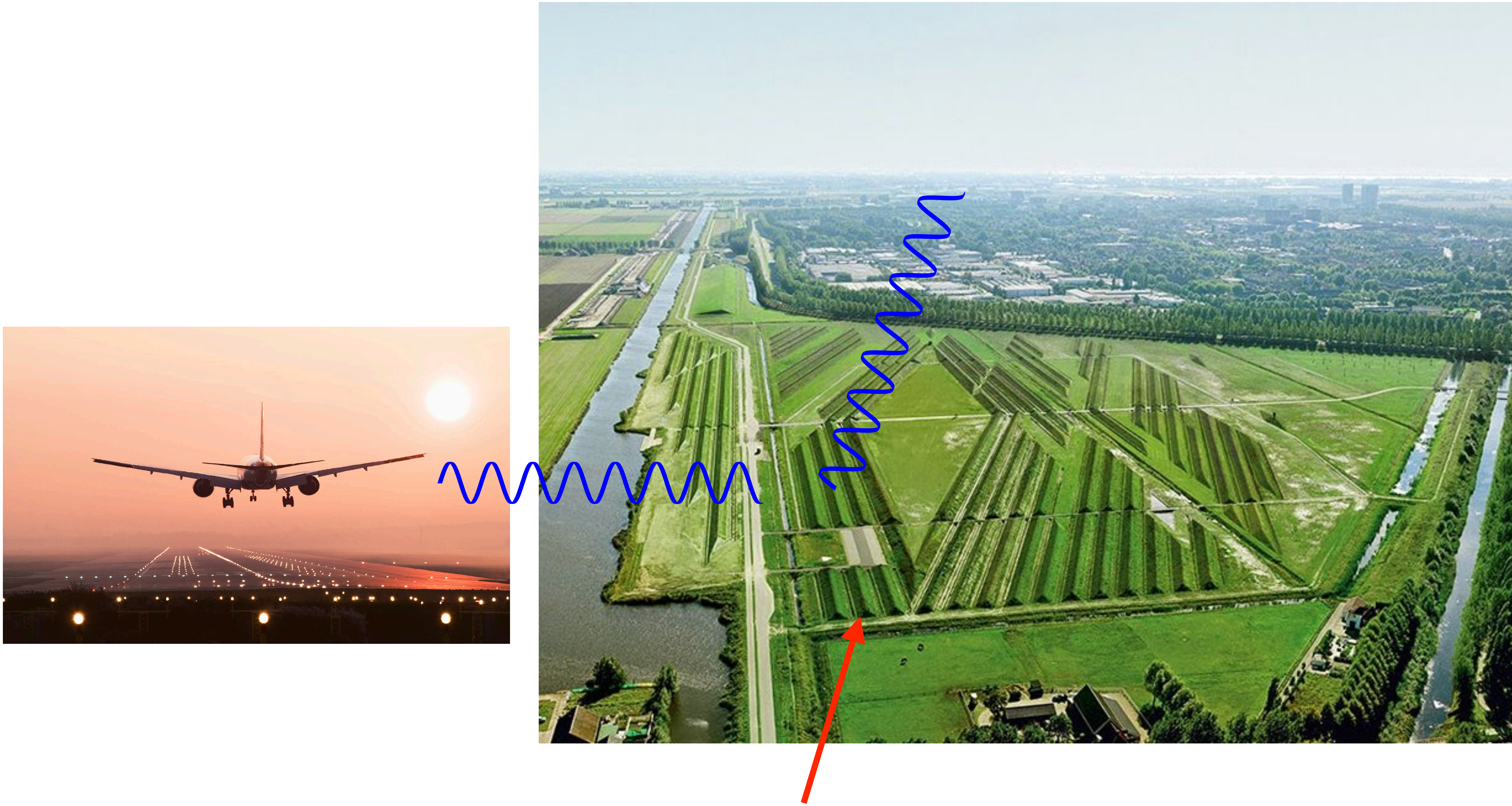


H. Wang and K-Q. Zhang,
Sensors 13, 4192 (2013)



V. Saranathan et al.,
J. R. Soc. Interface 9, 2563 (2012)

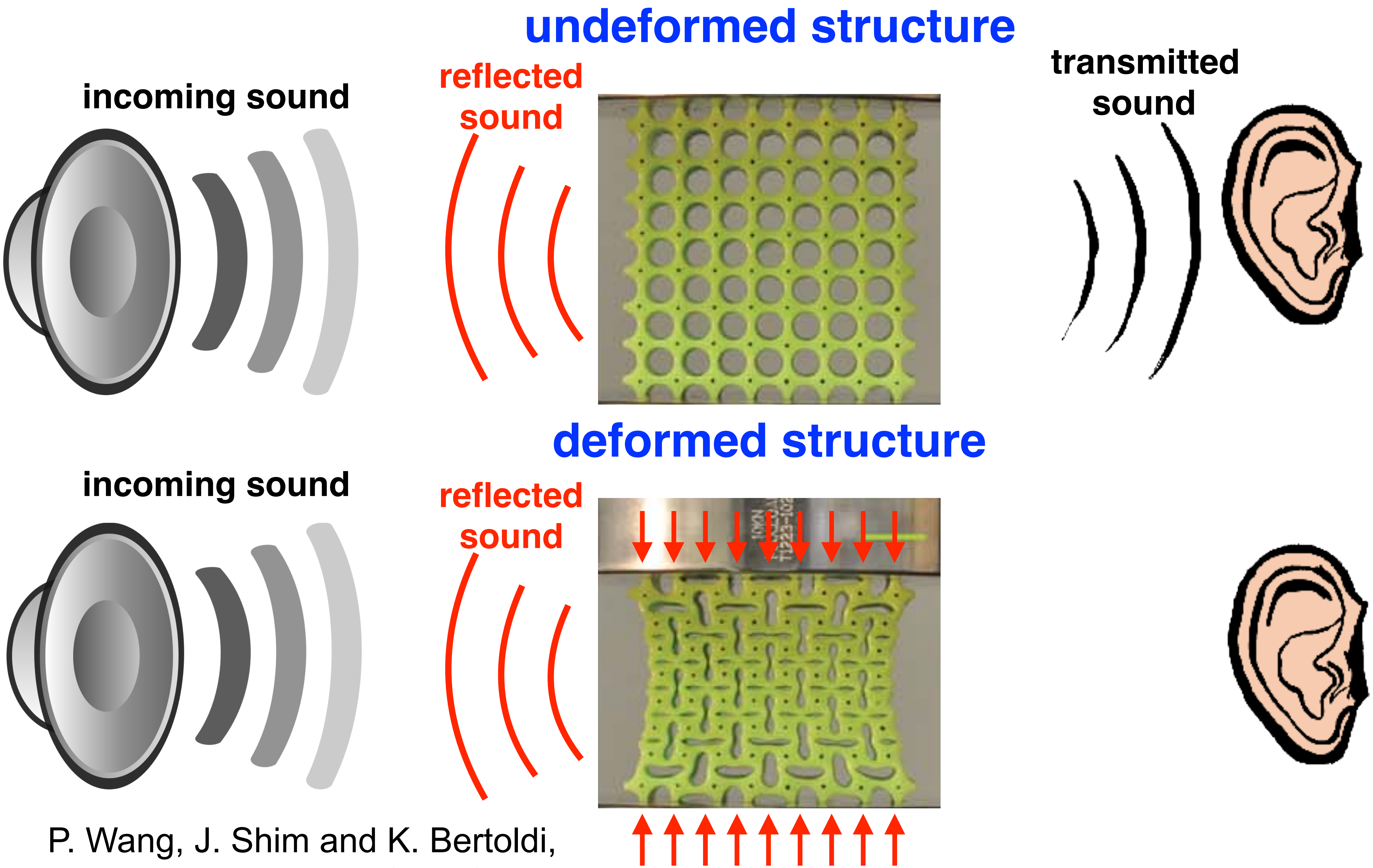
Noise barriers around the Amsterdam airport



Sound from airplanes that are landing and taking off is reflected from artificial barriers into the atmosphere.

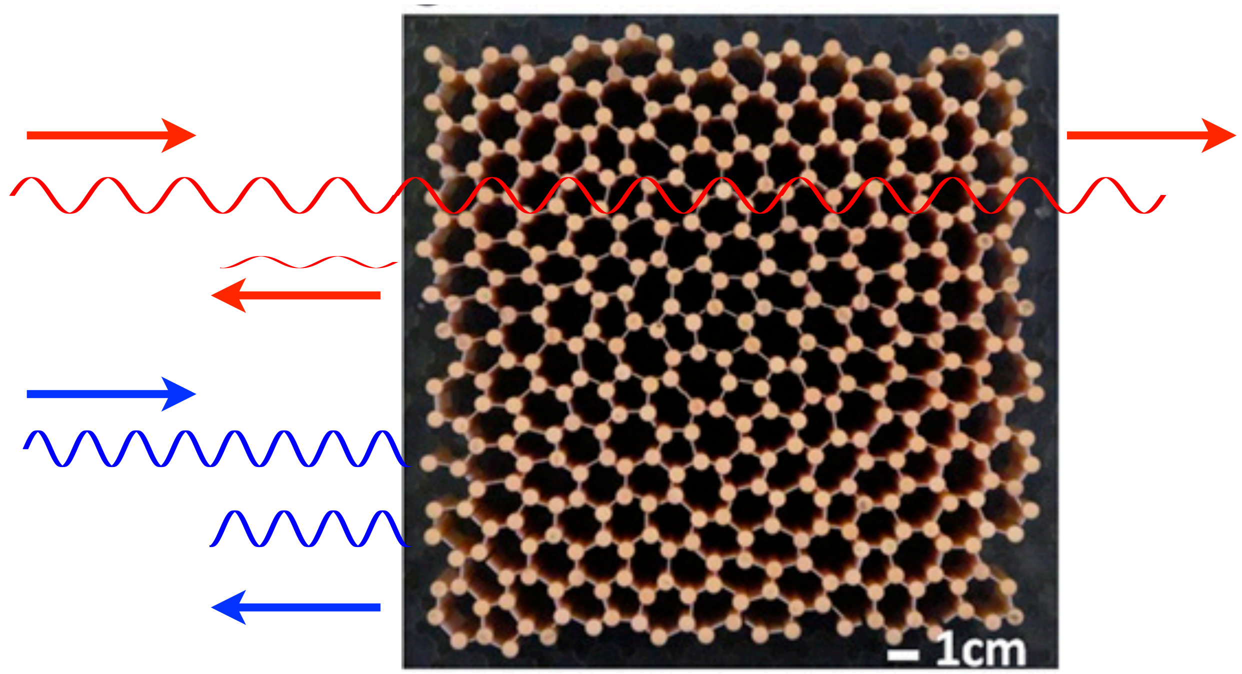
Controllable sound filters

In periodic structures sound waves of certain frequencies (within a “band gap”) cannot propagate. The range of “band gap” frequencies depends on material properties, the geometry of structure and the external load.



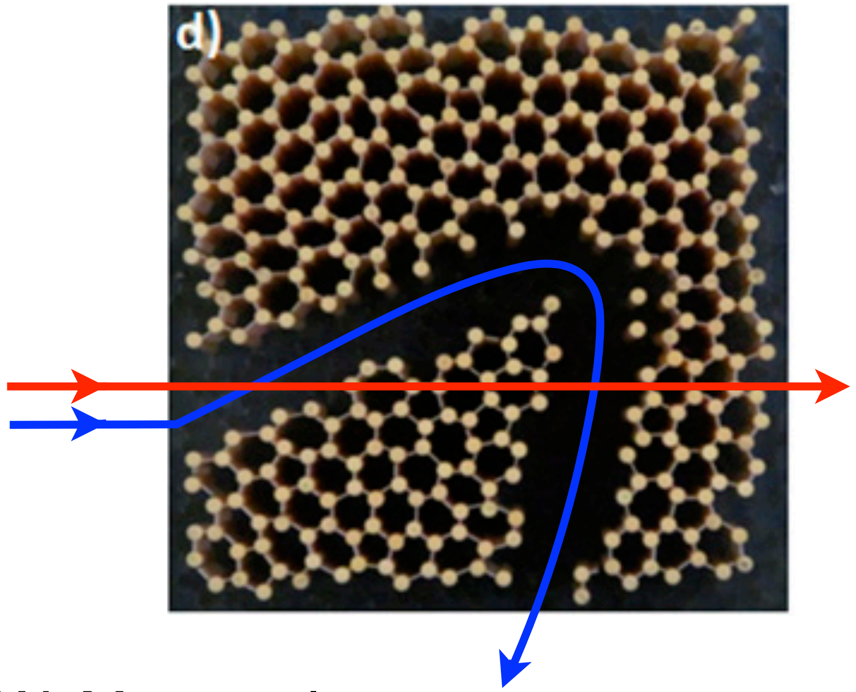
P. Wang, J. Shim and K. Bertoldi,
PRB **88**, 014304 (2013)

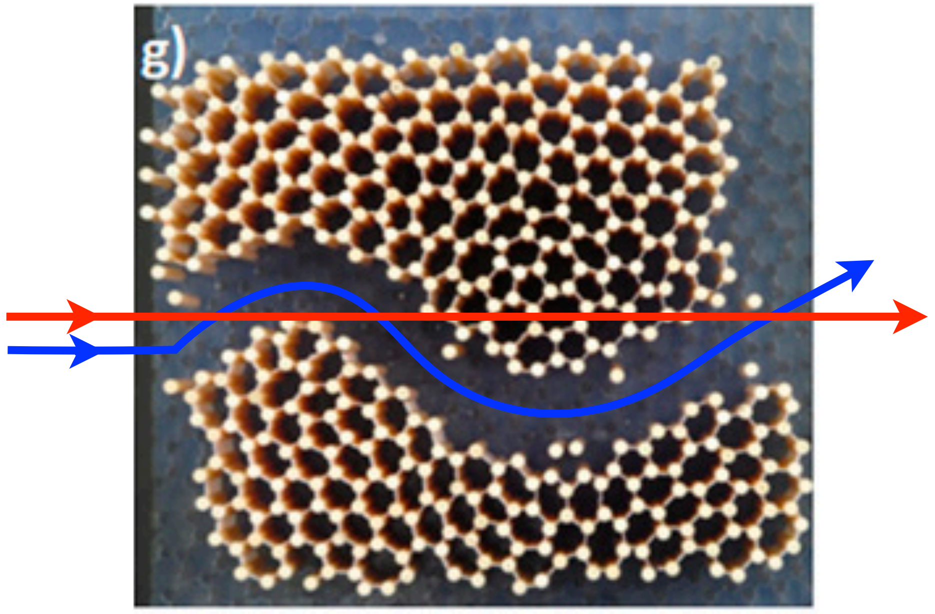
Waveguides in disordered structures



Channels inside structures can be used as guides for waves with wavelengths that are totally reflected from a complete structure!

