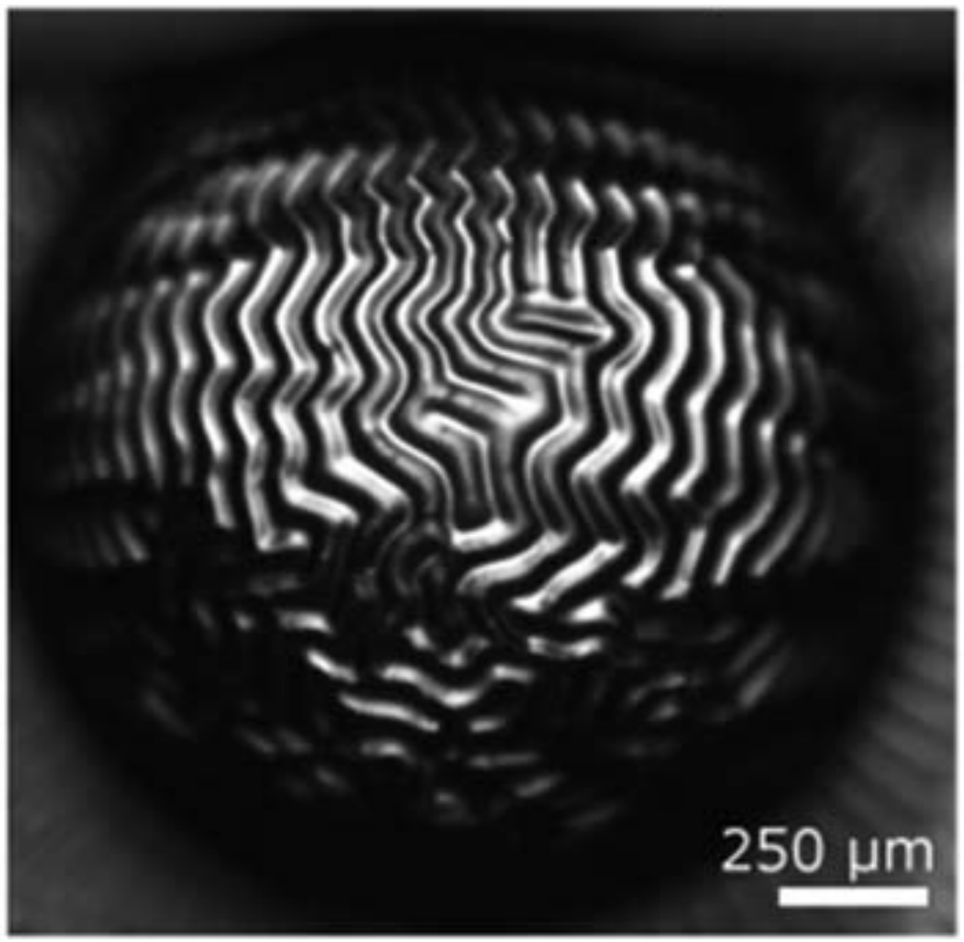
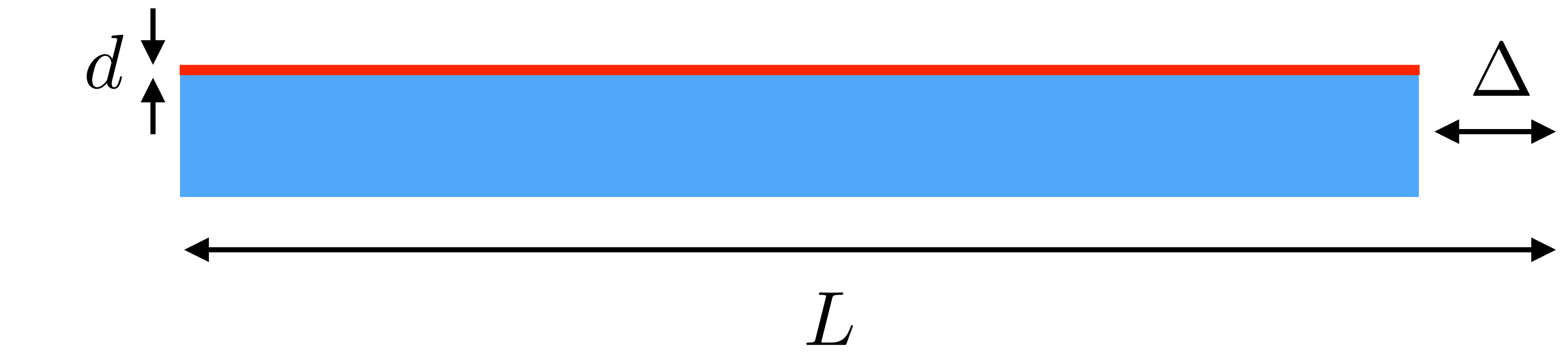
MAE 545: Lecture 6 (2/22)

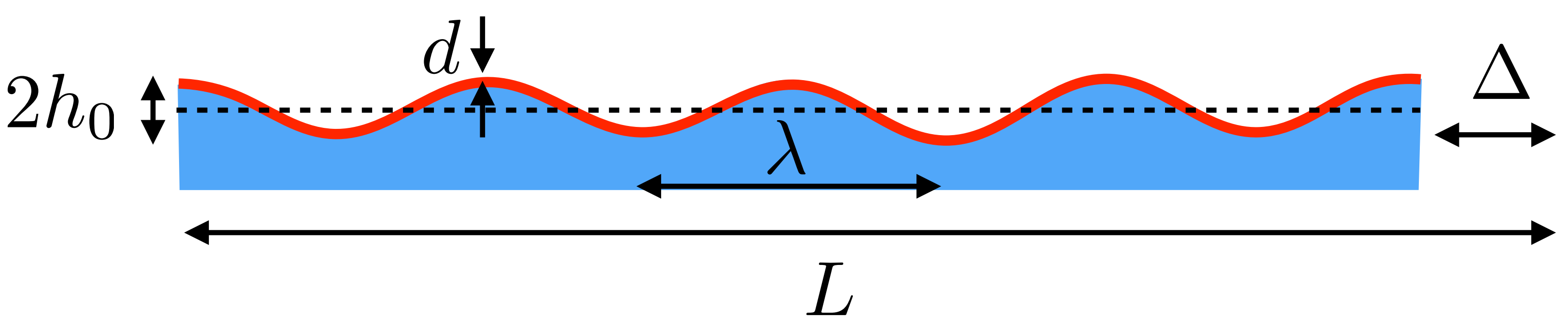
Wrinkled surfaces



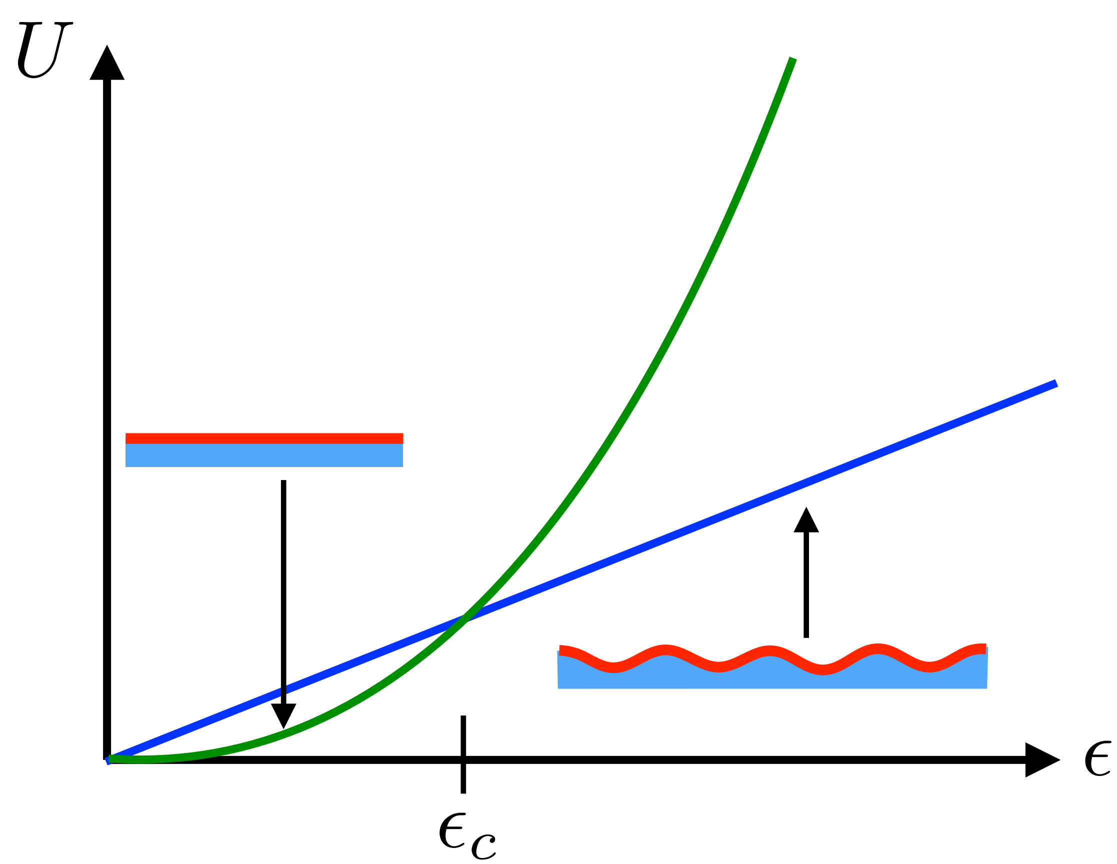
Compression of stiff thin membranes on liquid substrates



$$U_c \sim A \times E_m d \times \epsilon^2$$



$$U_b, U_p \sim A\epsilon\sqrt{E_m d^3 \rho g}$$



wrinkles are stable above the critical strain

$$\epsilon > \epsilon_c \sim \sqrt{\frac{\rho g d}{E_m}}$$

wavelength of wrinkles

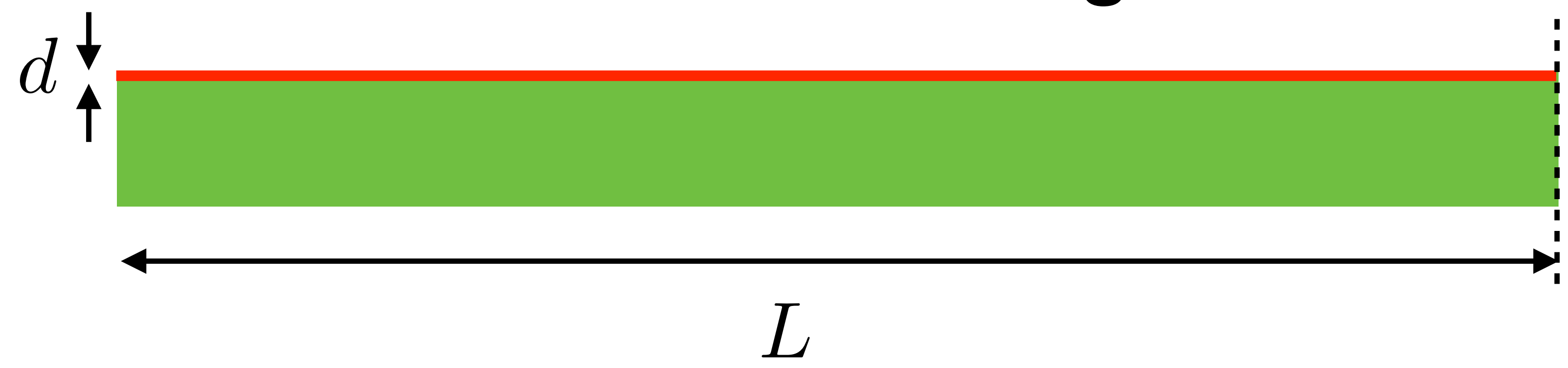
$$\lambda \sim \left(\frac{E_m d^3}{\rho g}\right)^{1/4}$$