Note: channels can have arbitrary bends!

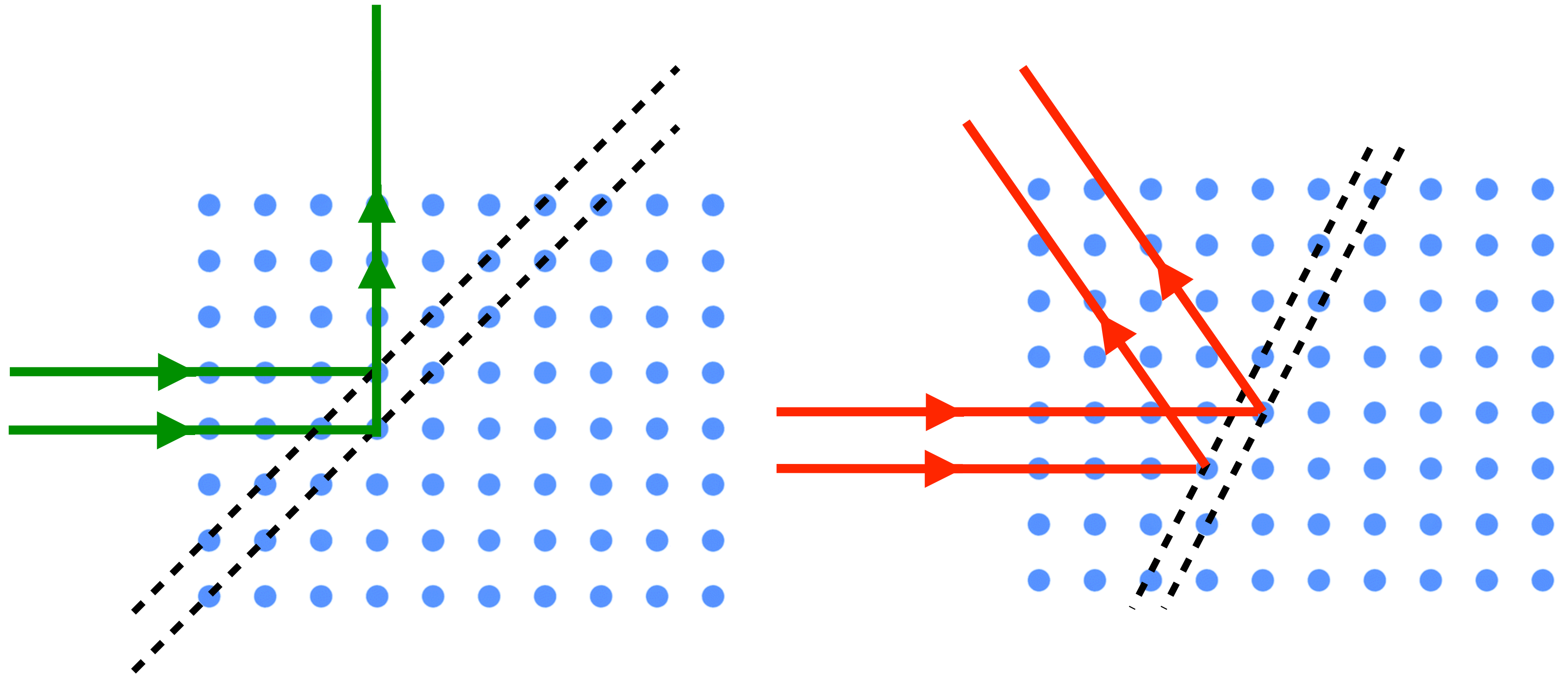




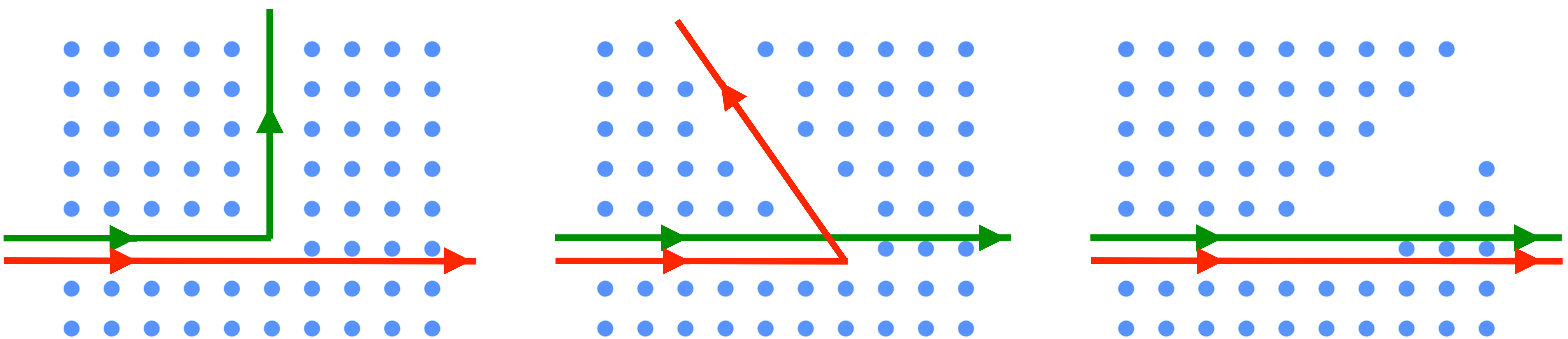
W. Man et al.,
PNAS **110**, 15886 (2013)

Waveguides in periodic structures

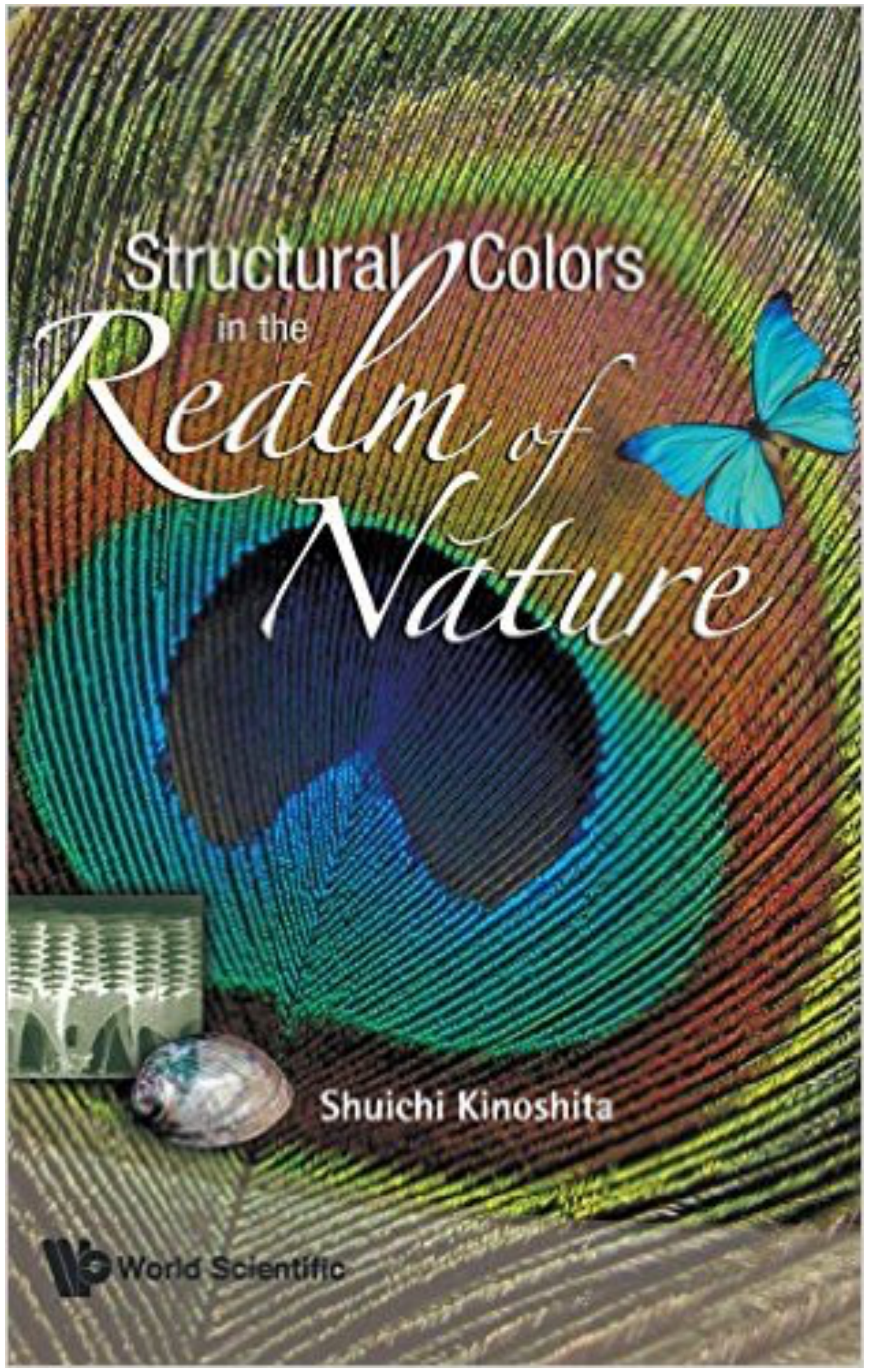
In periodic structures waves are completely reflected only at certain angles.

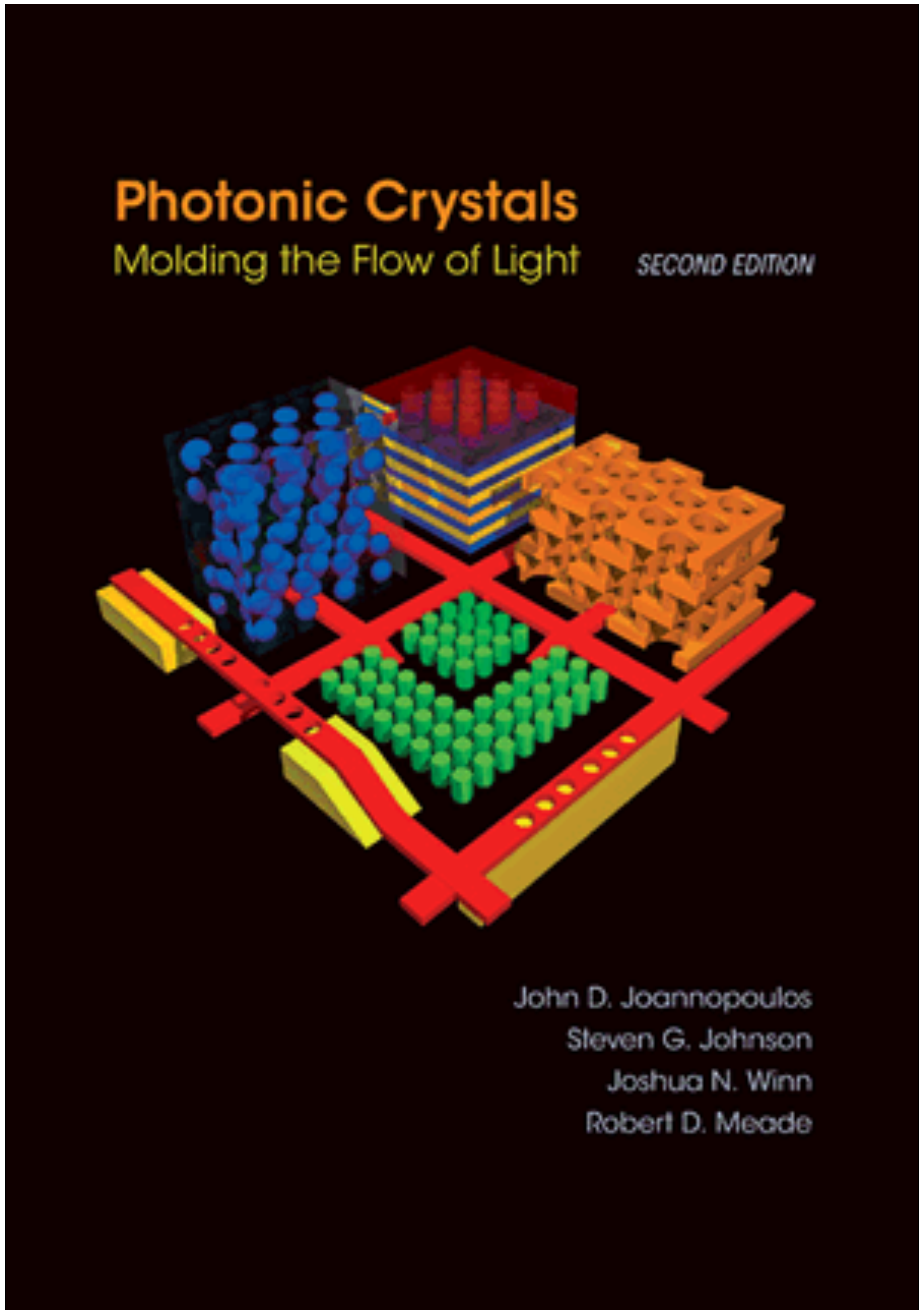


Note: channels with certain bends act as waveguides only for those waves that are completely reflected at these angles!