amplitude of wrinkles at the critical strain

$$h_0^* \sim \lambda\sqrt{\epsilon_c} \sim d$$

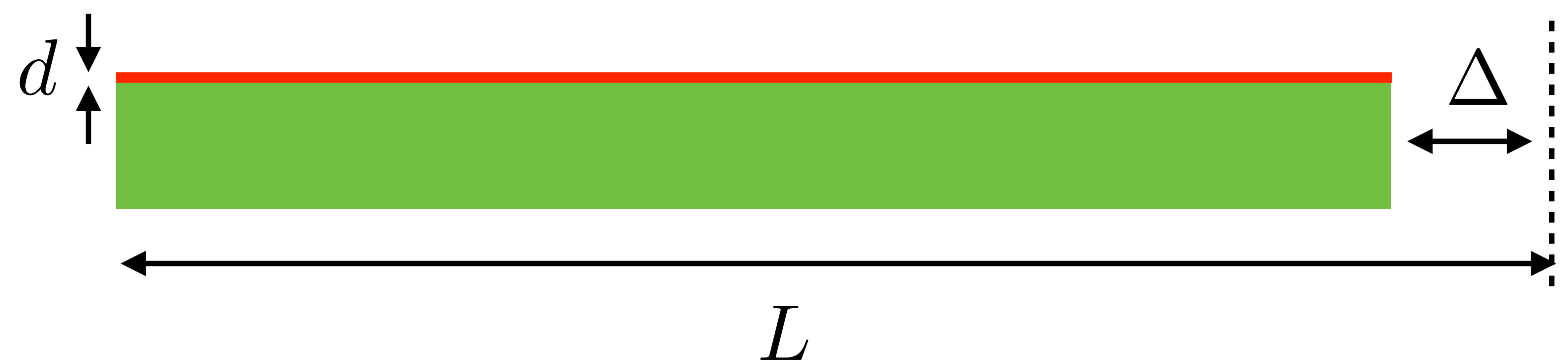
Compression of stiff thin membranes on soft elastic substrates

initial undeformed configuration



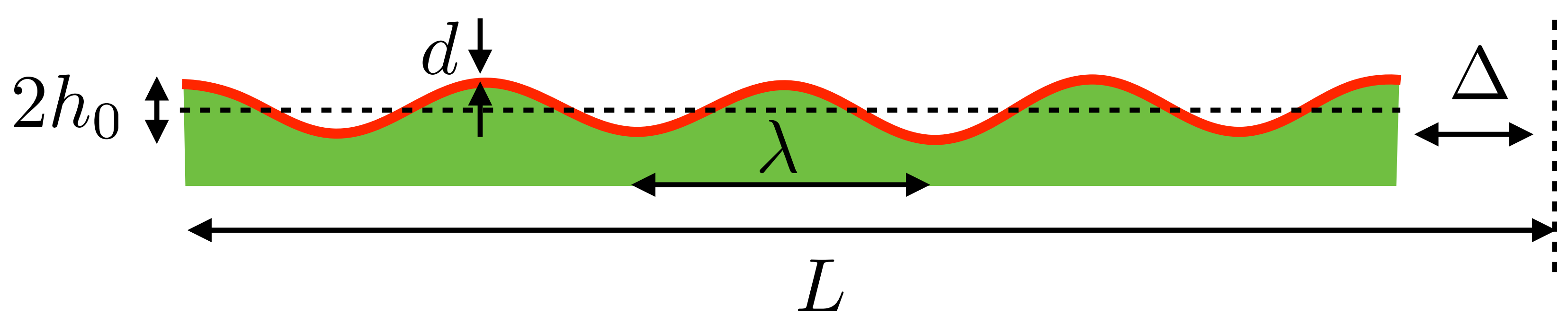
Consider the energy cost for two different scenarios:

1.) thin membrane is compressed (no bending)

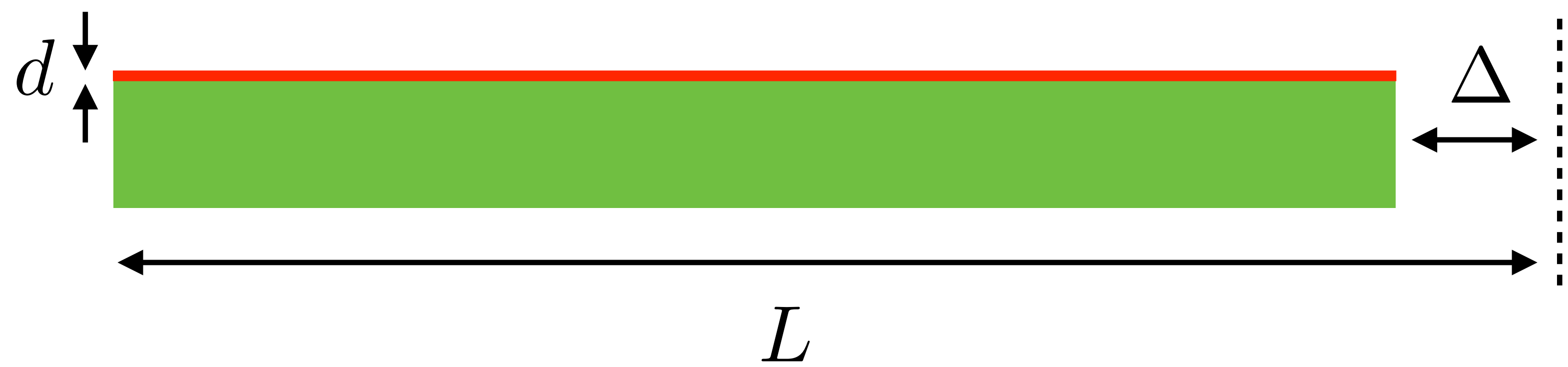


2.) thin membrane is wrinkled (no compression)

additional elastic energy for deformed substrate



Compression of stiff thin membranes on soft elastic substrates



compression energy of thin membrane

$$U_c \sim A \times E_m d \times \epsilon^2$$

membrane area	membrane 3D Young's modulus	strain	substrate 3D Young's modulus
$A = WL$	E_m	$\epsilon = \frac{\Delta}{L}$	E_s

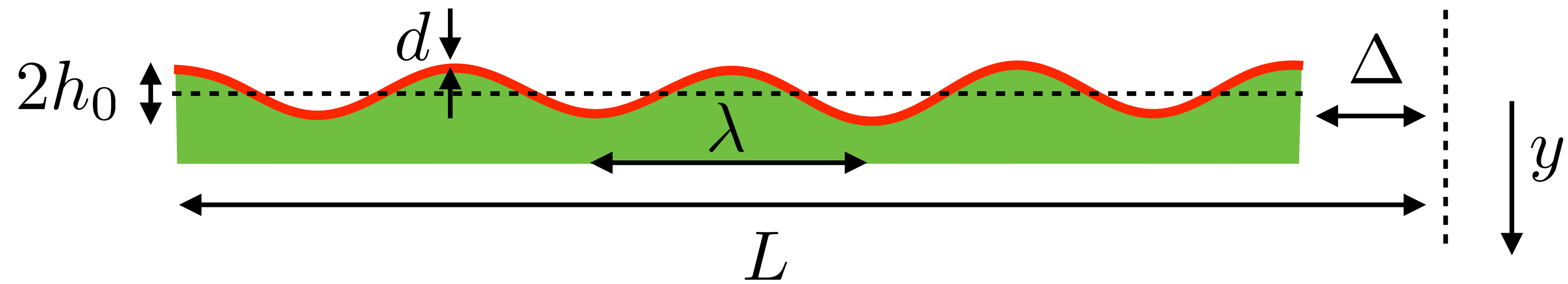
$$E_s \ll E_m$$

Note: soft elastic substrate is also compressed, but we will measure the substrate elastic energy relative to this base value!

Compression of stiff thin membranes on soft elastic substrates

assumed profile

$$h(s) = h_0 \cos(2\pi s/\lambda)$$

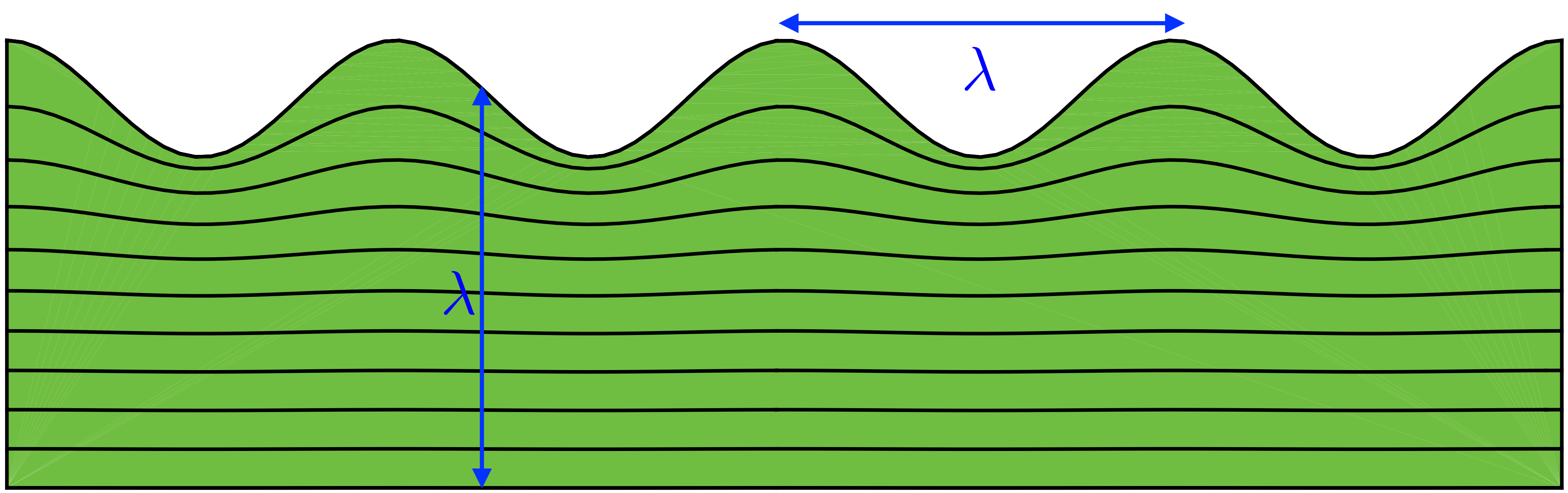


amplitude of wrinkles

$$h_0 = \frac{\lambda}{\pi}\sqrt{\frac{\Delta}{L}} = \frac{\lambda\sqrt{\epsilon}}{\pi}$$