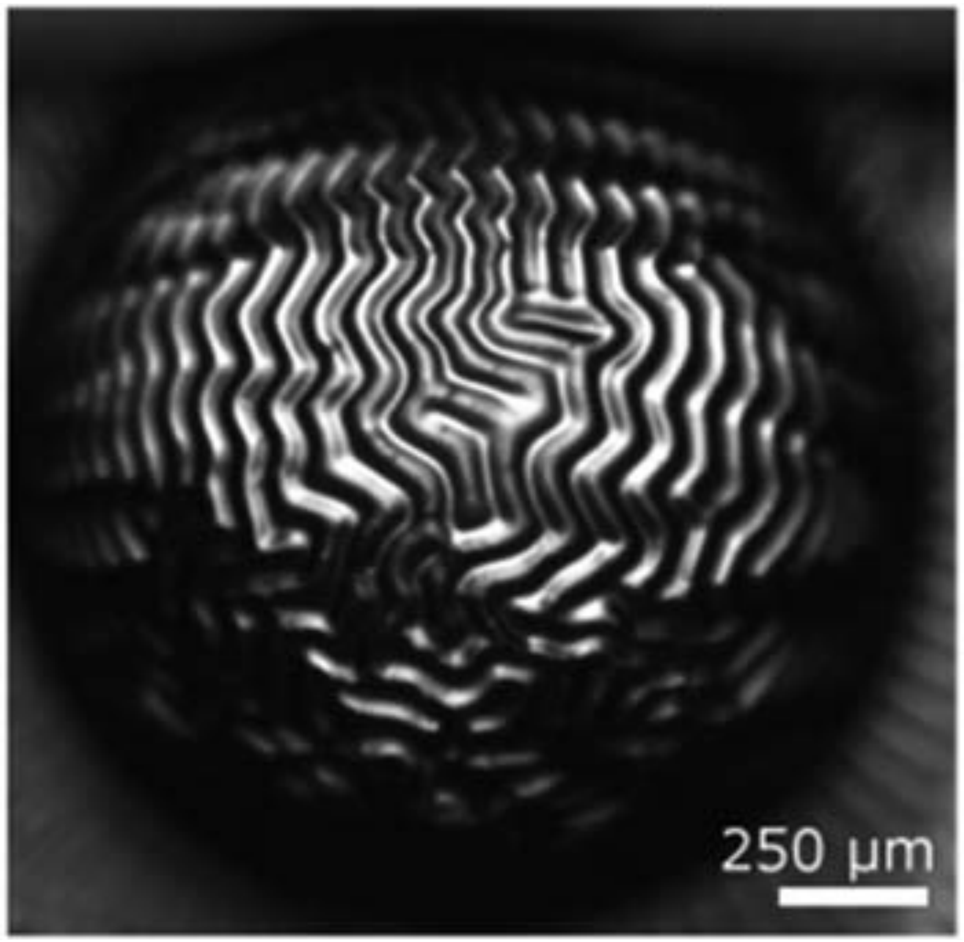
Further reading about structural colors and photonic crystals





http://ab-initio.mit.edu/book/

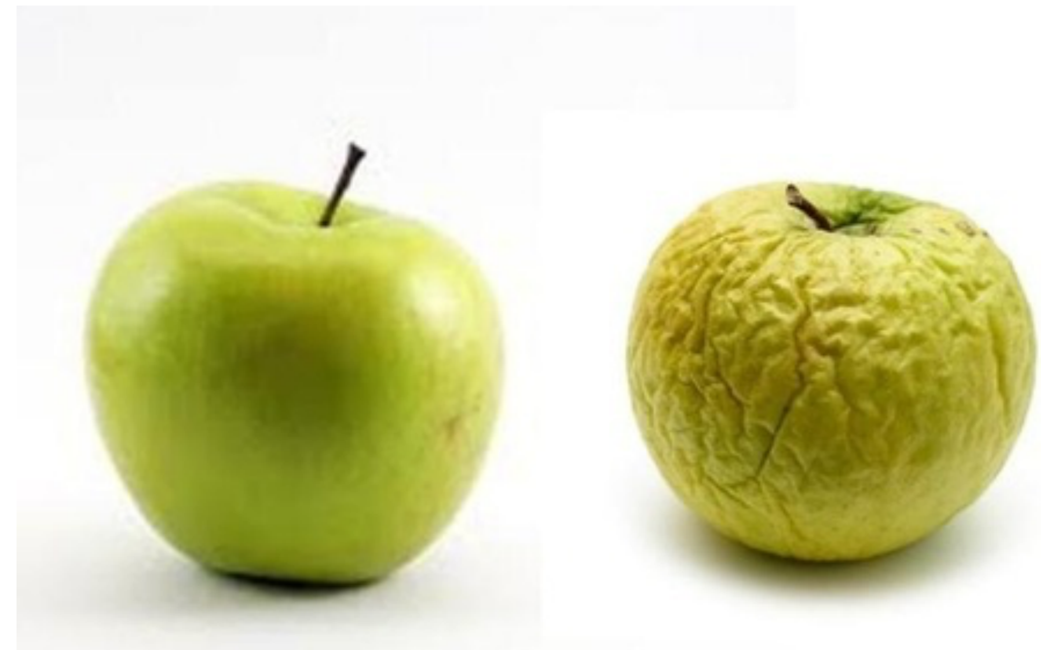
Wrinkled surfaces



Why do we get wrinkled surfaces?

Fingers after being exposed to water for some time

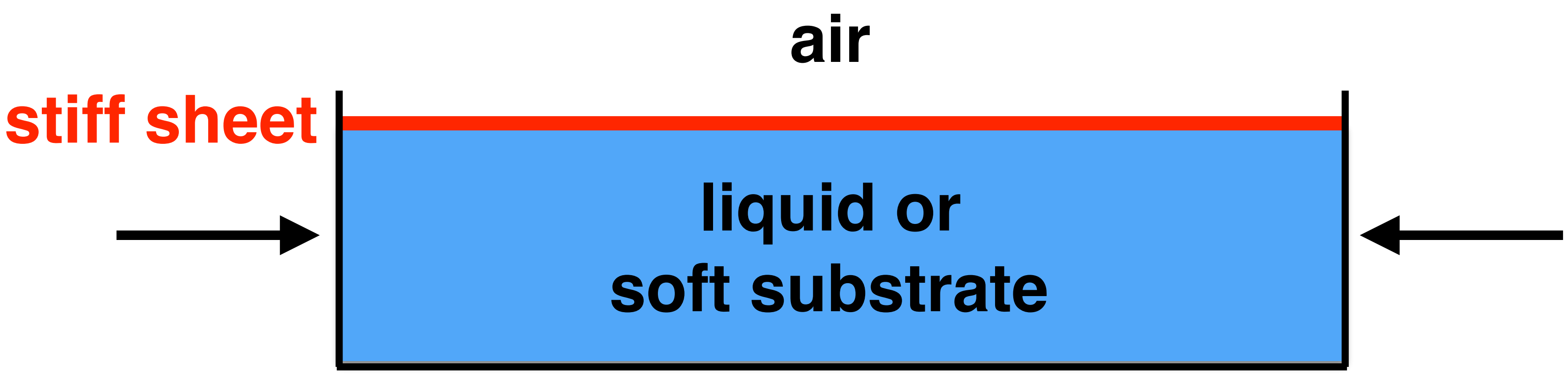
Old apple

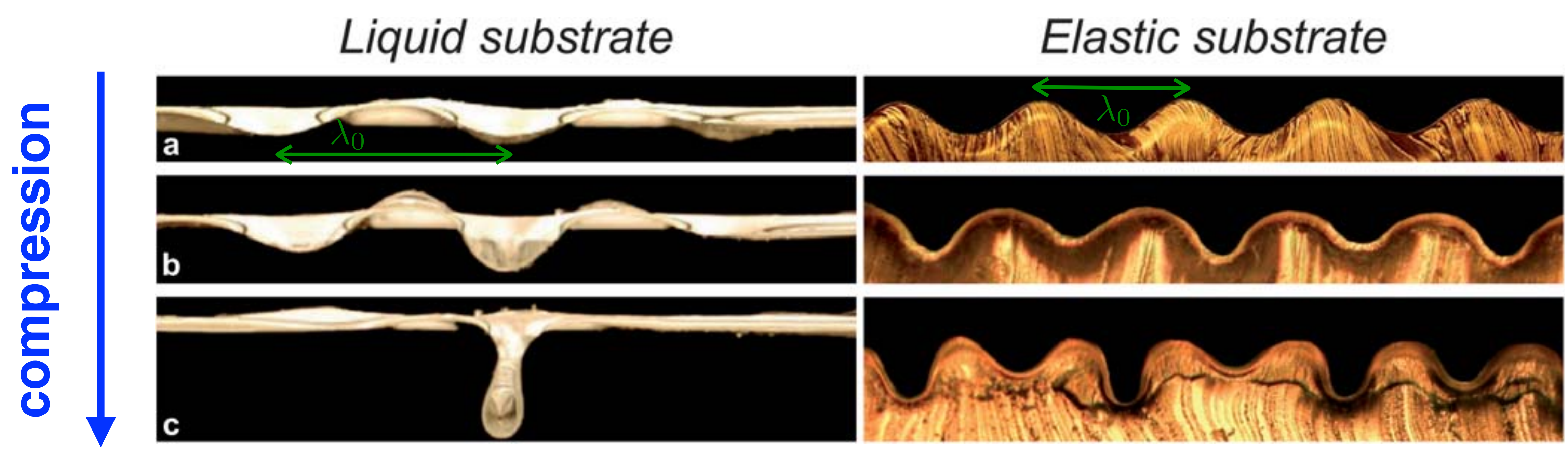


Brain

Rising dough

Compression of stiff thin sheets on liquid and soft elastic substrates





10 μm thin sheet of polyester on water

$$\lambda_0 = 1.6\,\text{cm}$$

L. Pocivavsek et al., Science **320**, 912 (2008)

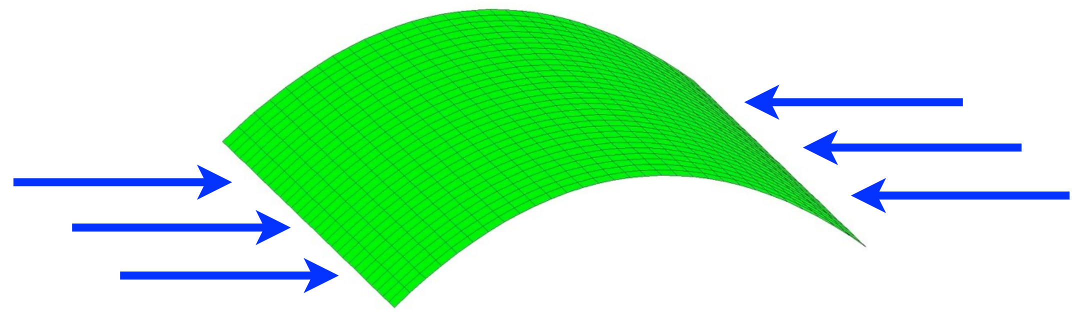
~10 μm thin PDMS (stiffer) sheet on PDMS (softer) substrate

$$\lambda_0 = 70\,\mu\text{m}$$

F. Brau et al., Soft Matter **9**, 8177 (2013)

Buckling vs wrinkling

Compressed thin sheets buckle



Compressed thin sheets on liquid and soft elastic substrates wrinkle

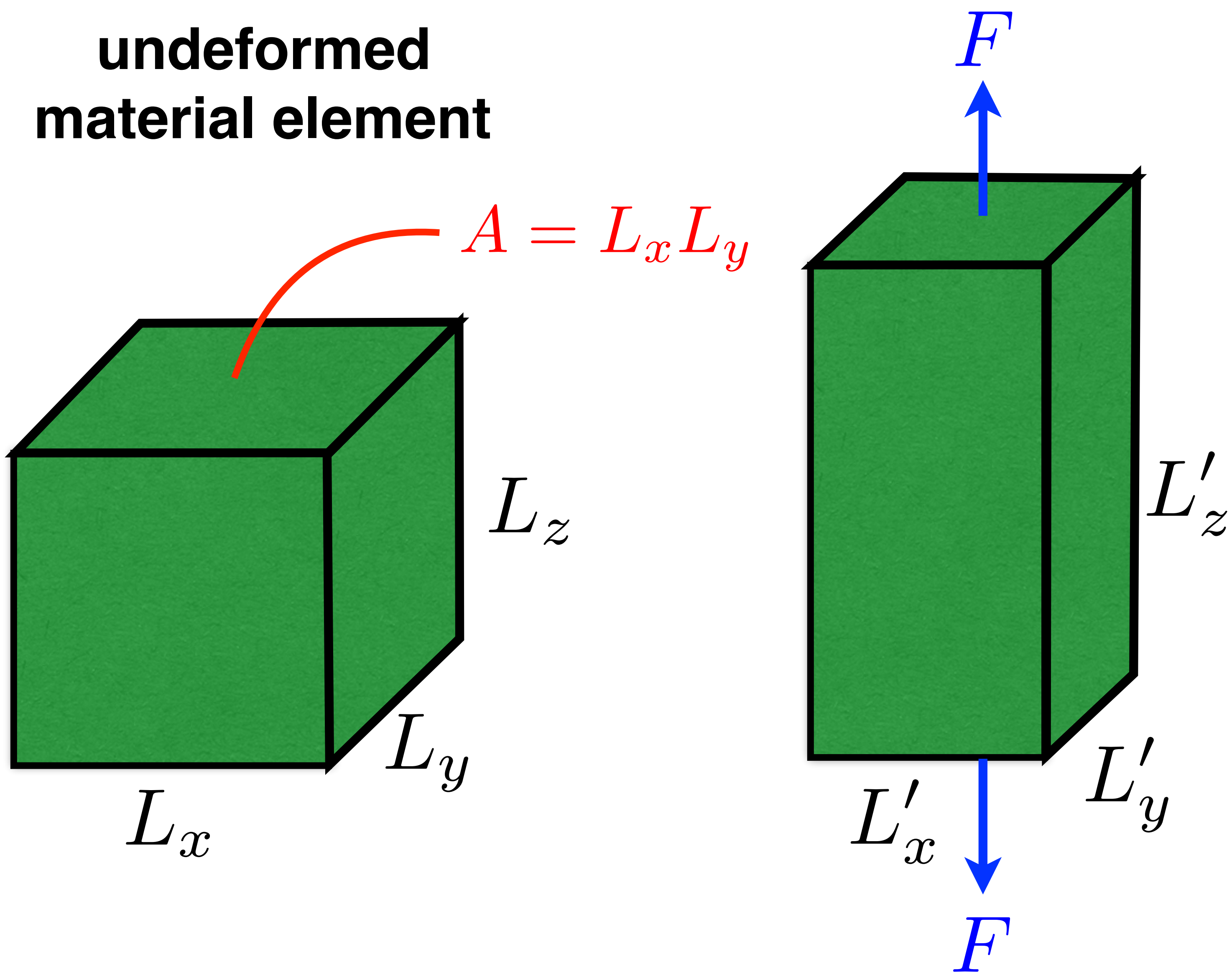
Liquid substrate

Elastic substrate

In compressed thin sheets on liquid and soft elastic substrates global buckling is suppressed, because it would result in very large energy cost associated with deformation of the liquid or soft elastic substrate!

Brief intro to mechanics: Young's modulus

undeformed material element



Hooke's law (small deformations)

$$\frac{F}{A} = E\,\frac{\Delta L_z}{L_z}$$

normal stress: $\sigma = F/A$

Young's modulus: E

normal strain: $\epsilon = \Delta L_z / L_z$

Robert Hooke (1635-1703)

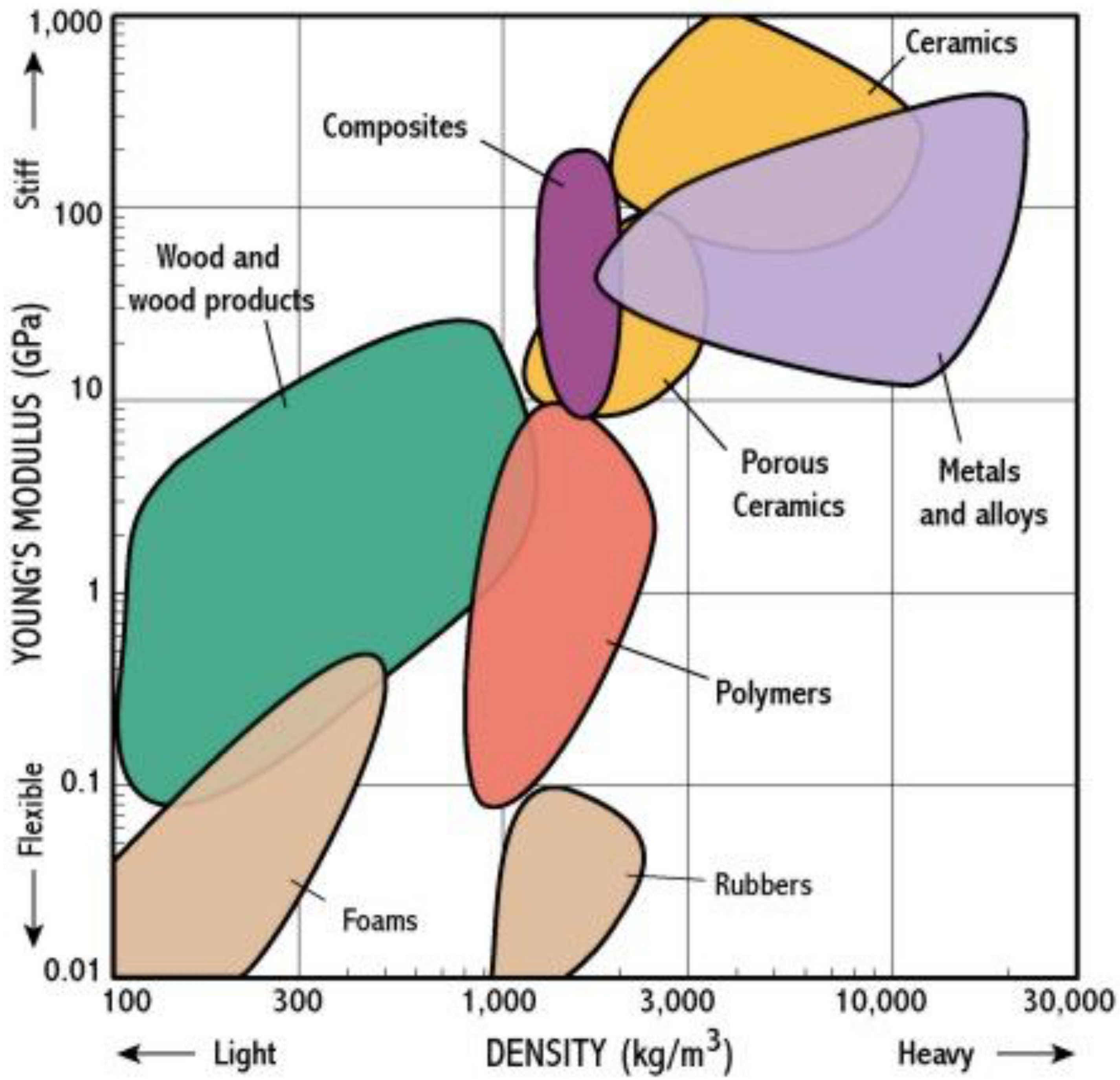
Thomas Young (1773-1829)

Elastic energy of deformation

$$U = \frac{1}{2} V E \epsilon^2$$

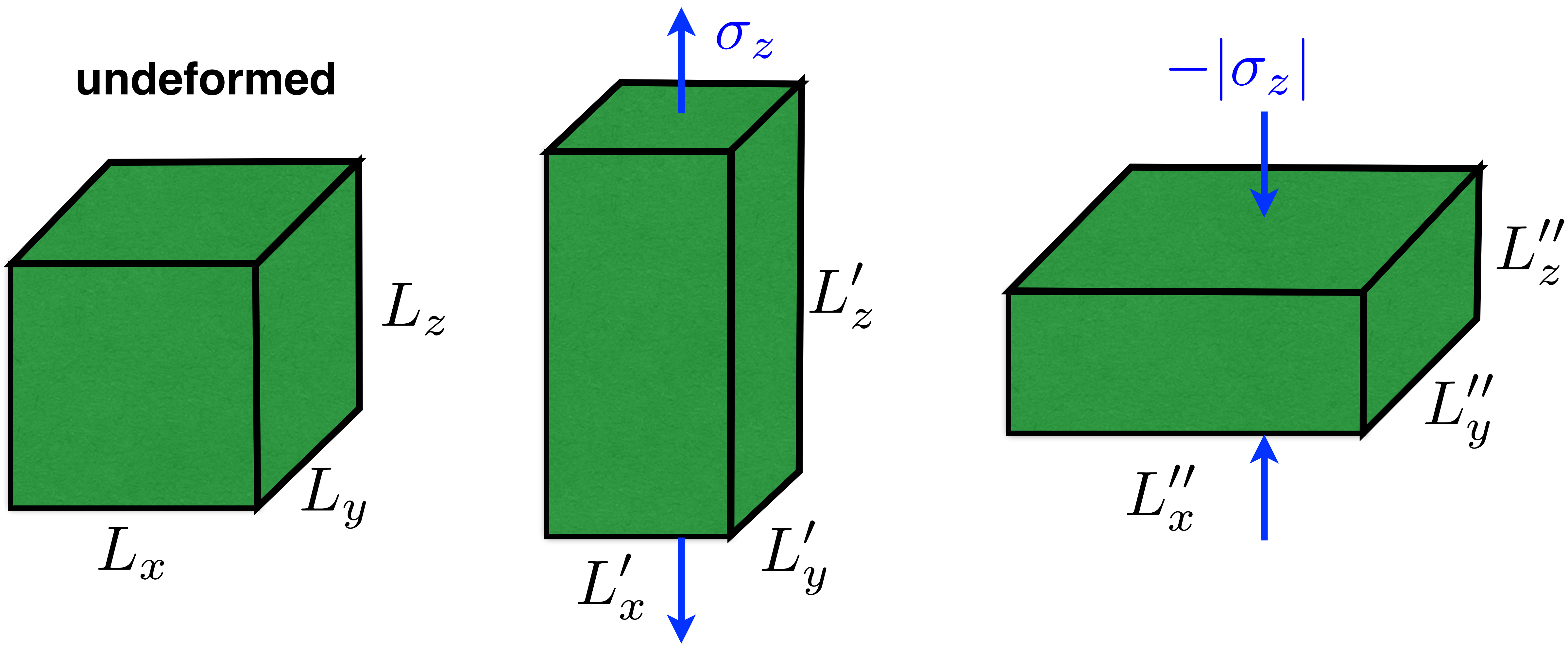
element volume: $V = L_x L_y L_z$

Young's modulus of materials



http://www-materials.eng.cam.ac.uk/mpsite/physics/introduction/

Poisson's ratio



Typically material shrinks (expands) in the transverse direction of the axial tension (compression)!

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

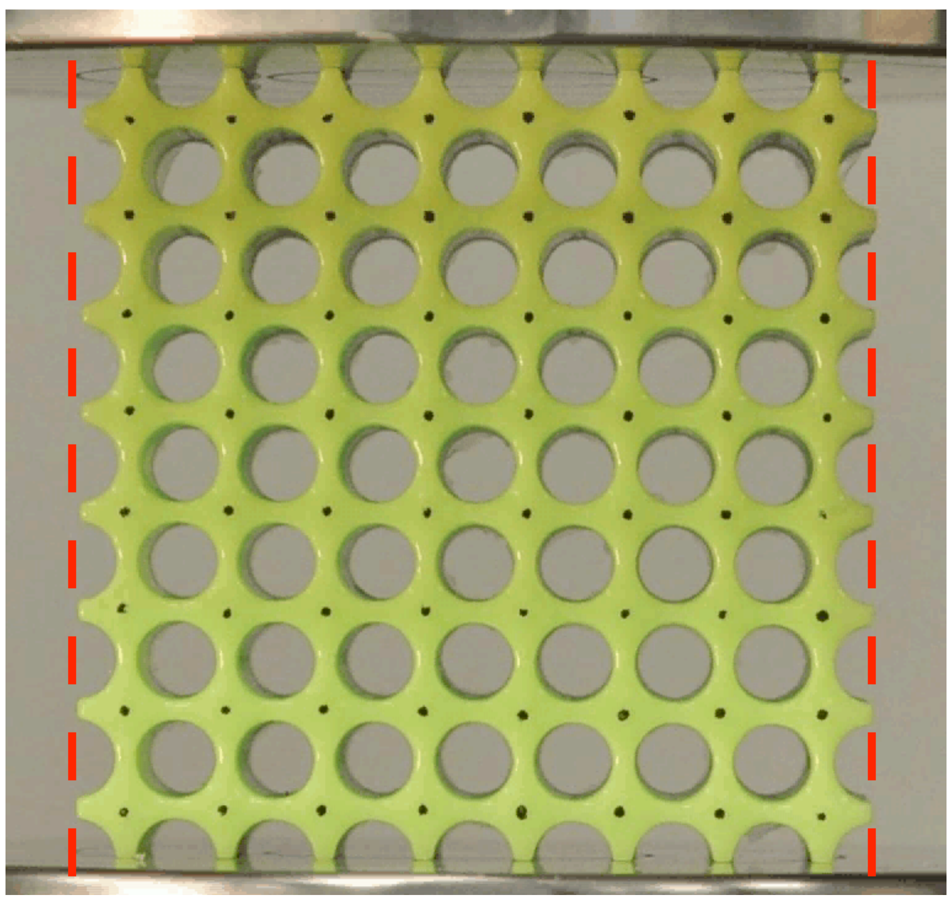
$$\epsilon_z = \frac{\sigma_z}{E}$$

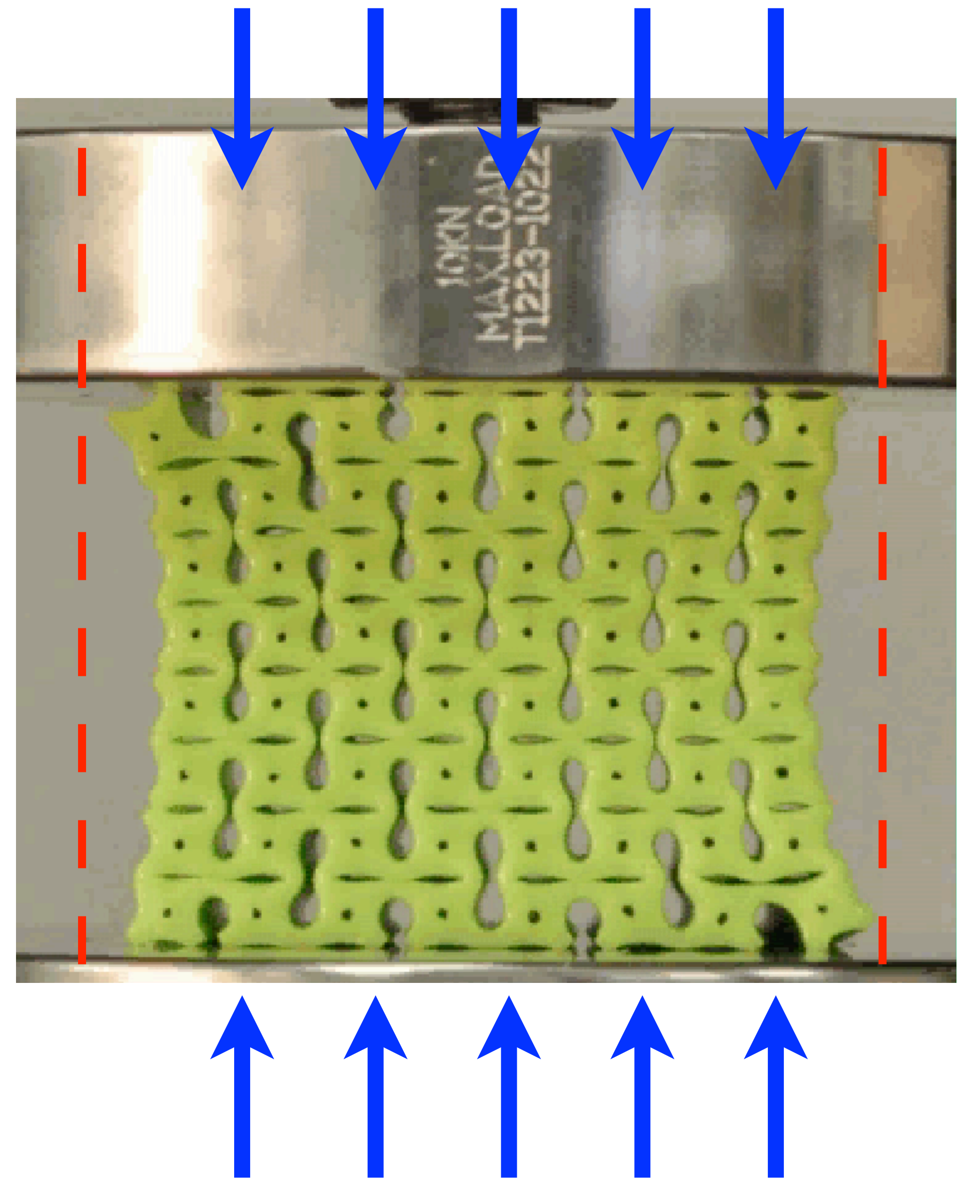
normal strains: $\epsilon_i = \dfrac{\Delta L_i}{L_i}$

Simeon Poisson (1781-1840)