deformation of the soft substrate decays exponentially away from the surface

$$h(s, y) \approx h_0 \cos(2\pi s/\lambda) e^{-2\pi y/\lambda}$$

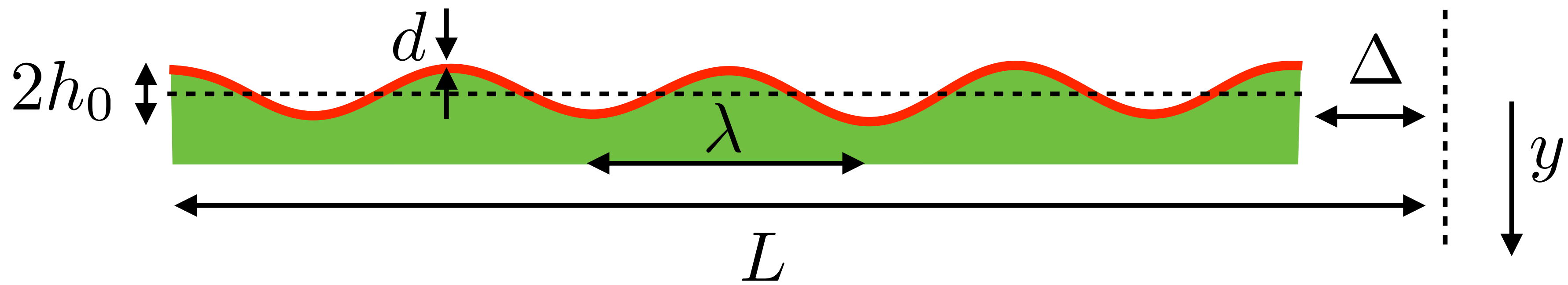


F. Brau et al., Nat. Phys. **7**, 56 (2010)

Compression of stiff thin membranes on soft elastic substrates

assumed profile

$$h(s) = h_0 \cos(2\pi s/\lambda)$$



amplitude of wrinkles

$$h_0 = \frac{\lambda}{\pi}\sqrt{\frac{\Delta}{L}} = \frac{\lambda\sqrt{\epsilon}}{\pi}$$

deformation of the soft substrate decays exponentially away from the surface

$$h(s, y) \approx h_0 \cos(2\pi s/\lambda) e^{-2\pi y/\lambda}$$

bending energy of stiff membrane

$$U_b \sim A \times \kappa \times \frac{1}{R^2} \sim A \times E_m d^3 \times \frac{h_0^2}{\lambda^4} \sim \frac{A E_m d^3 \epsilon}{\lambda^2}$$

deformation energy of soft substrate

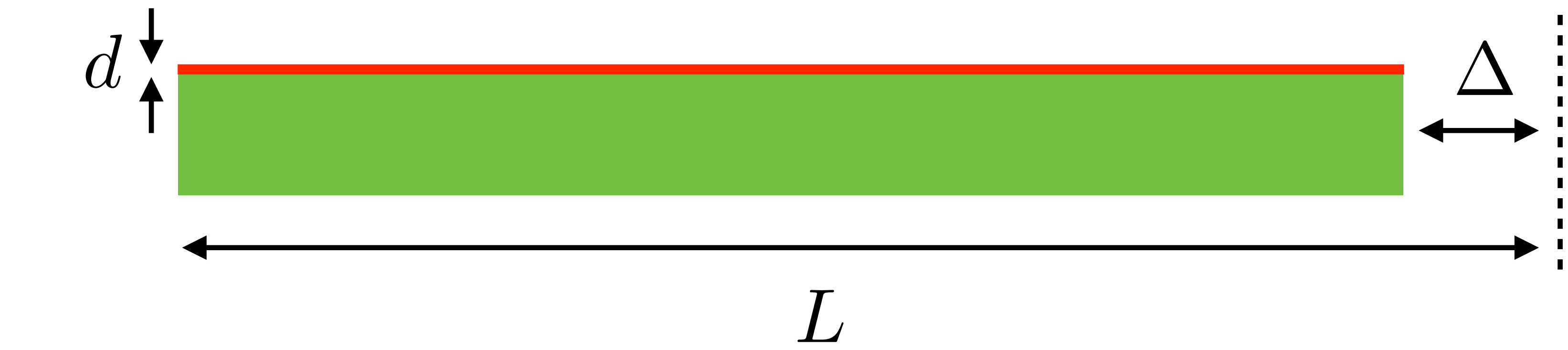
$$U_s \sim V \times E_s \times \epsilon_s^2 \sim A\lambda \times E_s \times \frac{h_0^2}{\lambda^2} \sim A E_s \lambda \epsilon$$

minimize total energy (U_b+U_s) with respect to λ

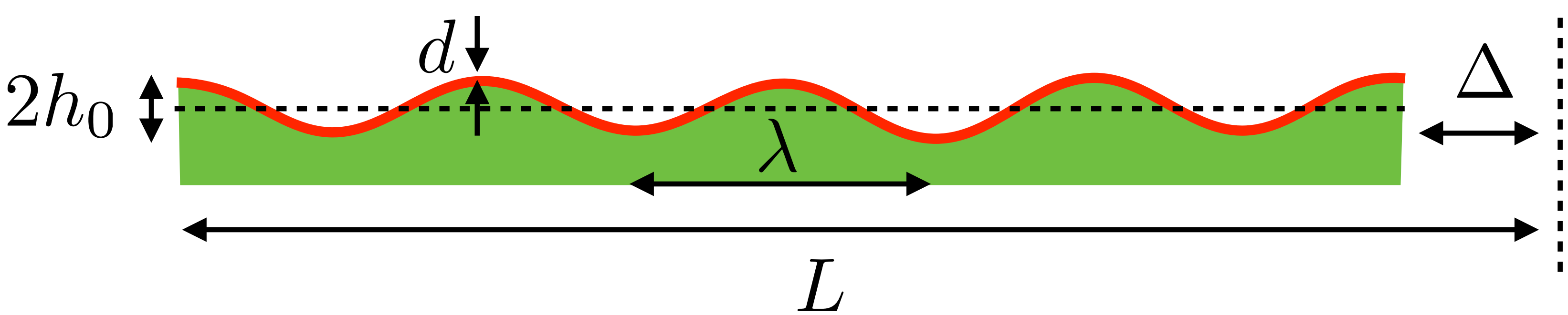
$$\lambda \sim d \left(\frac{E_m}{E_s}\right)^{1/3}$$

$$U_b, U_s \sim A d \epsilon \left(E_s^2 E_m\right)^{1/3}$$

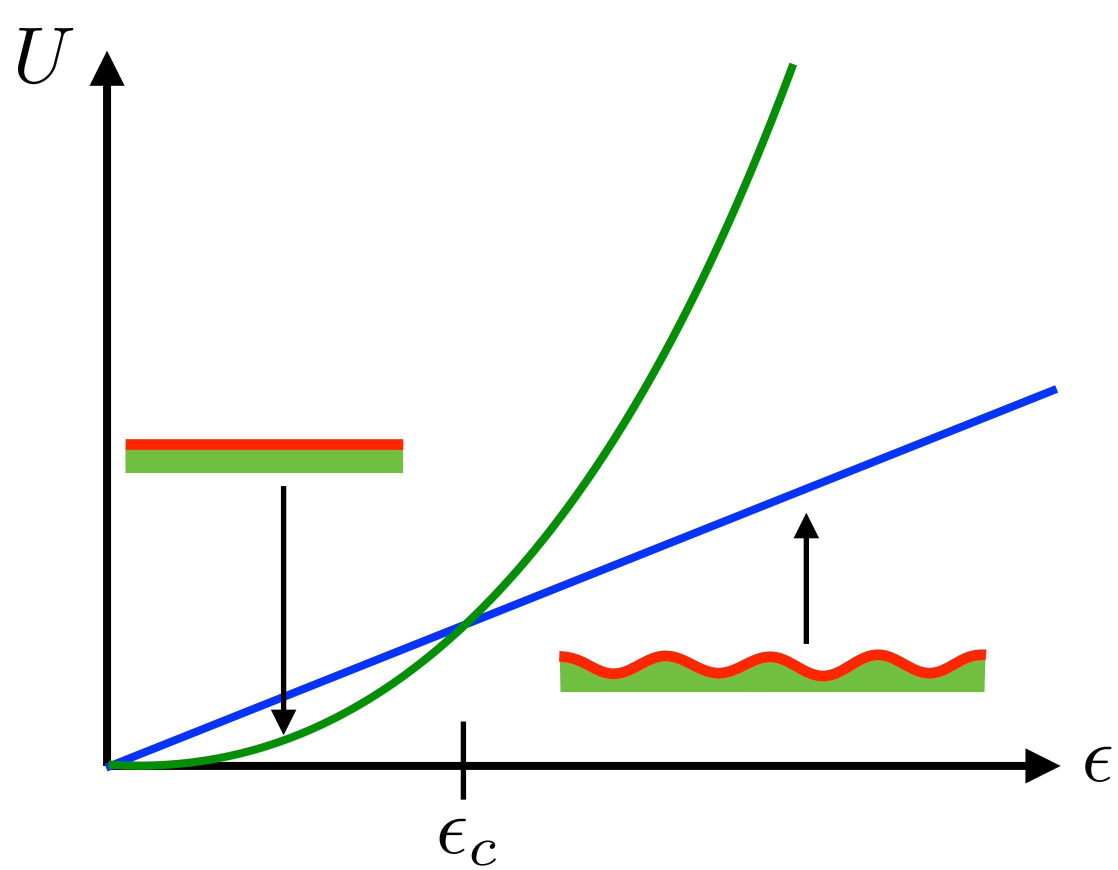
Compression of stiff thin membranes on soft elastic substrates