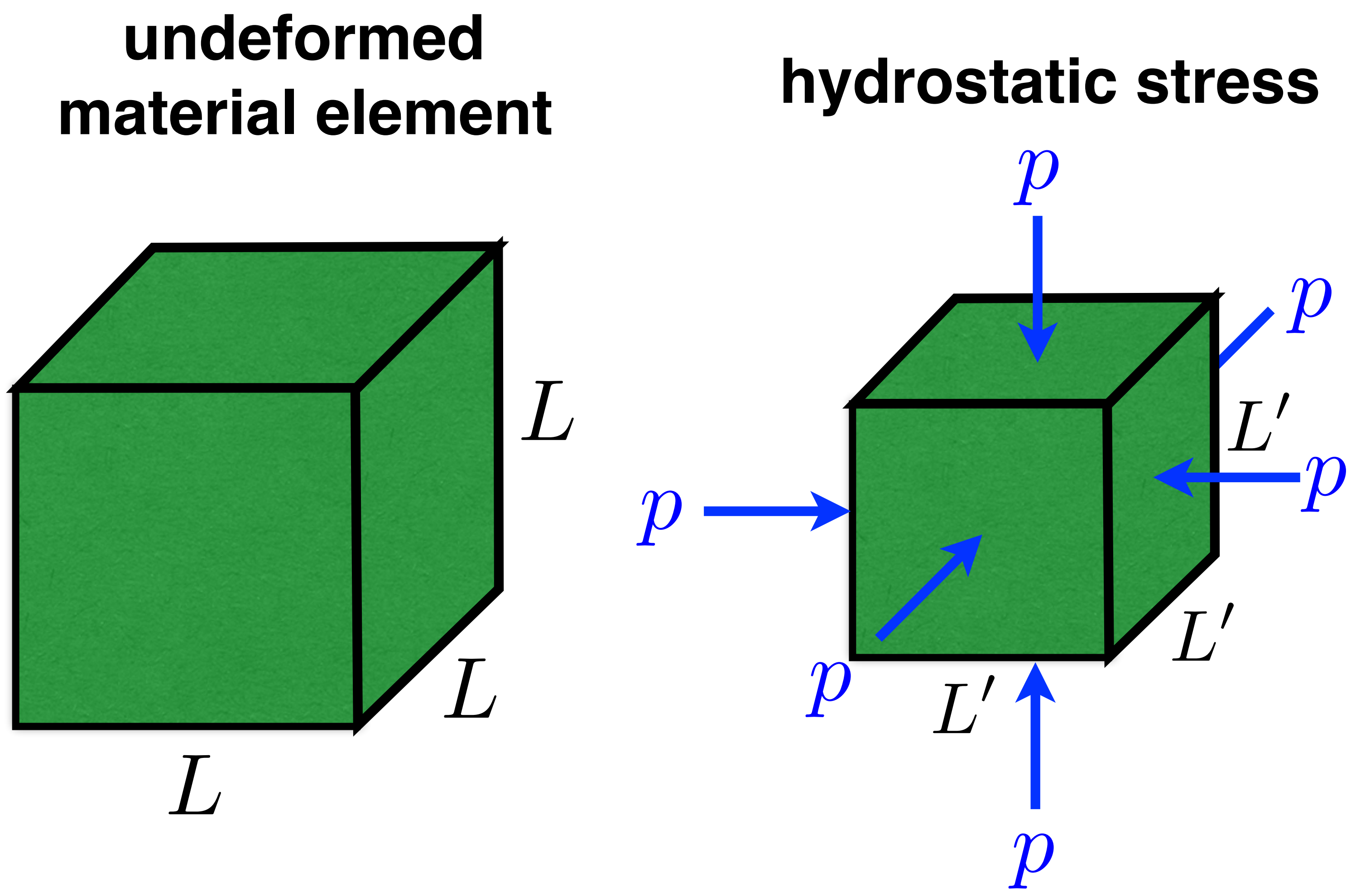
Effective negative Poisson’s ratio for structures

Certain structures behave like they have effective negative Poisson’s ratio, even though they are made of materials with positive Poisson’s ratio!





Bulk modulus



Hooke's law (small deformations)

$$\frac{\Delta V}{V} = -\frac{p}{K}$$

hydrostatic stress: p

bulk modulus: $K = \frac{E}{3(1-2\nu)}$

volumetric strain: $\frac{\Delta V}{V} \approx 3\frac{\Delta L}{L}$

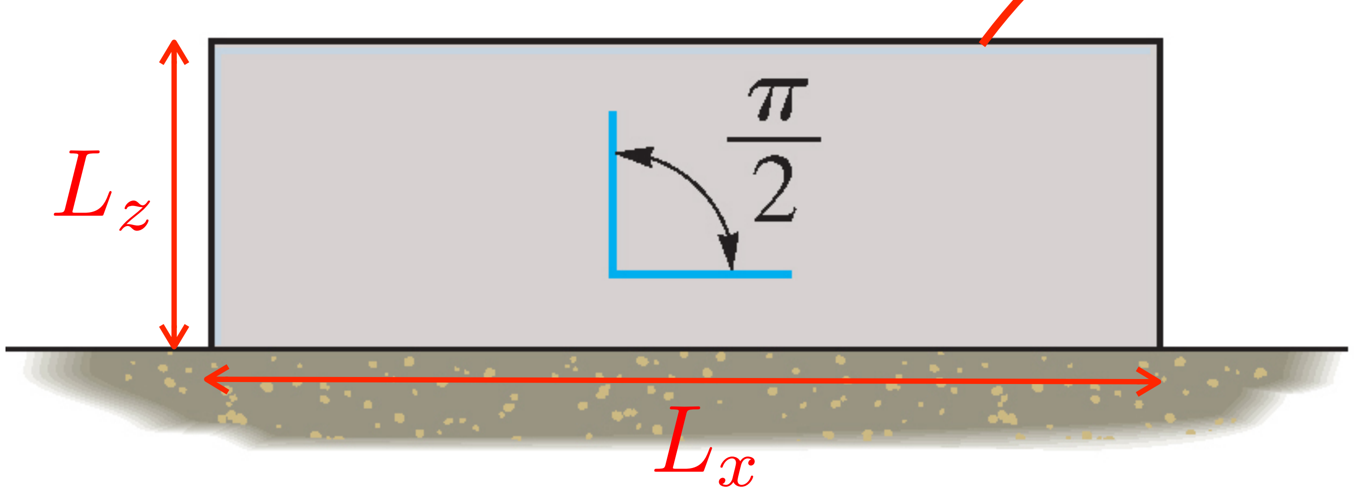
Elastic energy of deformation

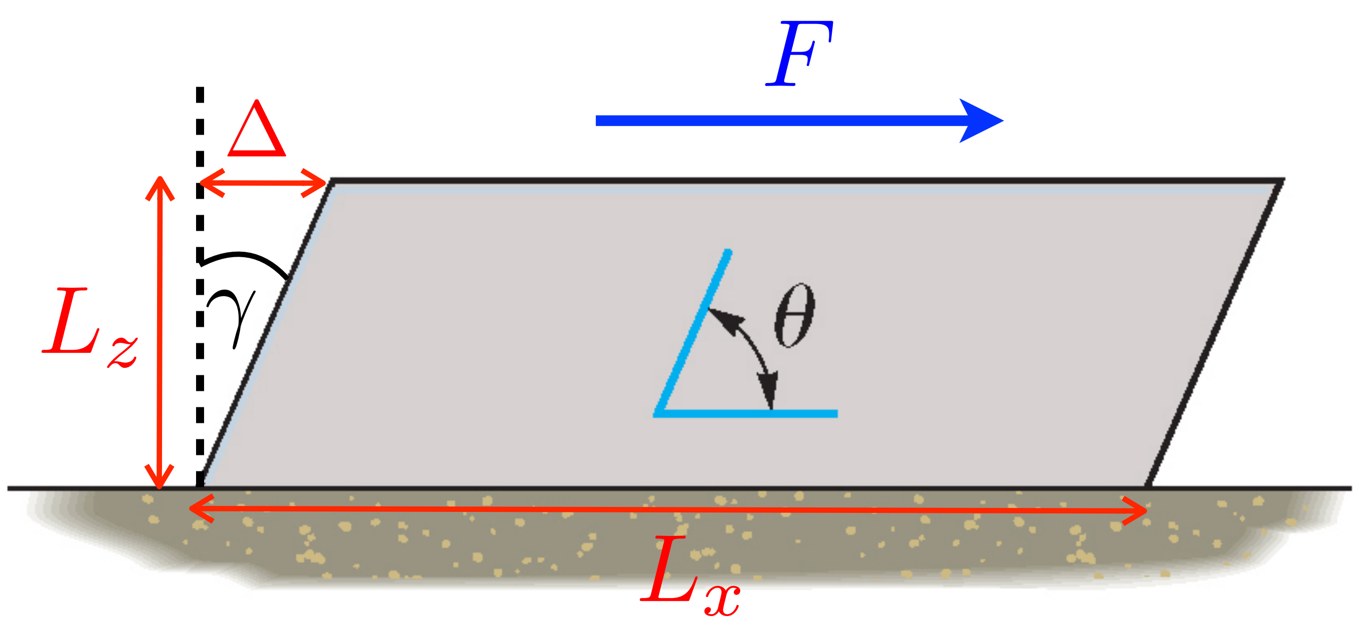
$$U = \frac{1}{2} V K \left(\frac{\Delta V}{V}\right)^2 \sim V E \left(\frac{\Delta L}{L}\right)^2$$

Shear

undeformed material element

$A = L_x L_y$





Note: shear does not change the volume of material element!

Hooke's law (small deformations)

$$\frac{F}{A} = G\gamma$$

shear stress: $\tau = F/A$

shear modulus: $G = \dfrac{E}{2(1+\nu)}$

shear strain: $\gamma = \arctan\left(\Delta/L_z\right)$

$$\gamma \approx \Delta/L_z$$

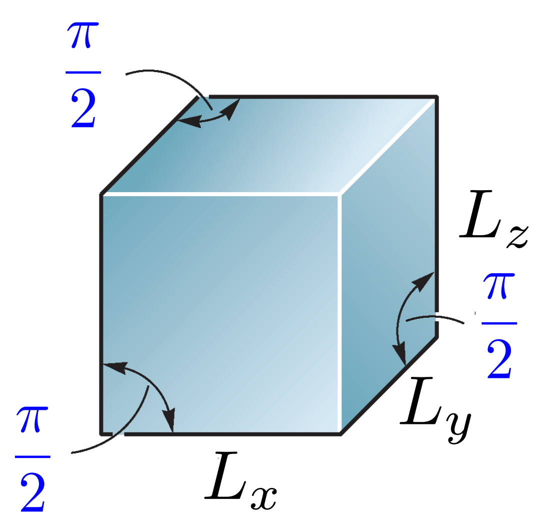
Elastic energy of deformation

$$U = \frac{1}{2} V G \gamma^2 \sim V E \left(\frac{\Delta}{L_z}\right)^2$$

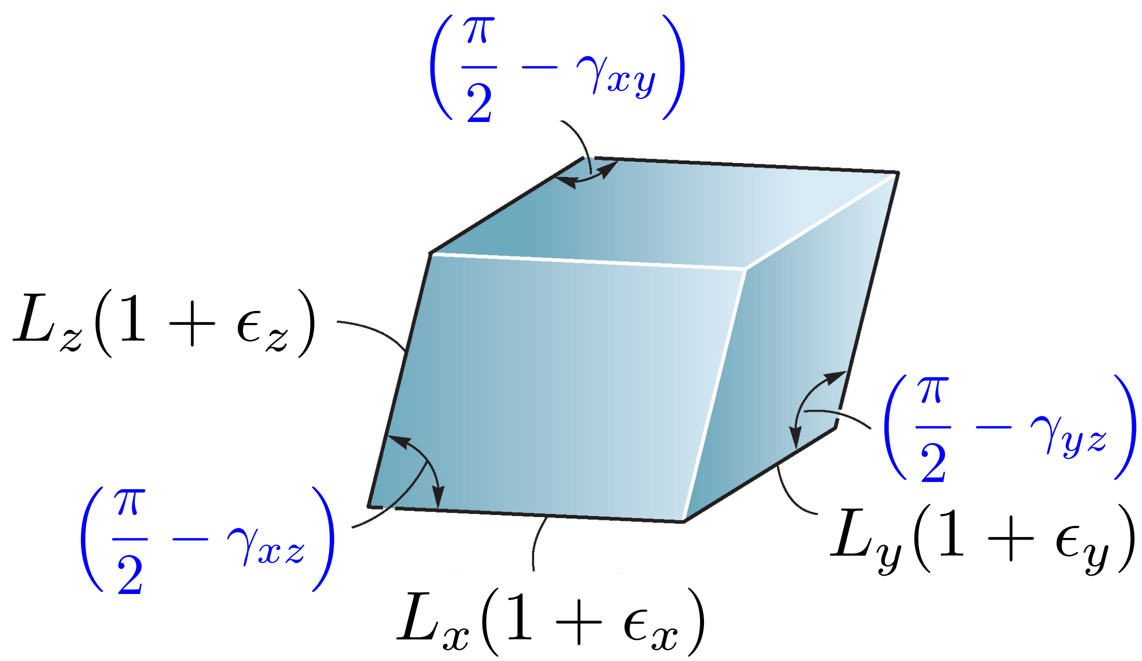
element volume: $V = L_x L_y L_z$

Arbitrary deformation of 3D solid element

undeformed element



deformed element

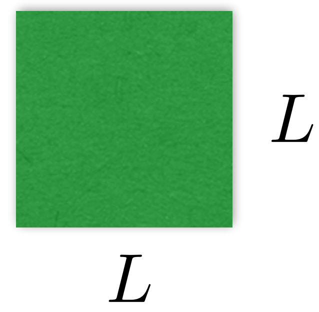


Arbitrary deformation can be decomposed to the volume change and the shear deformation.

$$U = U_{\text{bulk}} + U_{\text{shear}}$$

In plane deformations of thin sheets

undeformed square patch of thin sheet

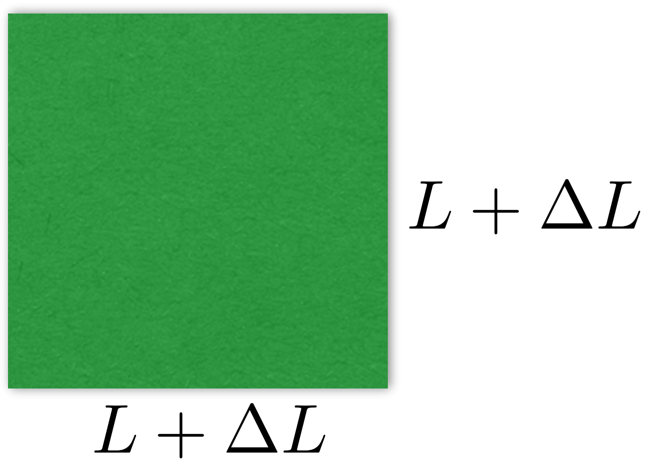


patch area

$A = L^2$

sheet thickness t
Young's modulus E
Poisson's ratio ν

isotropic deformation

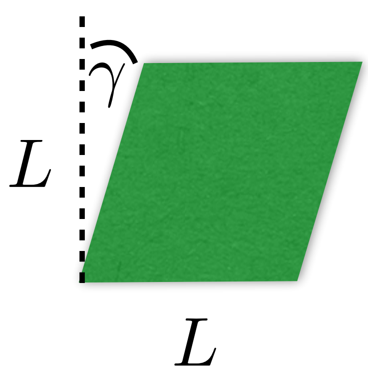


$$\frac{U}{A} = \frac{B}{2}\left(\frac{\Delta A}{A}\right)^2 \approx \frac{B}{2}\left(\frac{2\Delta L}{L}\right)^2$$