$$U_c \sim A \times E_m d \times \epsilon^2$$



$$U_b, U_s \sim A d \epsilon \left(E_s^2 E_m\right)^{1/3}$$



wrinkles are stable for large strains

$$\epsilon > \epsilon_c \sim \left(\frac{E_s}{E_m}\right)^{2/3}$$

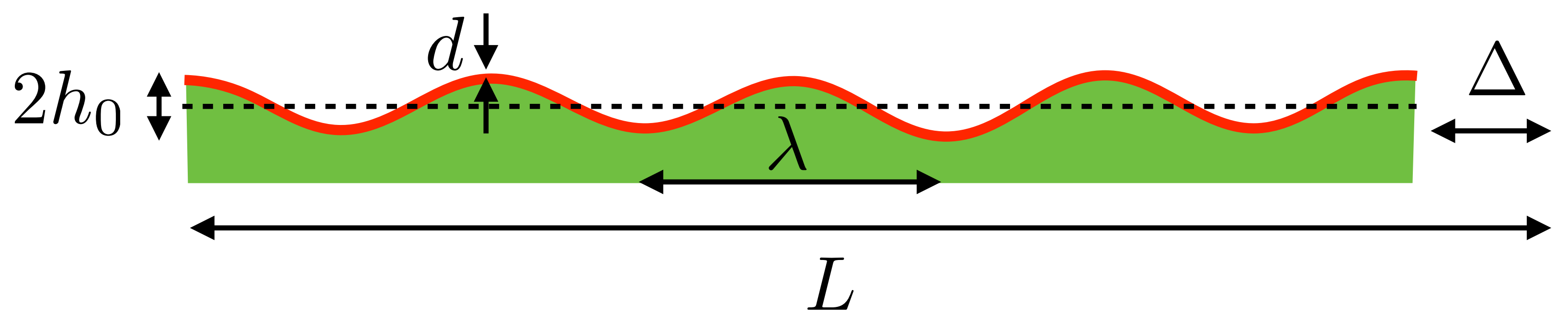
wavelength of wrinkles

$$\lambda \sim d \left(\frac{E_m}{E_s}\right)^{1/3}$$

amplitude of wrinkles at the critical strain

$$h_0^* \sim \lambda \sqrt{\epsilon_c} \sim d$$

Compression of stiff thin membranes on liquid and soft elastic substrates

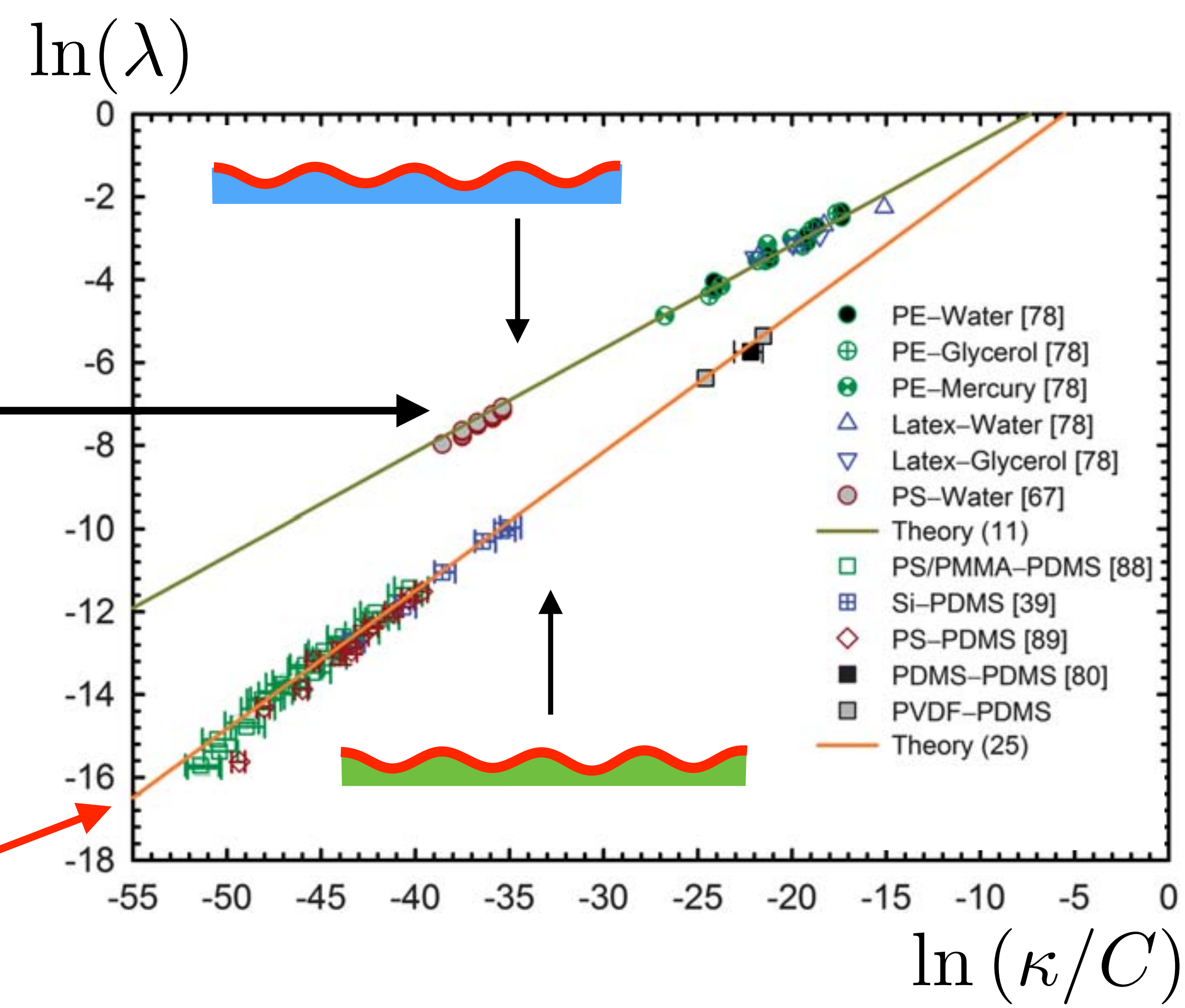


wavelength of wrinkles on liquid substrates

$$\lambda = 2\pi \left(\frac{\kappa}{\rho g}\right)^{1/4}$$

wavelength of wrinkles on soft elastic substrates

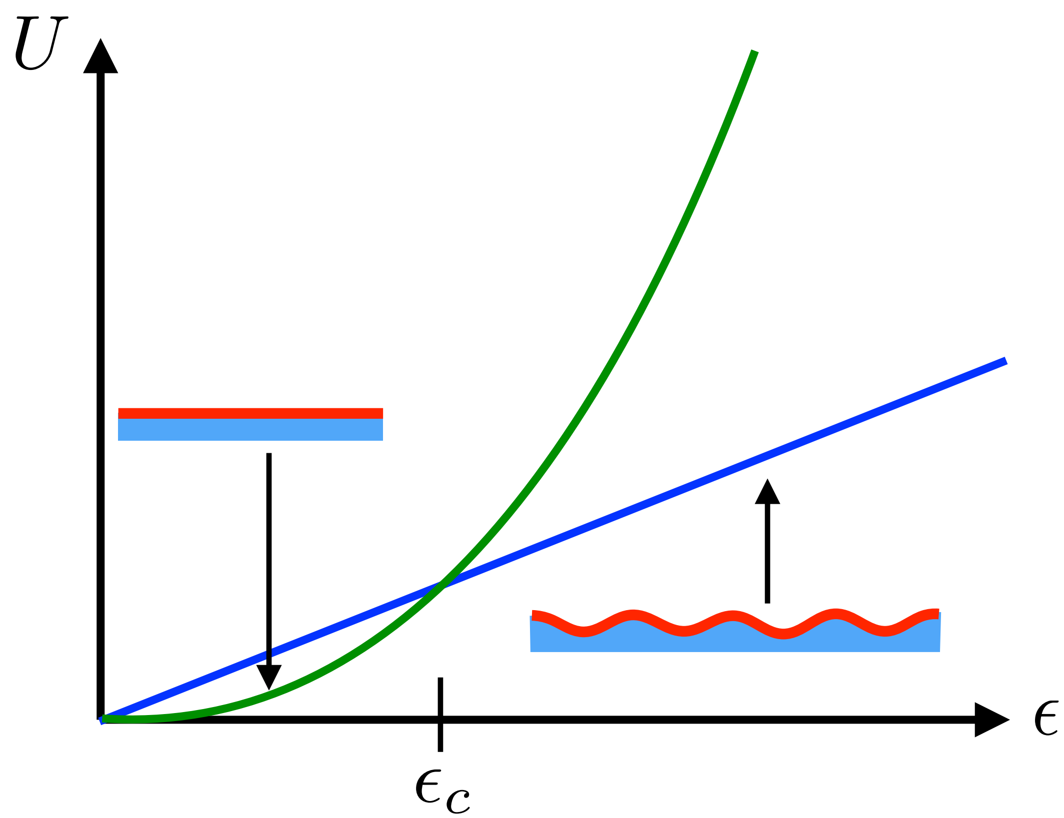
$$\lambda = 2\pi \left(\frac{3\kappa}{E_s}\right)^{1/3}$$



F. Brau et al., Soft Matter **9**, 8177 (2013)

Compression of stiff thin sheets on liquid and soft elastic substrates

liquid substrate



wrinkles are stable above the critical strain

$$\epsilon > \epsilon_c \sim \sqrt{\frac{\rho g d}{E_m}}$$

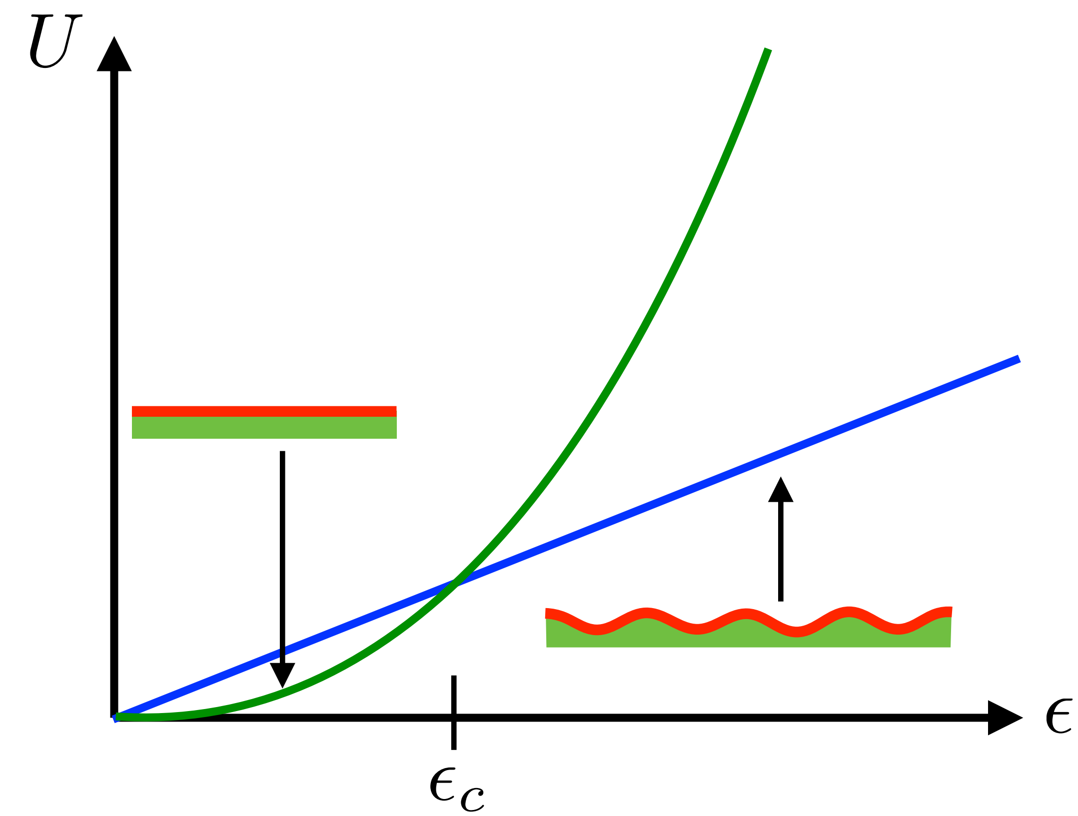
wavelength of wrinkles

$$\lambda \sim \left(\frac{E_m d^3}{\rho g}\right)^{1/4}$$

amplitude of wrinkles

$$h_0 \sim \lambda\sqrt{\epsilon}$$

soft elastic substrate $E_s \ll E_m$



wrinkles are stable for large strains

$$\epsilon > \epsilon_c \sim \left(\frac{E_s}{E_m}\right)^{2/3}$$

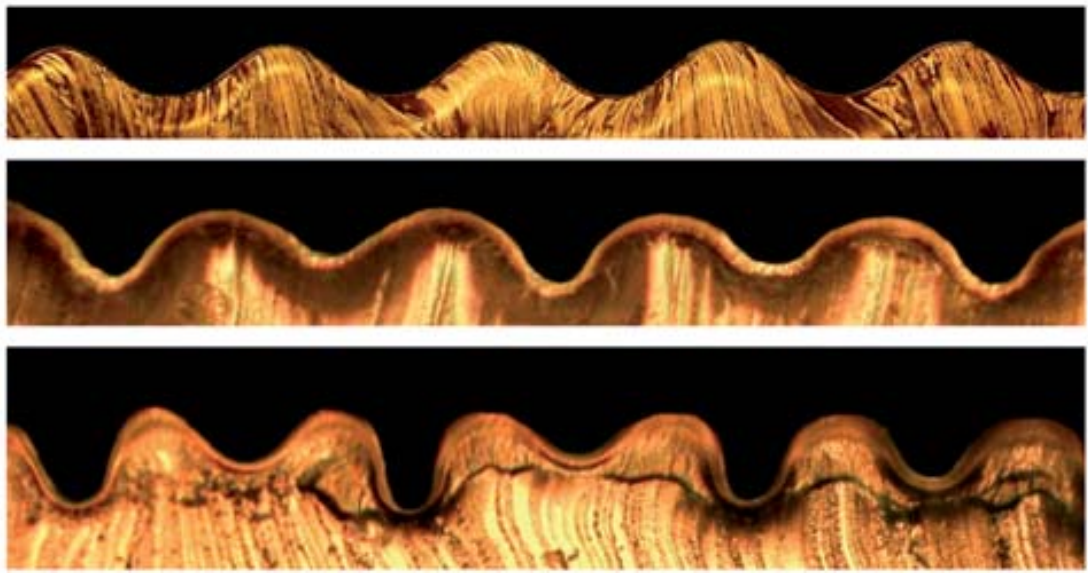
wavelength of wrinkles

$$\lambda \sim d\left(\frac{E_m}{E_s}\right)^{1/3}$$

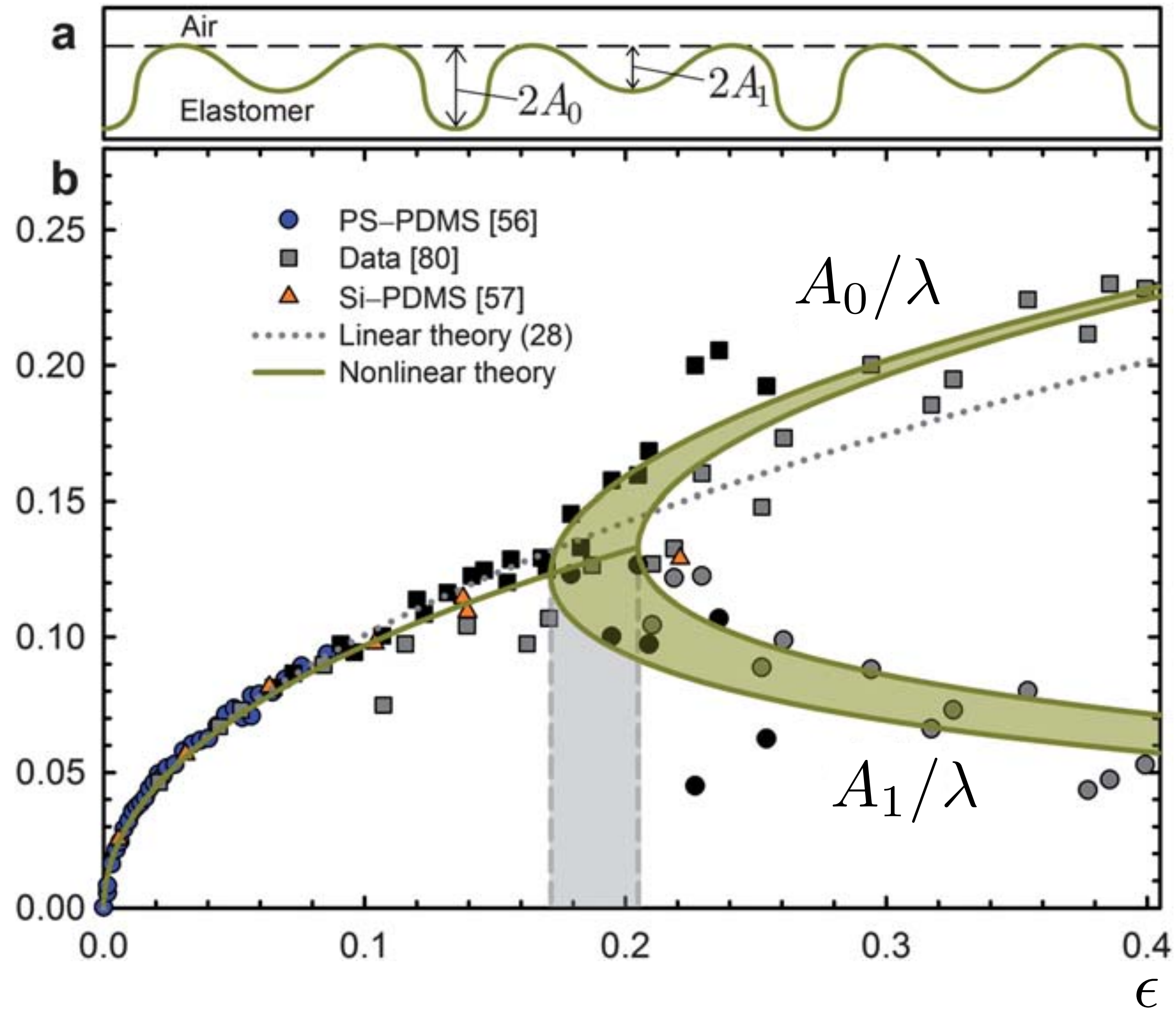
amplitude of wrinkles

$$h_0 \sim \lambda\sqrt{\epsilon}$$

Compression of stiff thin membranes on soft elastic substrates



In order to explain period doubling (quadrupling, ...) one has to take into account the full nonlinear deformation of the soft substrate!

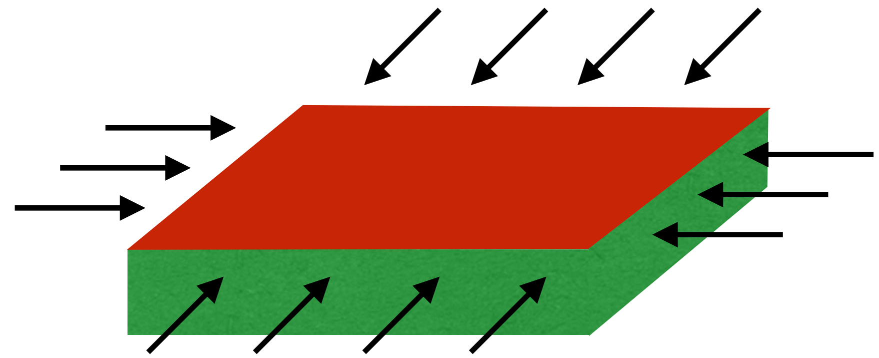


F. Brau et al., Soft Matter **9**, 8177 (2013)

Uniform compression of stiff thin membranes on soft elastic substrates

critical strain

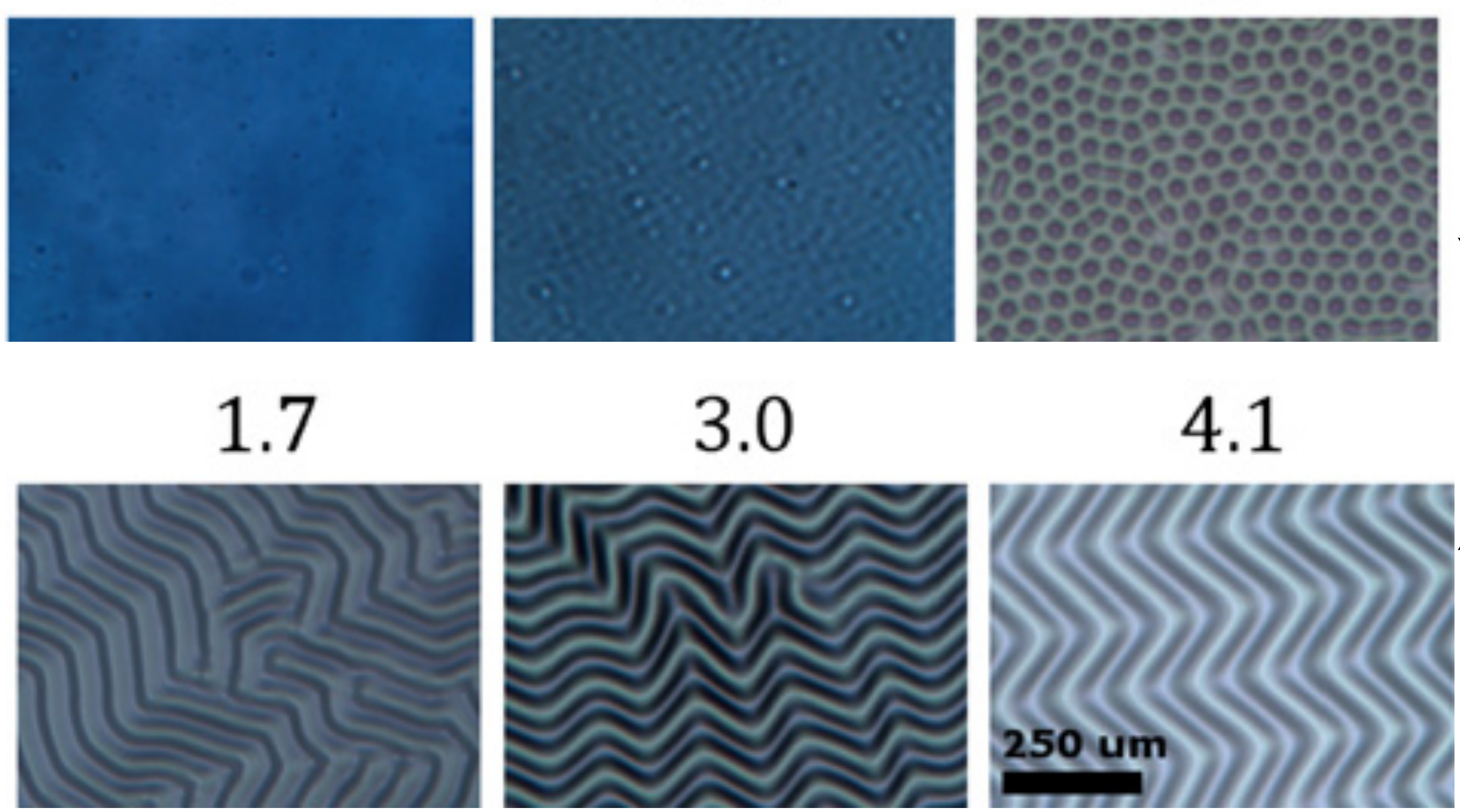
$$\epsilon_c \sim \left(\frac{E_s}{E_m}\right)^{2/3}$$



hexagonal crystal of bumps

ϵ/ϵ_c $\lesssim 1$ $\gtrsim 1$ 1.3

ϵ/ϵ_c 1.7 3.0 4.1



characteristic wavelength

$$\lambda \sim d \left(\frac{E_m}{E_s}\right)^{1/3}$$

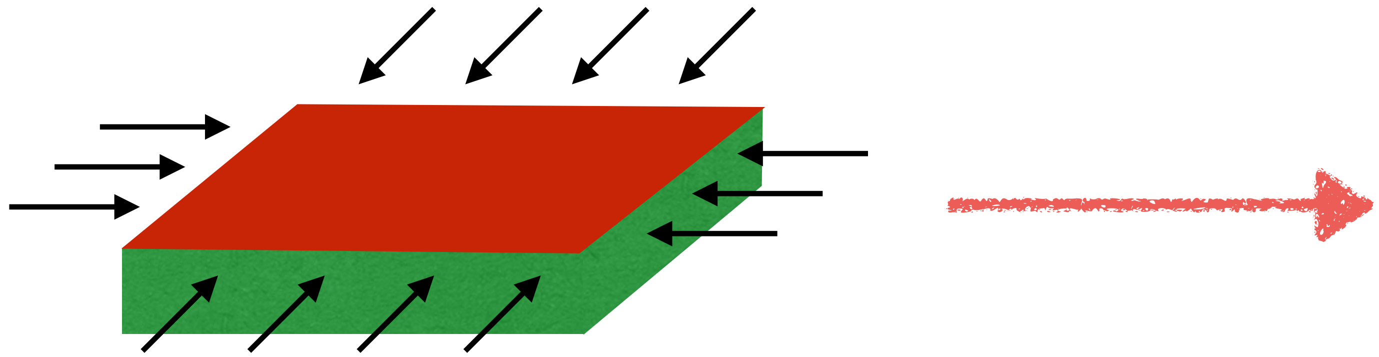
herringbone pattern

S. Cai et al., J. Mech. Phys. Solids **59**, 1094 (2011)

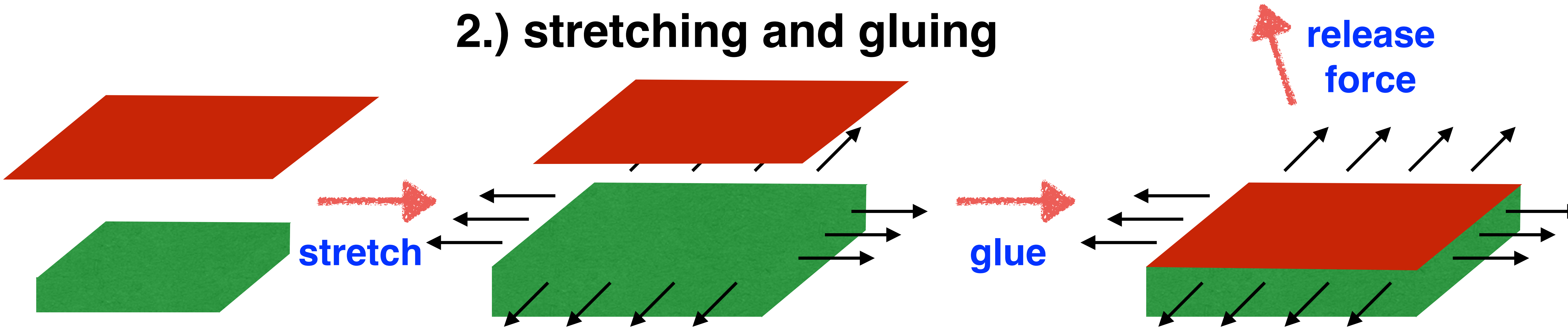
Experimental protocols

All protocols produce equivalent results for small strains!