2D bulk modulus

$$B = \frac{Et}{2(1-\nu)}$$

shear deformation

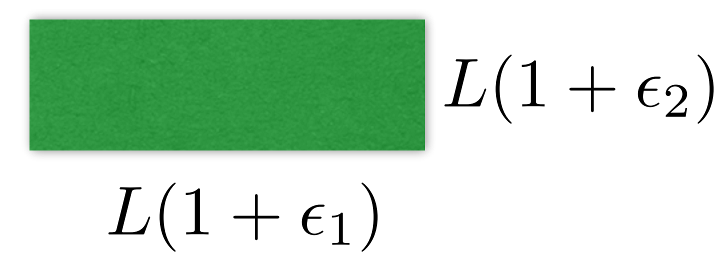


$$\frac{U}{A} = \frac{\mu \gamma^2}{2}$$

2D shear modulus

$$\mu = Gt = \frac{Et}{2(1+\nu)}$$

anisotropic stretching

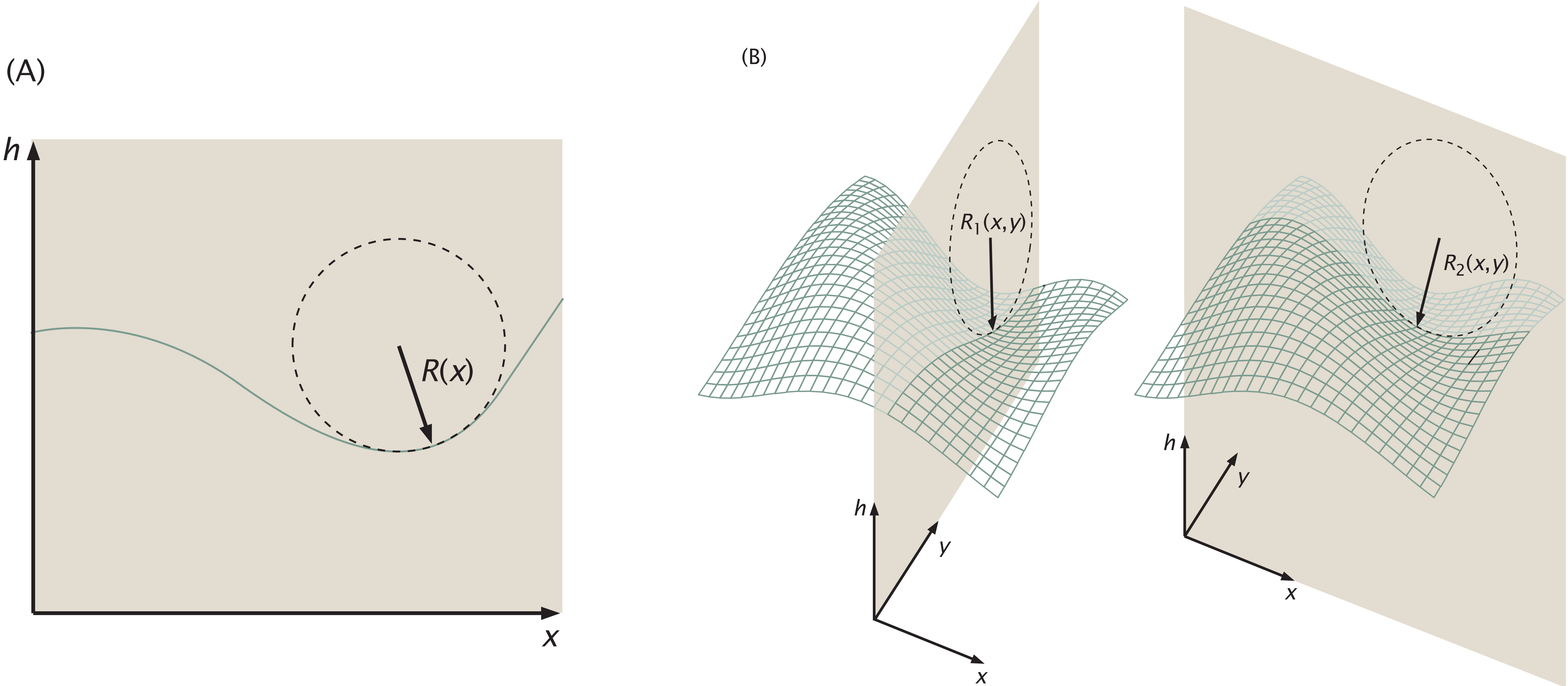


$$\frac{U}{A} = \frac{B}{2}(\epsilon_1 + \epsilon_2)^2 + \frac{\mu}{2}(\epsilon_1 - \epsilon_2)^2$$

$\epsilon_1, \epsilon_2 \ll 1$

(shearing can be interpreted as anisotropic stretching)

Curvature of surfaces



**curvature
of space curves**

$$\frac{1}{R} = \frac{h''}{\left(1 + h'^2\right)^{3/2}} \approx h''$$

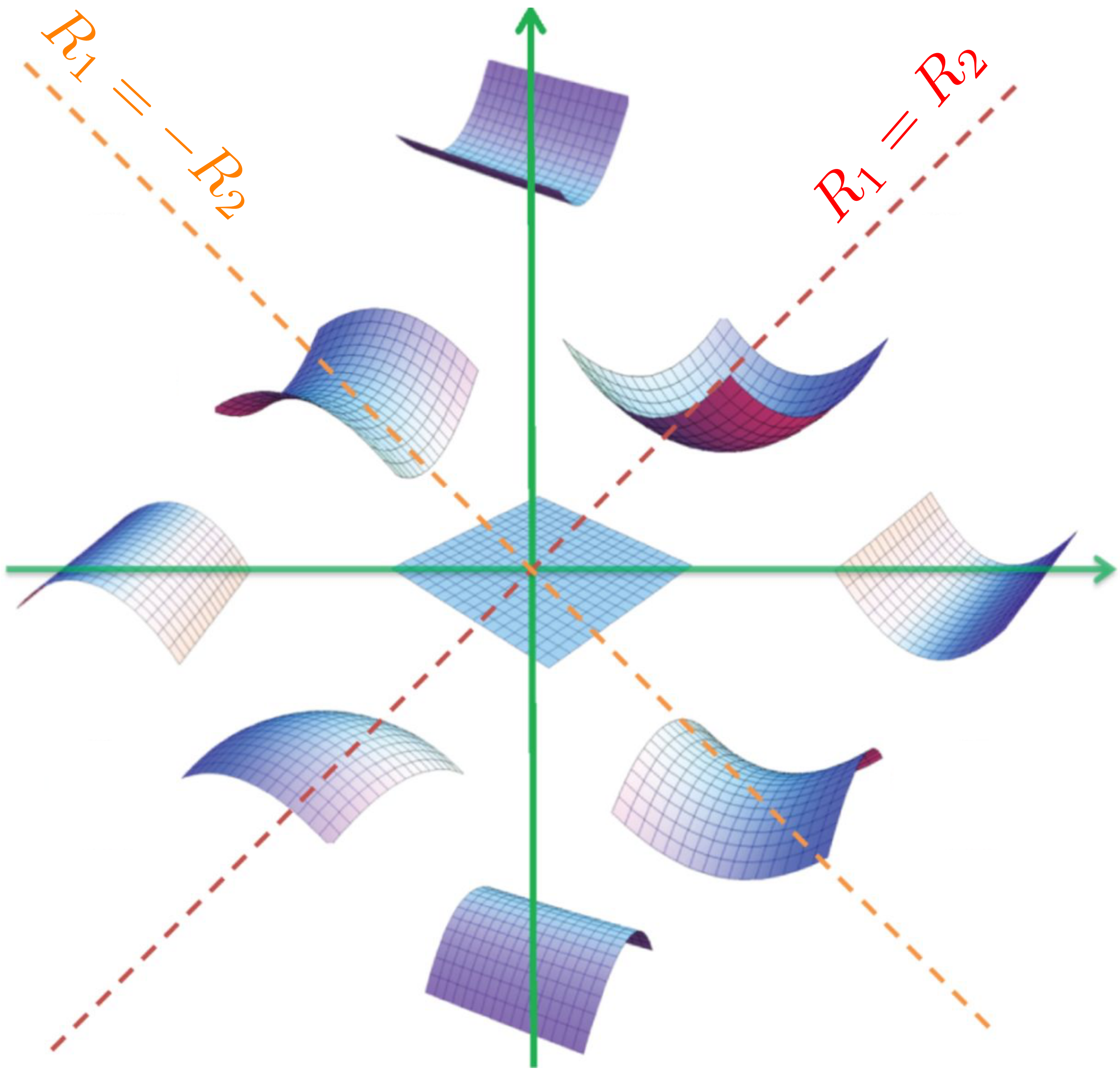
R. Phillips et al., Physical Biology of the Cell

**curvature tensor
for surfaces**

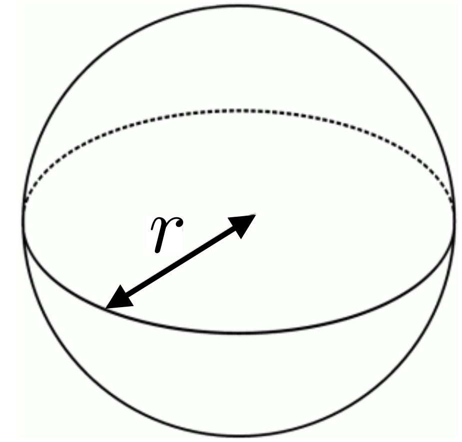
$$K_{ij} \approx \begin{pmatrix} \frac{\partial^2 h}{\partial x^2}, & \frac{\partial^2 h}{\partial x \partial y} \\ \frac{\partial^2 h}{\partial x \partial y}, & \frac{\partial^2 h}{\partial y^2} \end{pmatrix}$$

**maximal and
minimal curvatures
(principal curvatures)
correspond to the
eigenvalues of
curvature tensor**

Surfaces of various principal curvatures

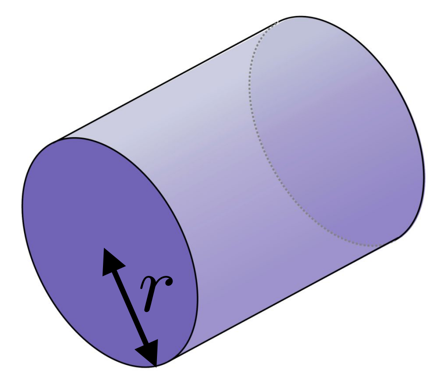


sphere



$$\frac{1}{R_1} = \frac{1}{R_2} = \frac{1}{r}$$

cylinder



$$\frac{1}{R_1} = \frac{1}{r}$$

$$\frac{1}{R_2} = 0$$

potato chips = “saddle”

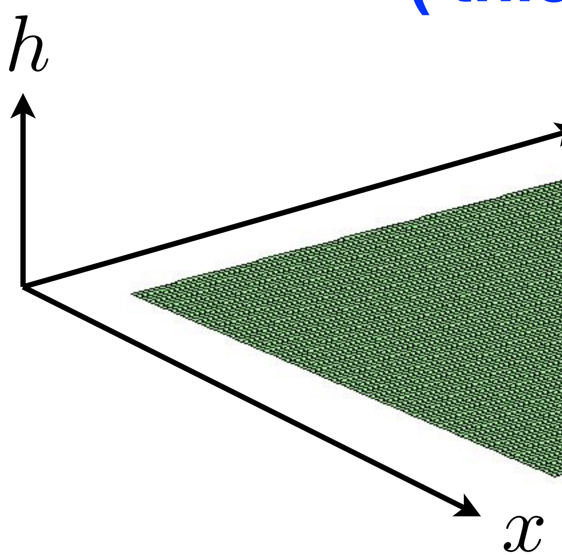
$$\frac{1}{R_1} > 0$$

$$\frac{1}{R_2} < 0$$

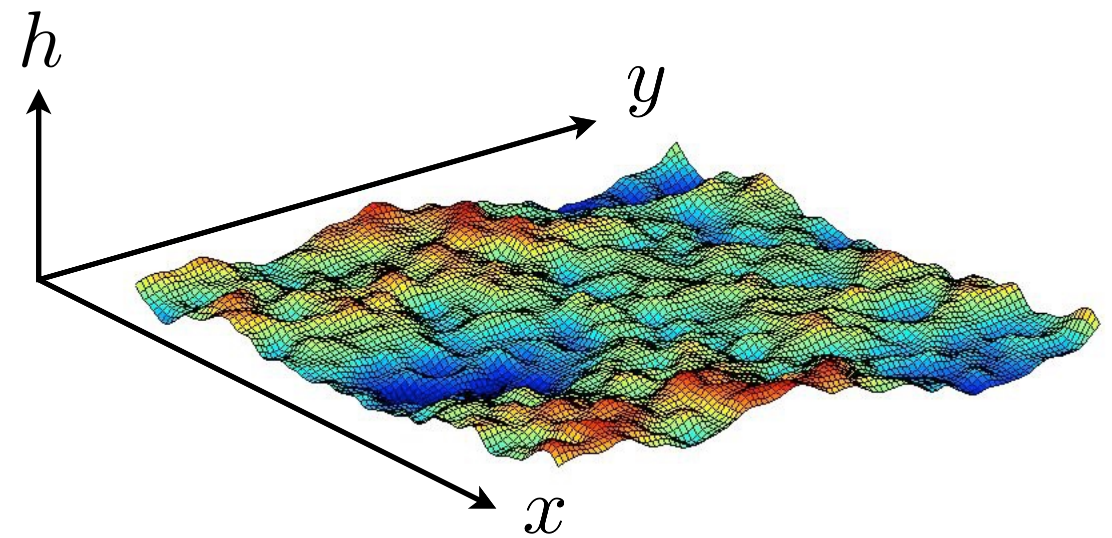
Bending energy cost for thin sheets

undeforced thin sheet

(thickness t)