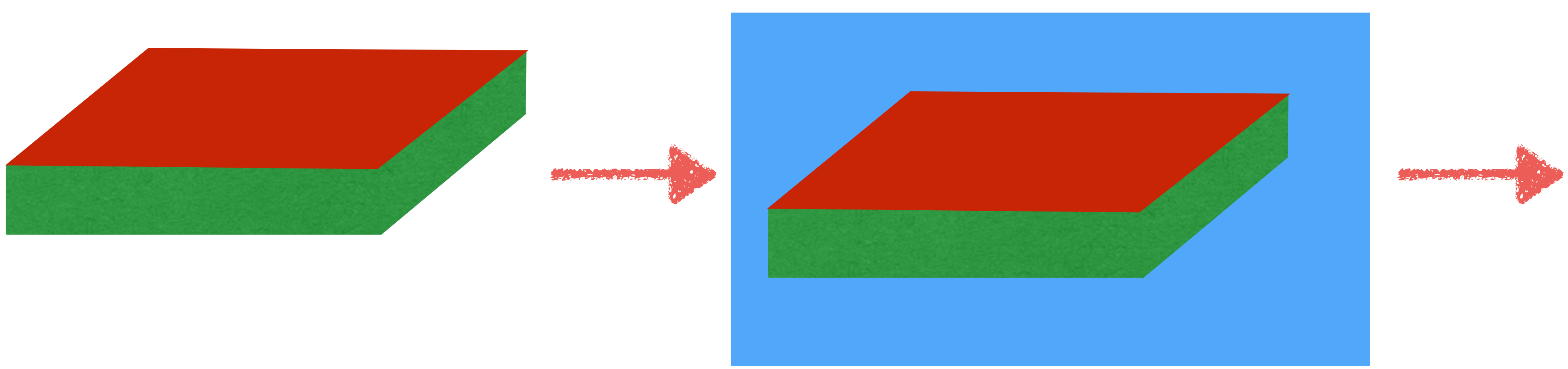
1.) compression



2.) stretching and gluing



3.) differential swelling of gels



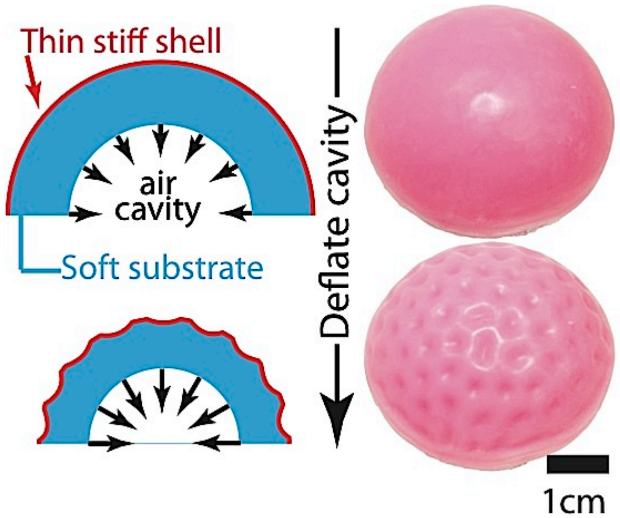
red gel swells more than the green gel

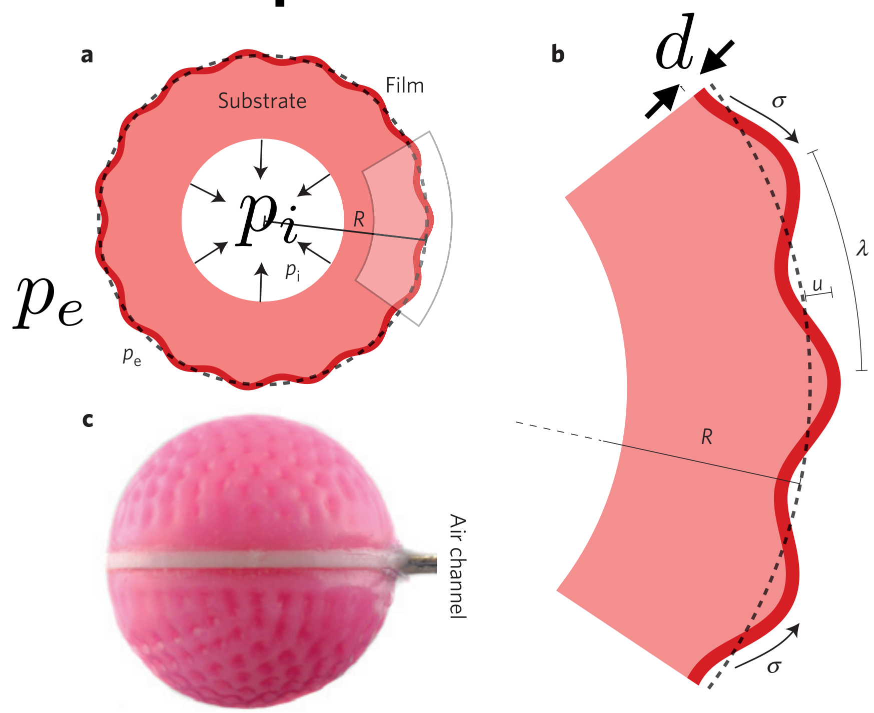
4.) differential growth in biology

5.) differential expansion due to temperature, electric field, etc.

Compression of stiff thin membranes on a spherical soft substrates

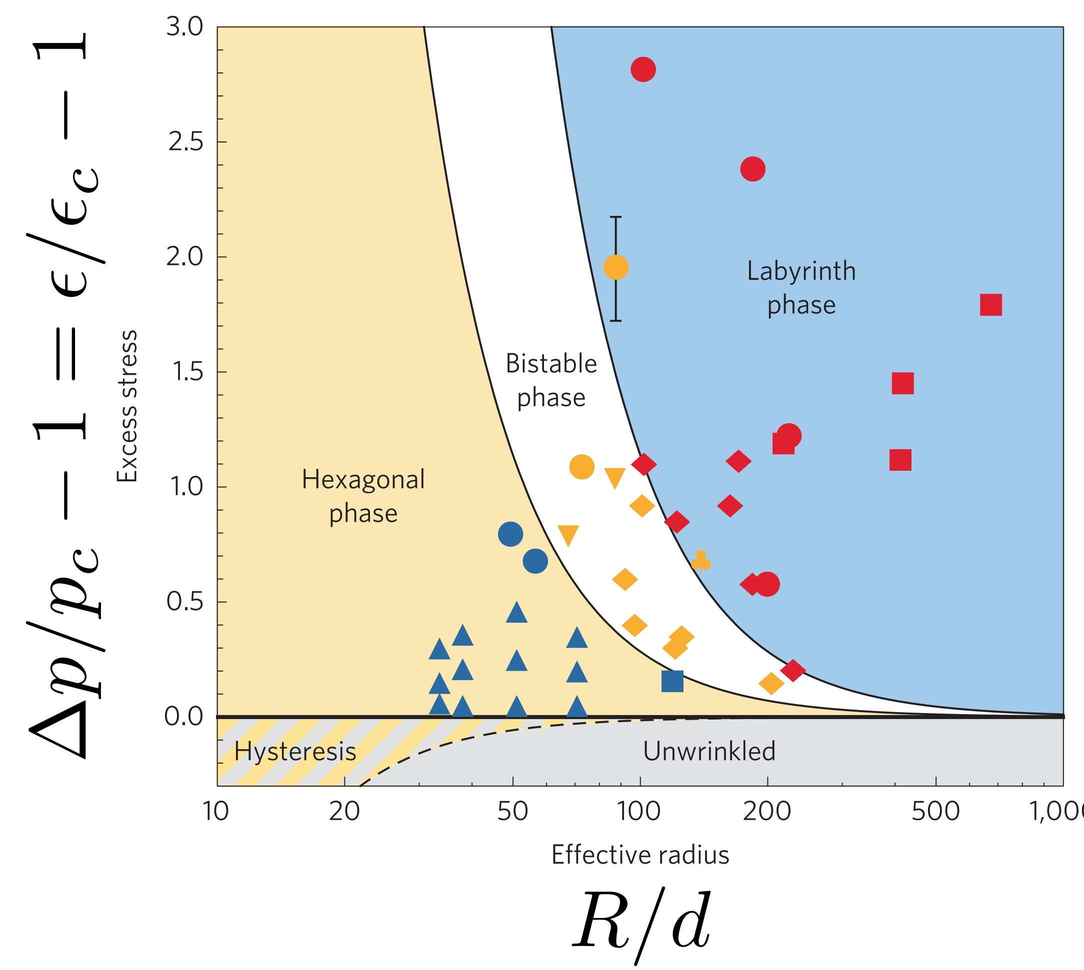
Spherical shells are compressed by reducing internal pressure



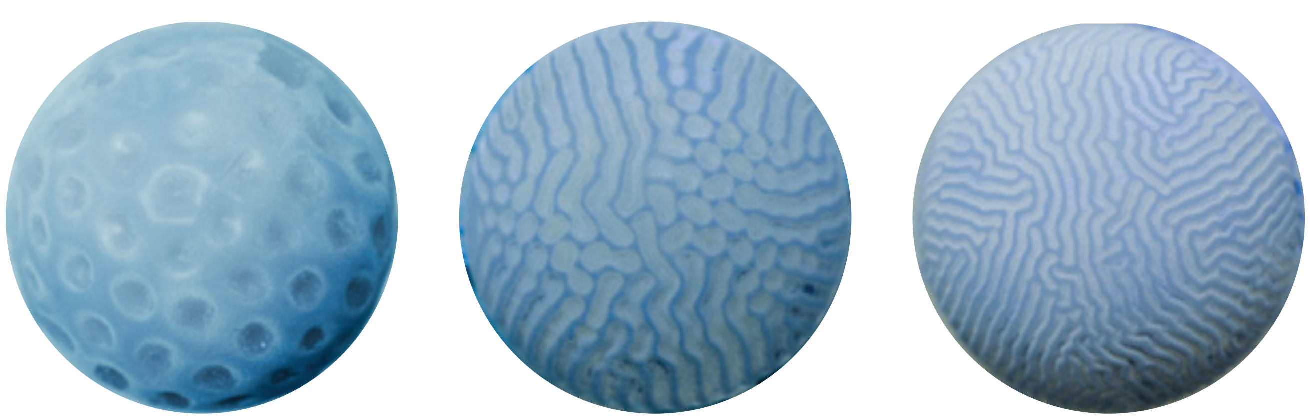


$R = 20\text{mm}$

Phase diagram of dimples/wrinkles



hexagonal phase **bistable phase** **labyrinth phase**



R/d

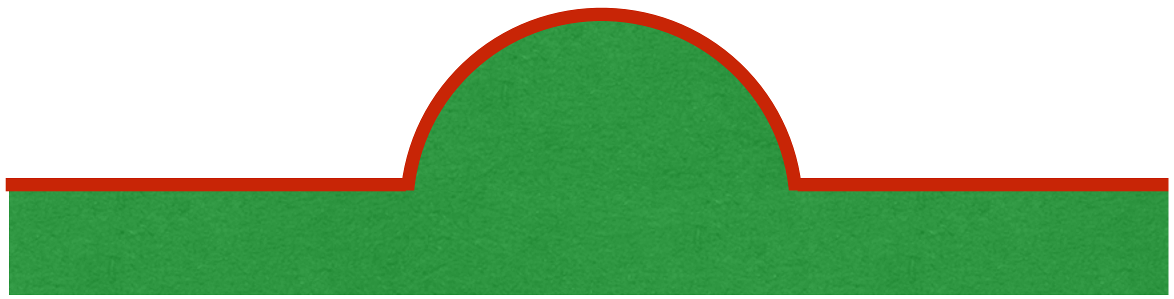
characteristic wavelength is almost independent of radius R

$$\lambda \sim d \left(\frac{E_m}{E_s} \right)^{1/3}$$

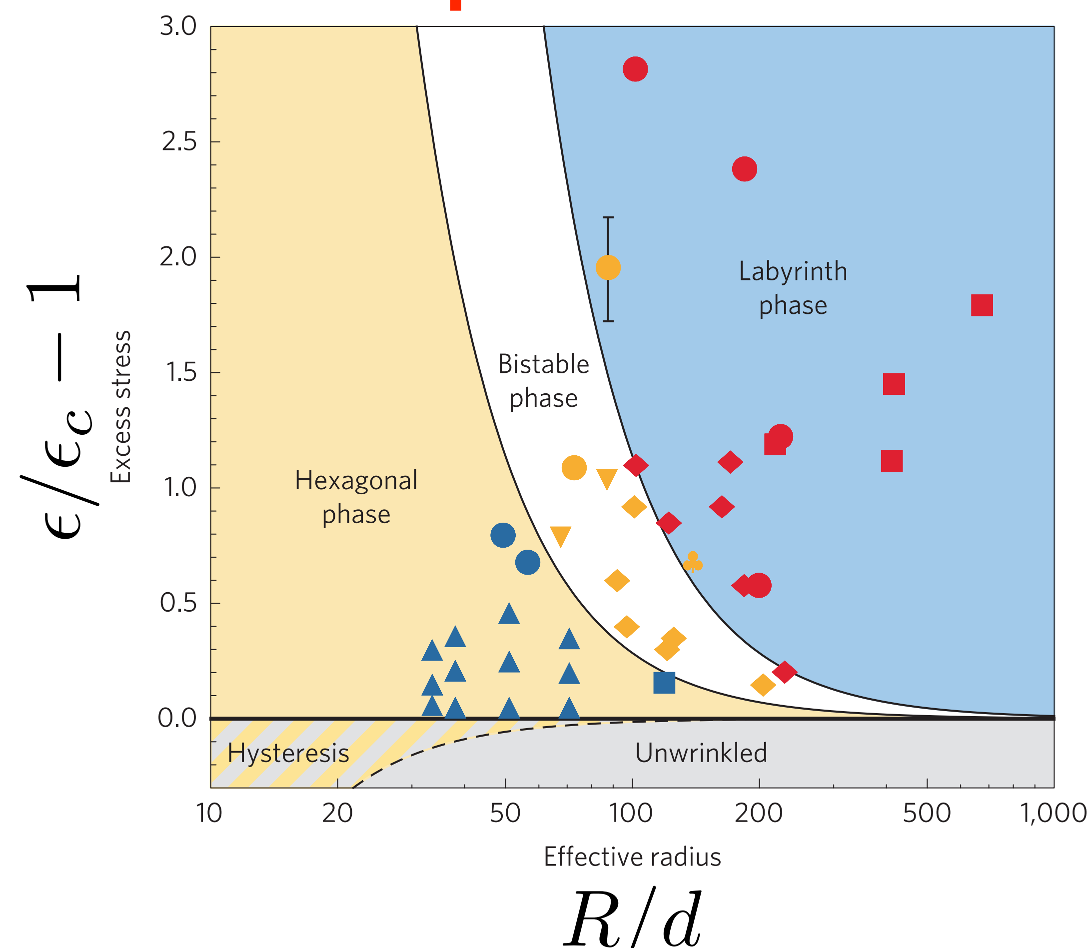
N. Stoop et al., Nat. Materials **14**, 337 (2015)

Compression of stiff thin membranes on a spherical soft substrates

Swelling of gels (red gel swells more than the green one)

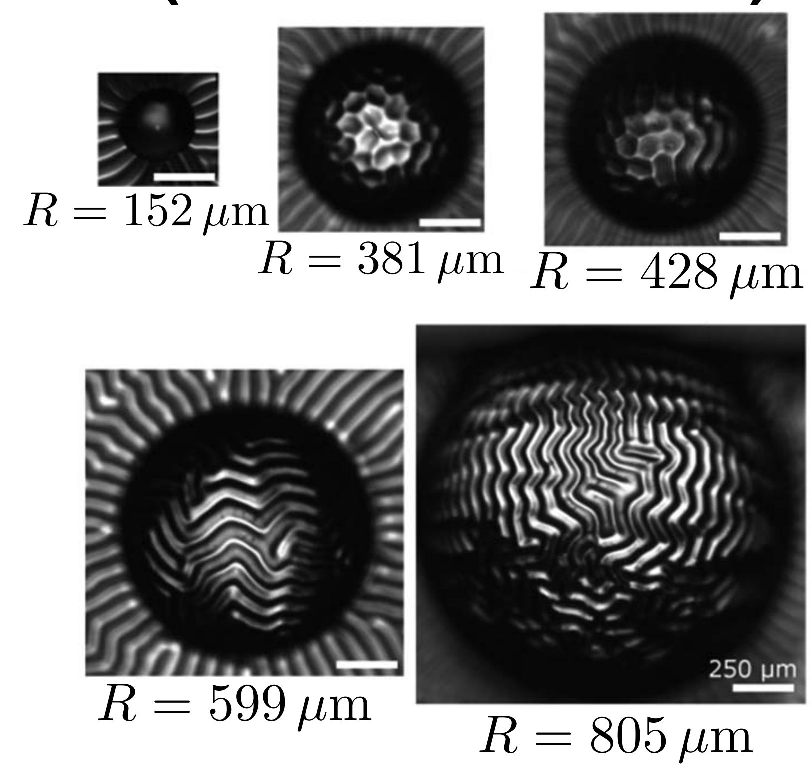


Phase diagram of dimples/wrinkles



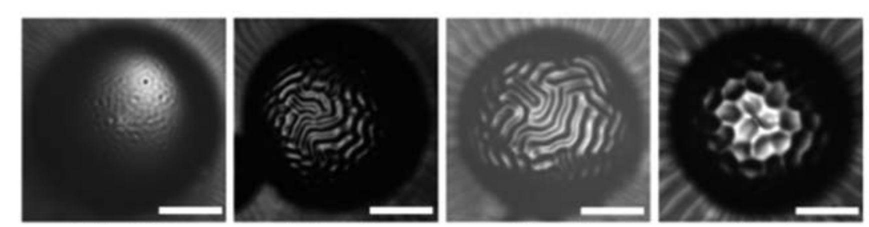
D. Breid and A.J. Crosby, Soft Matter **9**, 3624 (2013)

Modifying radius *R* (fixed thickness *d*)