deformed thin sheet



total mean curvature **Gaussian curvature**

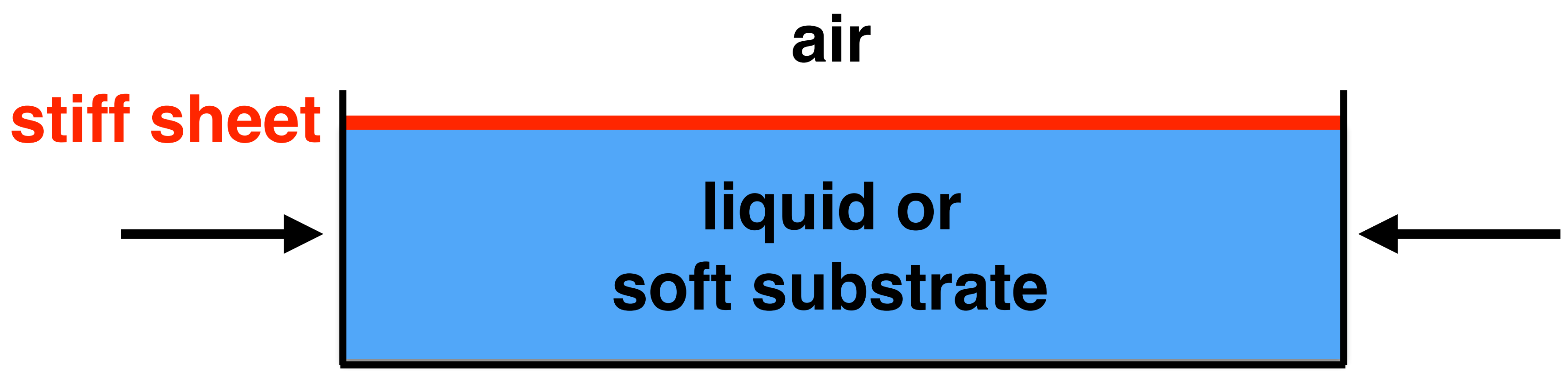
$$U = \int dA \left[\frac{\kappa}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^2 + \kappa_G \frac{1}{R_1 R_2} \right]$$

$$U \approx \int dx dy \left[\frac{\kappa}{2} \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right)^2 + \kappa_G \det \left(\frac{\partial^2 h}{\partial x_i \partial x_j} \right) \right] \qquad x_i, x_j \in \{x, y\}$$

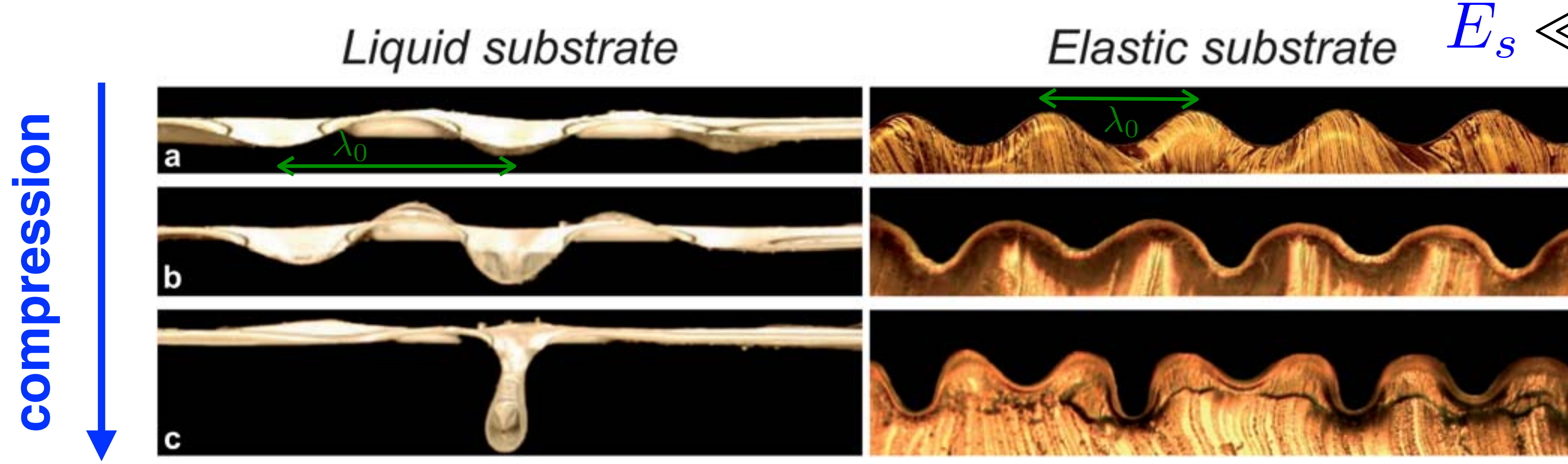
bending rigidity (flexural rigidiy) $\kappa = \dfrac{Et^3}{12(1-\nu^2)}$

Gauss bending rigidity $\kappa_G = -\dfrac{Et^3}{12(1+\nu)}$

Compression of stiff thin sheets on liquid and soft elastic substrates



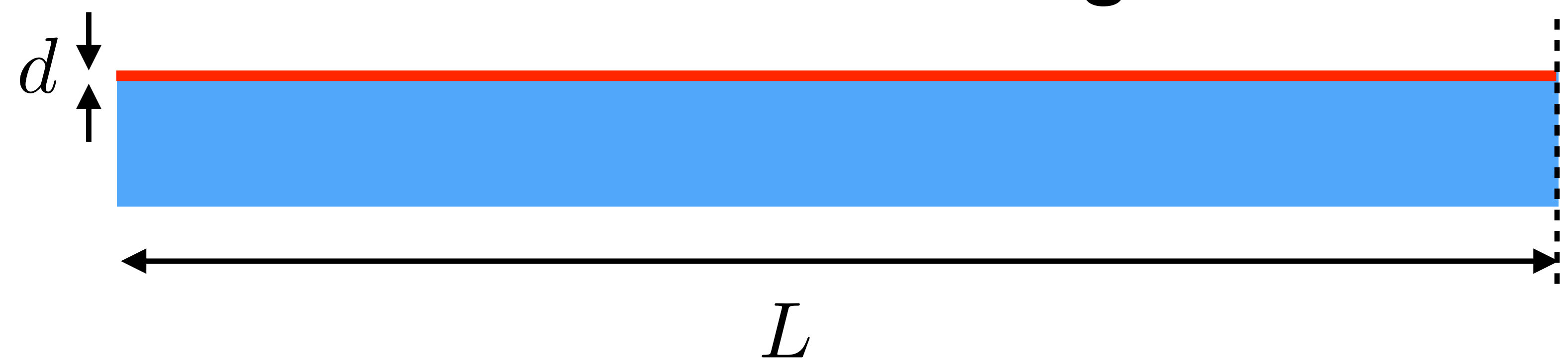
$E_s \ll E_m$



10 μm thin sheet of polyester on water	~10 μm thin PDMS (stiffer) sheet on PDMS (softer) substrate
$\lambda_0 = 1.6\,\mathrm{cm}$	$\lambda_0 = 70\,\mu\mathrm{m}$
L. Pocivavsek et al., Science **320**, 912 (2008)	F. Brau et al., Soft Matter **9**, 8177 (2013)

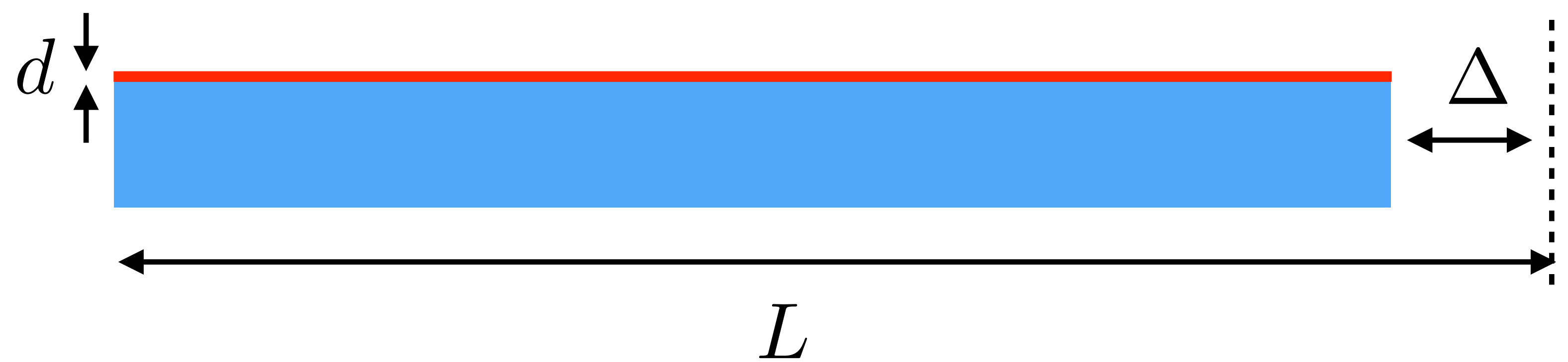
Compression of stiff thin membranes on liquid substrates

initial undeformed configuration



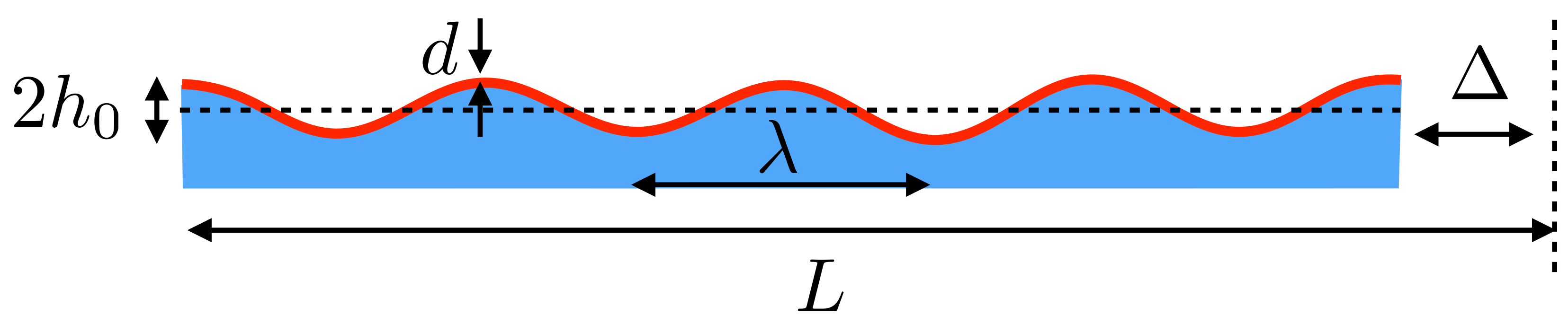
Consider the energy cost for two different scenarios:

1.) thin membrane is compressed (no bending)

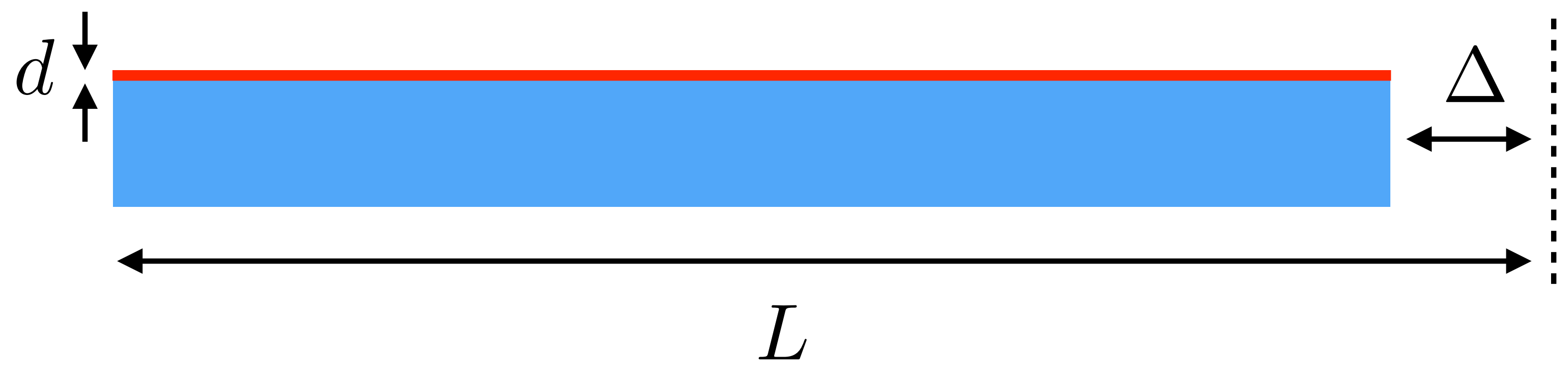


2.) thin membrane is wrinkled (no compression)

+ additional potential energy of liquid



Compression of stiff thin membranes on liquid substrates



compression energy of thin membrane

$$U_c \sim A \times E_m d \times \epsilon^2$$

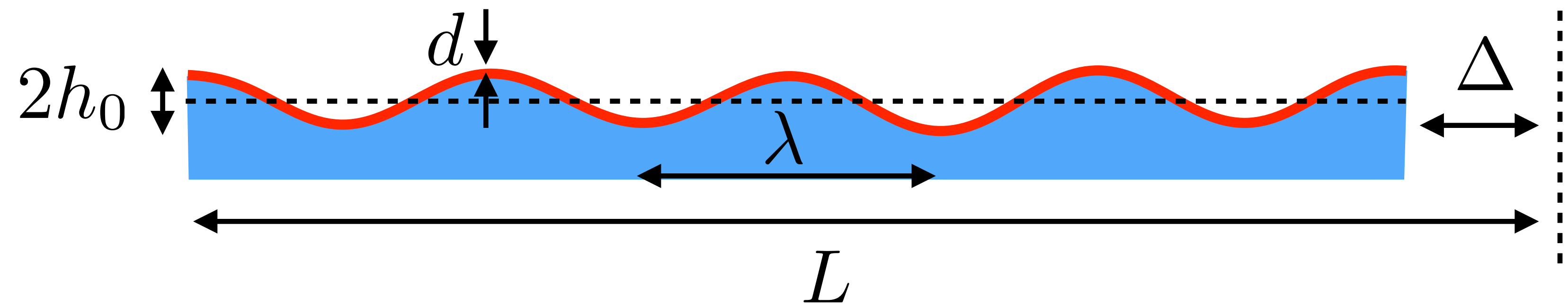
membrane area	membrane 3D Young's modulus	strain	liquid density
$A = WL$	E_m	$\epsilon = \frac{\Delta}{L}$	ρ

Note: upon compression the liquid surface also raises, but we will measure the potential energy relative to this new height!

Compression of stiff thin membranes on liquid substrates

assumed profile

$$h(s) = h_0 \cos(2\pi s/\lambda)$$



projected length assuming that membrane doesn't stretch

$$L - \Delta = \int_0^L ds \sqrt{1 - h'(s)^2} \approx \int_0^L ds \left(1 - h'(s)^2/2\right) \approx L \left(1 - \frac{\pi^2 h_0^2}{\lambda^2}\right)$$

amplitude of wrinkles

$$h_0 = \frac{\lambda}{\pi}\sqrt{\frac{\Delta}{L}} = \frac{\lambda\sqrt{\epsilon}}{\pi}$$

bending energy of stiff membrane

$$U_b \sim A \times \kappa \times \frac{1}{R^2} \sim A \times E_m d^3 \times \frac{h_0^2}{\lambda^4} \sim \frac{A E_m d^3 \epsilon}{\lambda^2}$$

potential energy of liquid

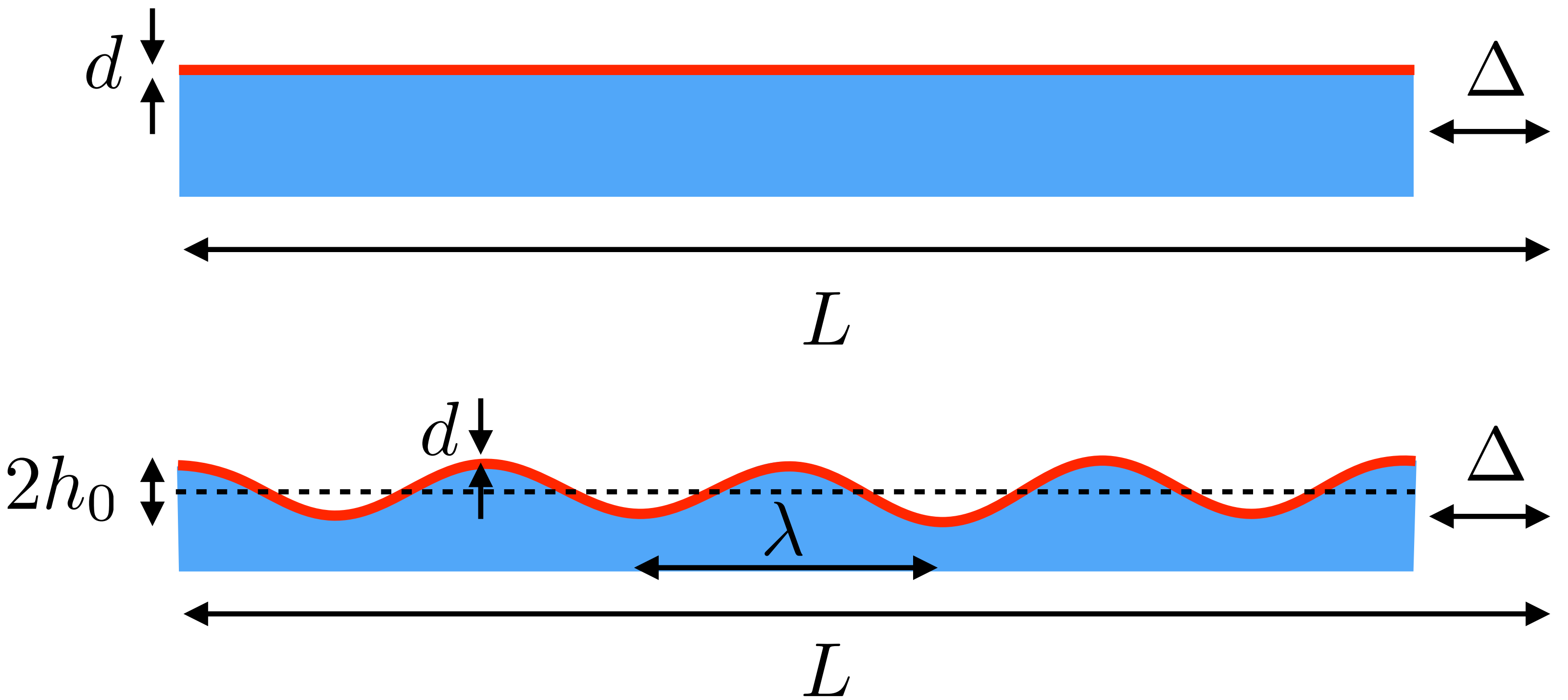
$$U_p \sim m \times g \times \Delta h \sim \rho \times A h_0 \times g \times h_0 \sim A \rho g \lambda^2 \epsilon$$

minimize total energy (U_b+U_p) with respect to λ

$$\lambda \sim \left(\frac{E_m d^3}{\rho g}\right)^{1/4}$$

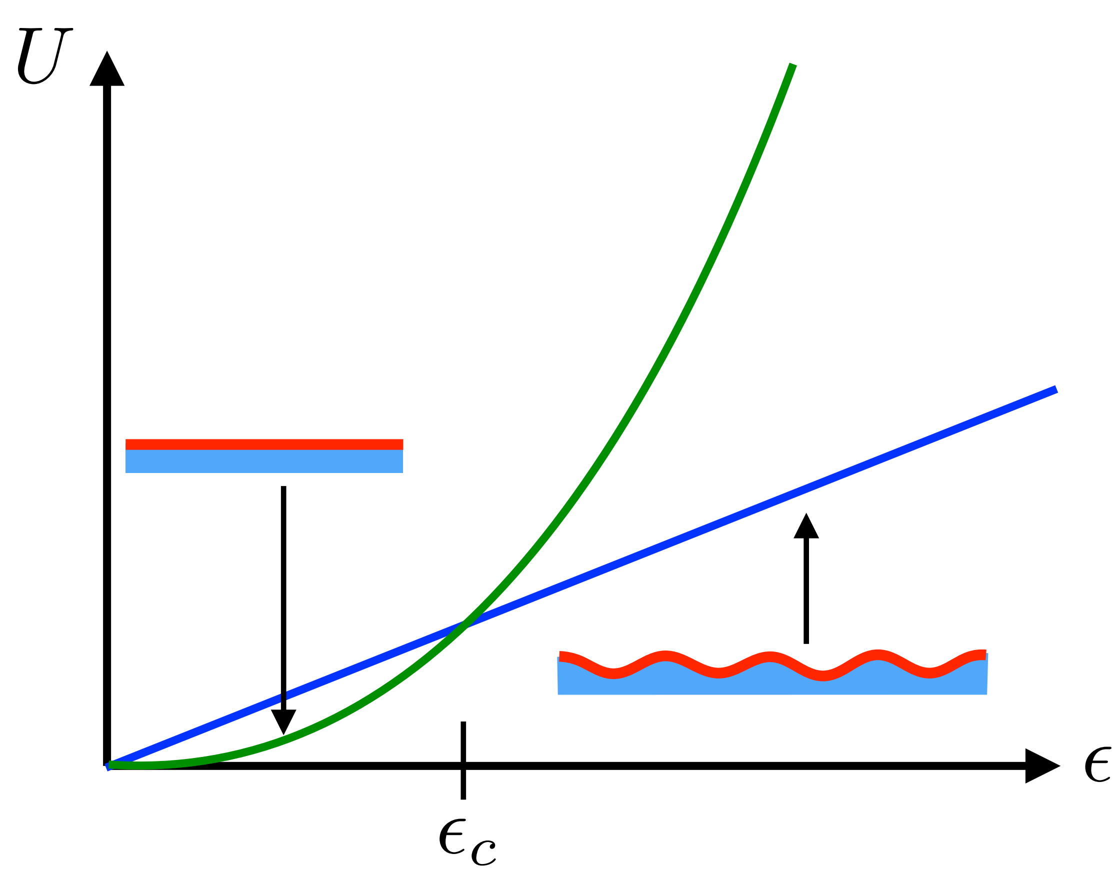
$$U_b, U_p \sim A\epsilon\sqrt{E_m d^3 \rho g}$$

Compression of stiff thin membranes on liquid substrates



$$U_c \sim A \times E_m d \times \epsilon^2$$

$$U_b, U_p \sim A\epsilon\sqrt{E_m d^3 \rho g}$$



wrinkles are stable above the critical strain

$$\epsilon > \epsilon_c \sim \sqrt{\frac{\rho g d}{E_m}}$$

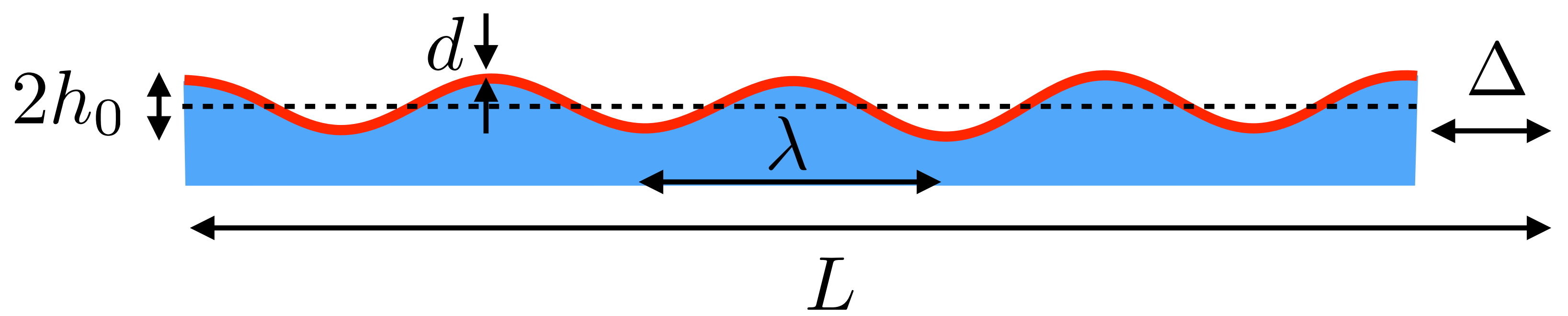
wavelength of wrinkles

$$\lambda \sim \left(\frac{E_m d^3}{\rho g}\right)^{1/4}$$

amplitude of wrinkles at the critical strain

$$h_0^* \sim \lambda\sqrt{\epsilon_c} \sim d$$

Compression of stiff thin membranes on liquid substrates



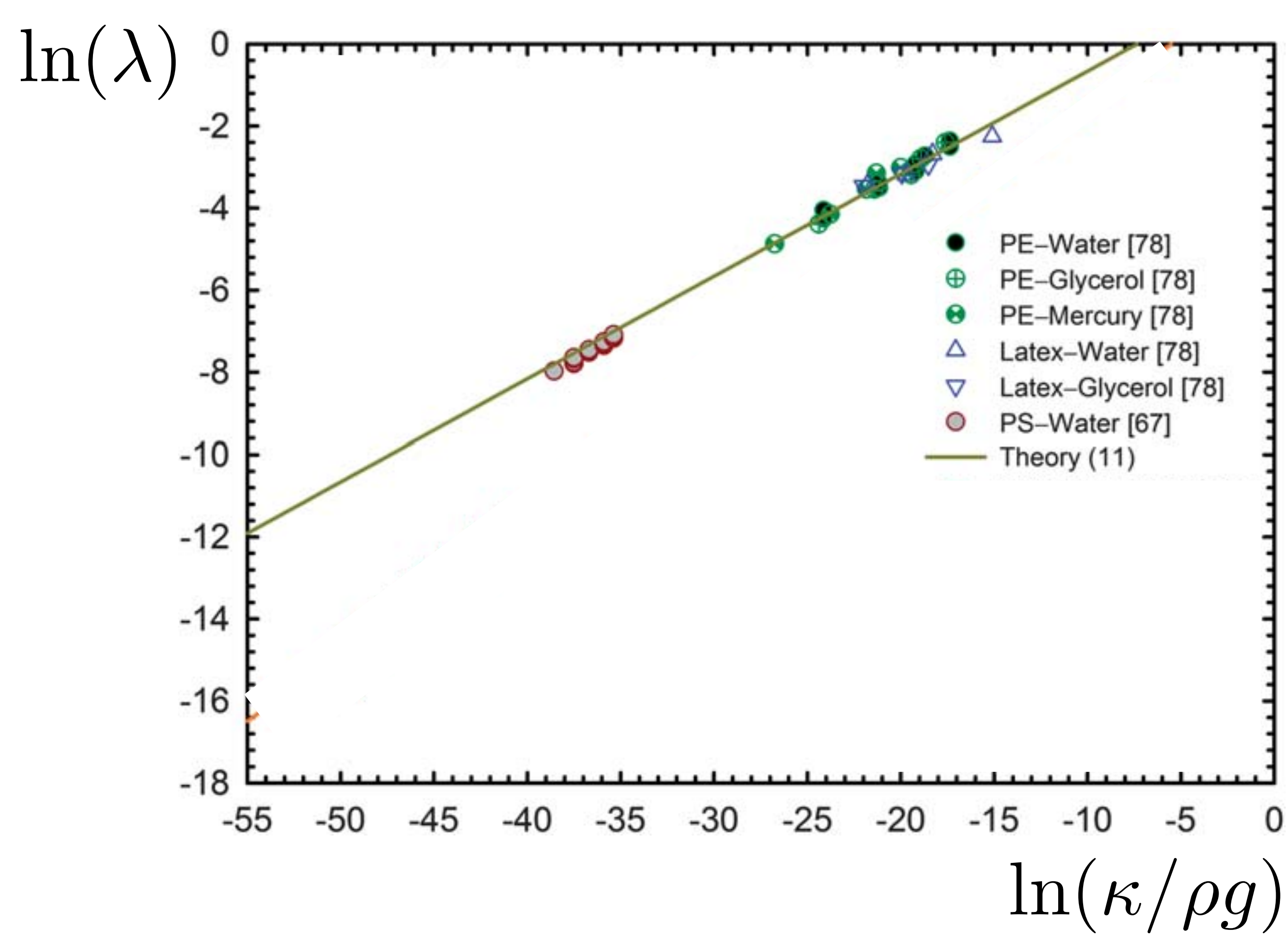
scaling analysis

$$\lambda \sim \left(\frac{E_m d^3}{\rho g}\right)^{1/4}$$

exact result

$$\lambda = 2\pi \left(\frac{\kappa}{\rho g}\right)^{1/4}$$

$$\kappa = \frac{E_m d^3}{12(1-\nu_m^2)}$$



F. Brau et al., Soft Matter **9**, 8177 (2013)