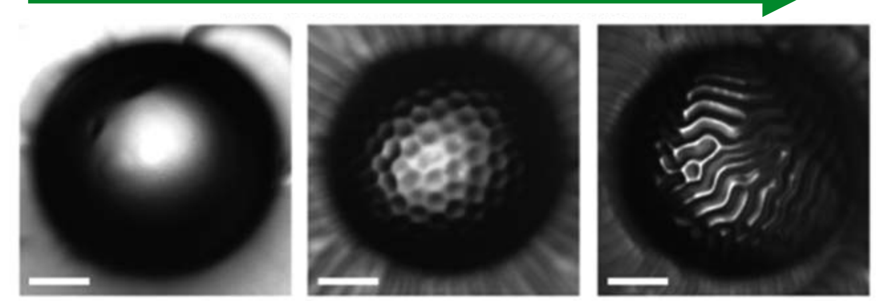
Modifying membrane thickness *d*

$R = 381\mu m$

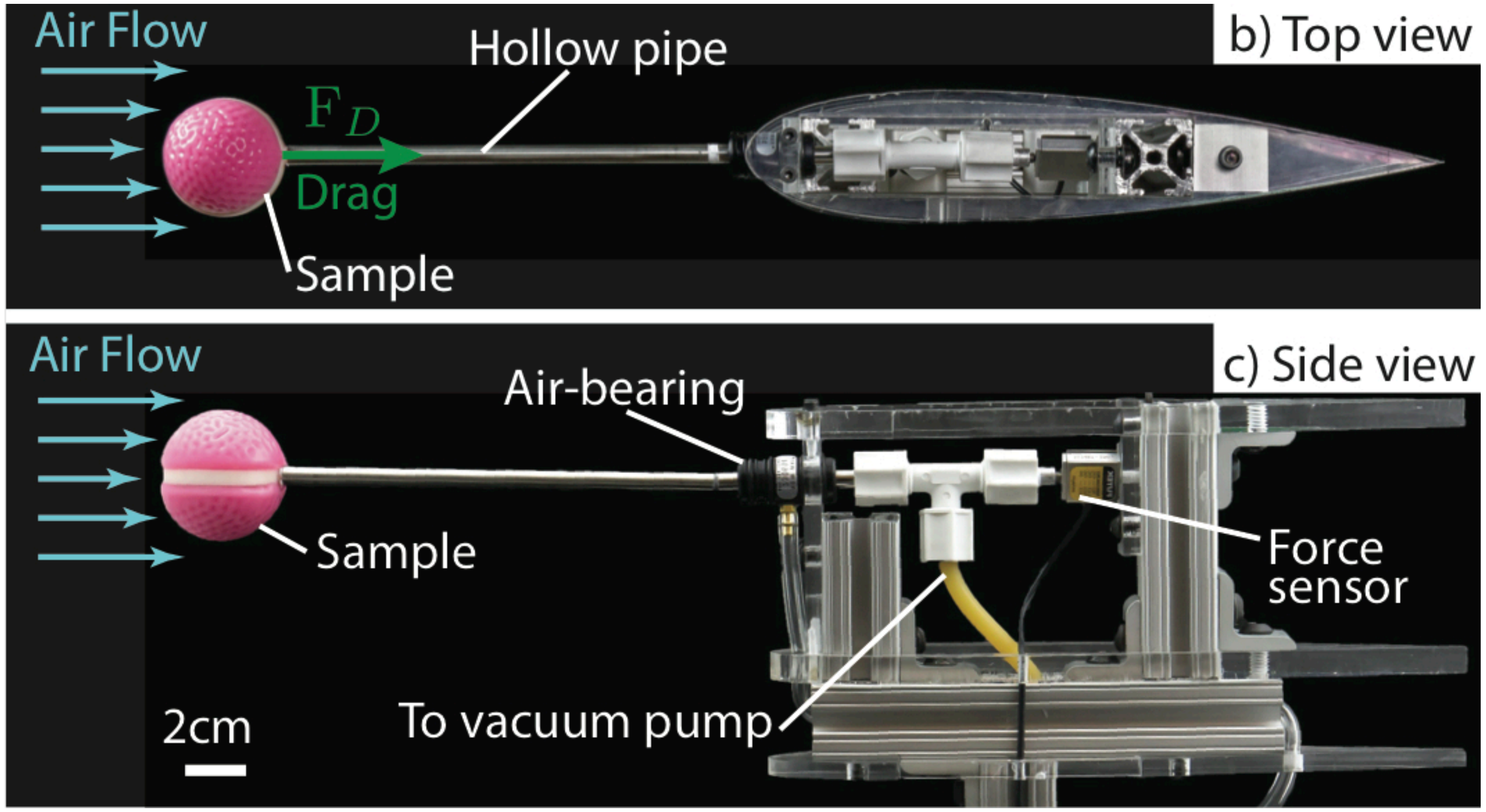


Modifying swelling strain ϵ

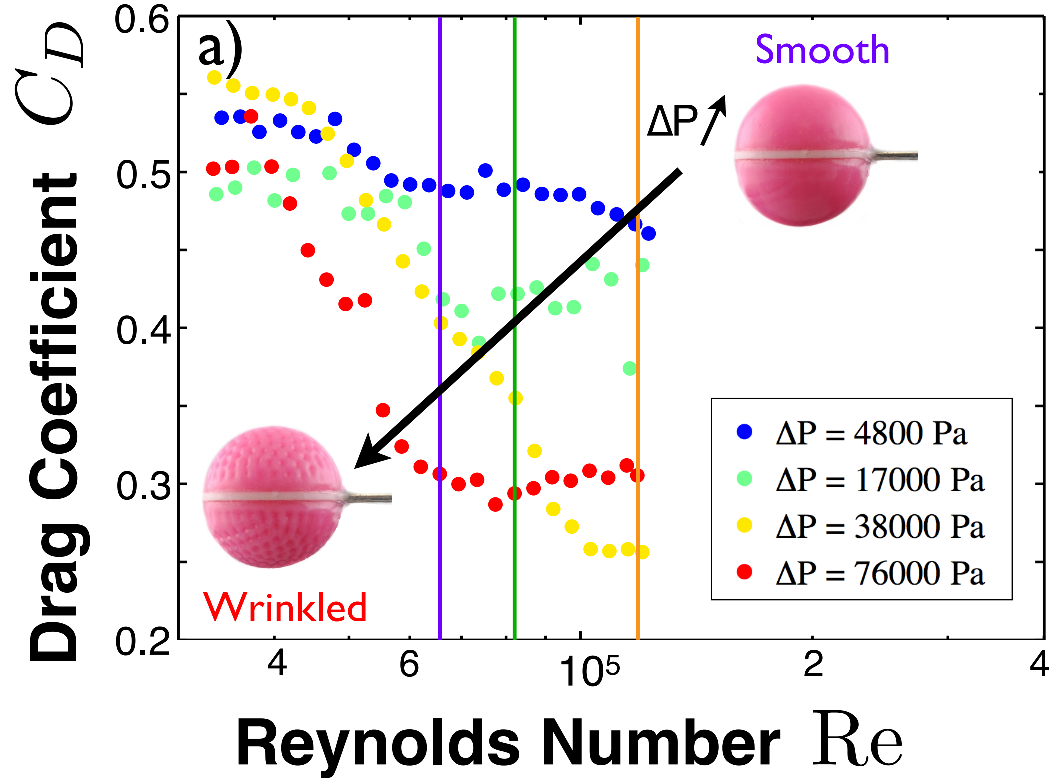
$R = 522\mu m$



Tuning drag coefficient via wrinkling



Wrinkling reduces drag coefficient



Drag Force

$$F_d = \frac{1}{2} C_D \rho u^2 A$$

ρ **air density**

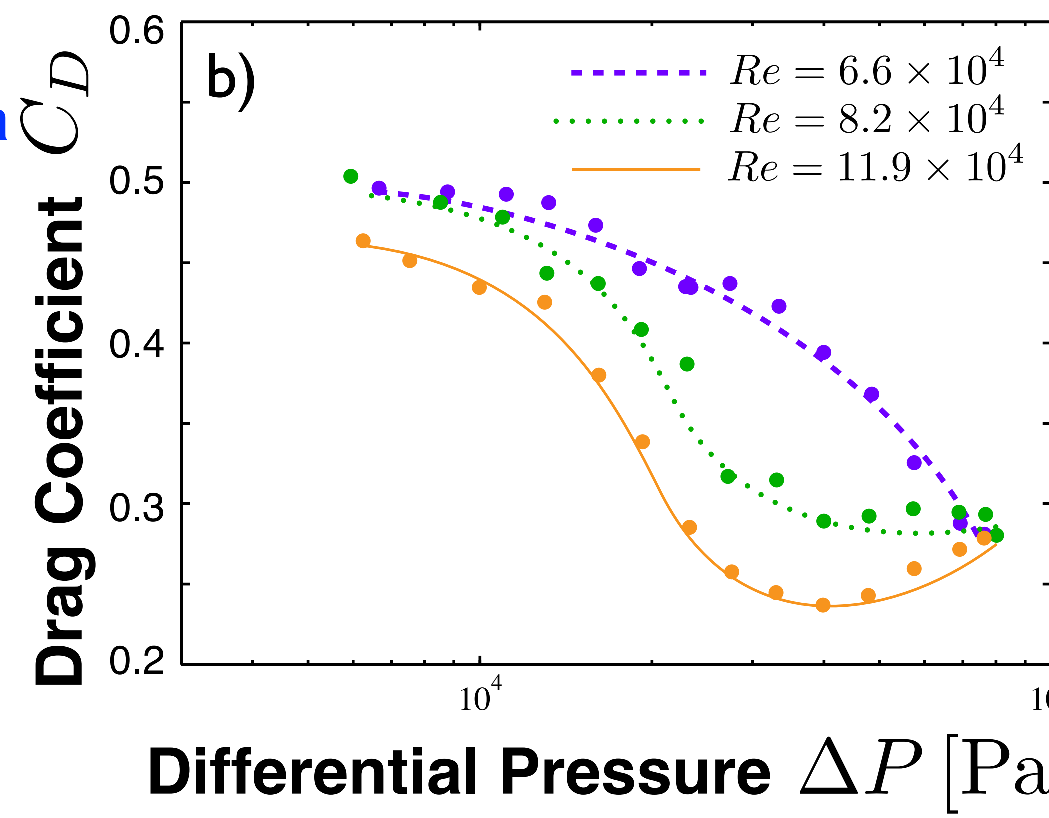
u **air flow speed**

R **sphere radius**

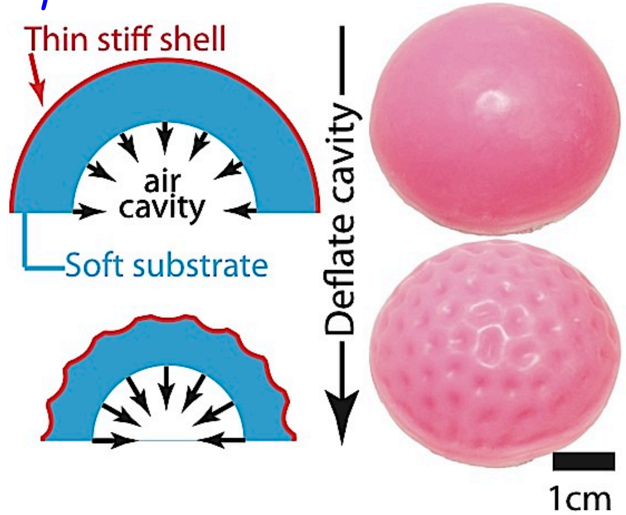
$A = \pi R^2$ **sphere cross-section area**

μ **air viscosity**

$$\mathrm{Re} = \frac{\rho u (2R)}{\mu} \gg 1$$ **Reynolds Number**



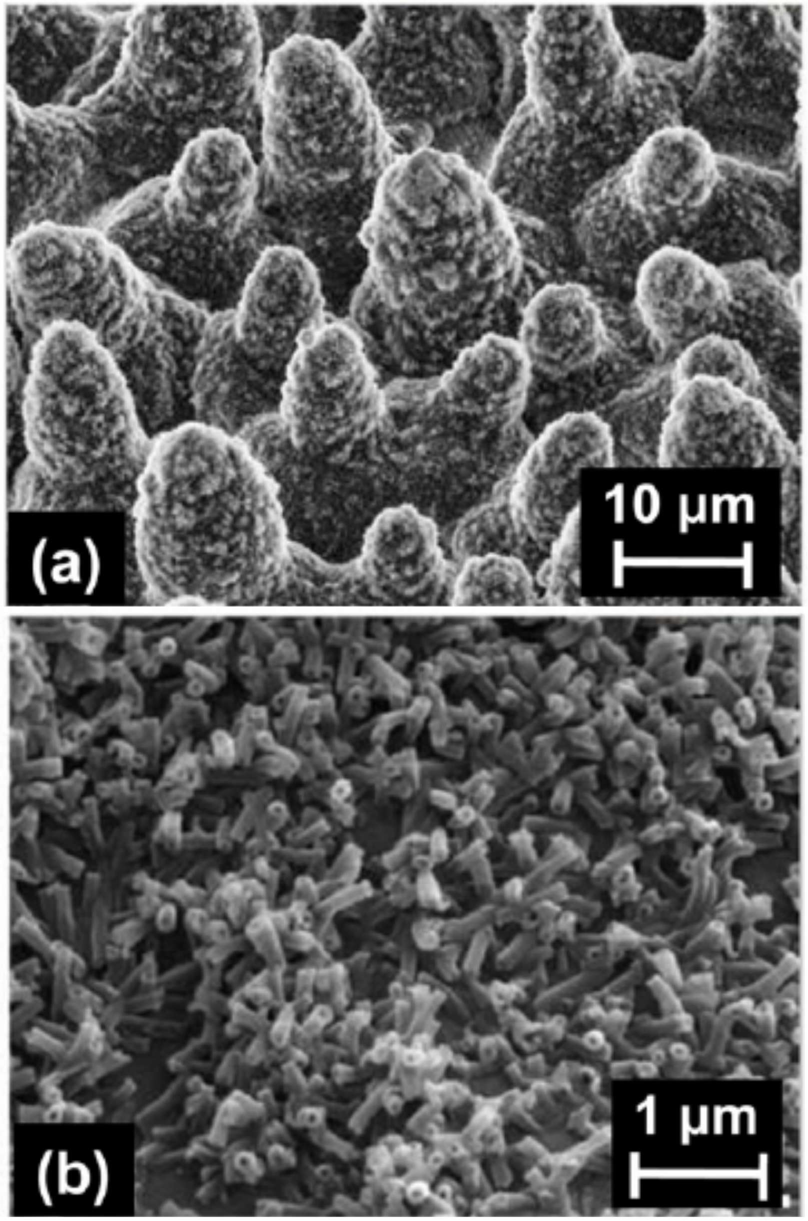
Depth of wrinkling is controlled via the reduction of internal pressure ΔP.



D. Terwagne et al., Adv. Mater. **26**, 6608 (2014)

Self-cleaning property of lotus leaves

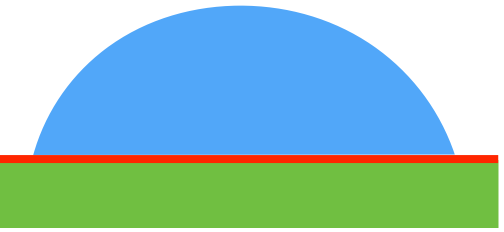
Lotus leaves repel water (hydrophobicity) due to the rough periodic microstructure

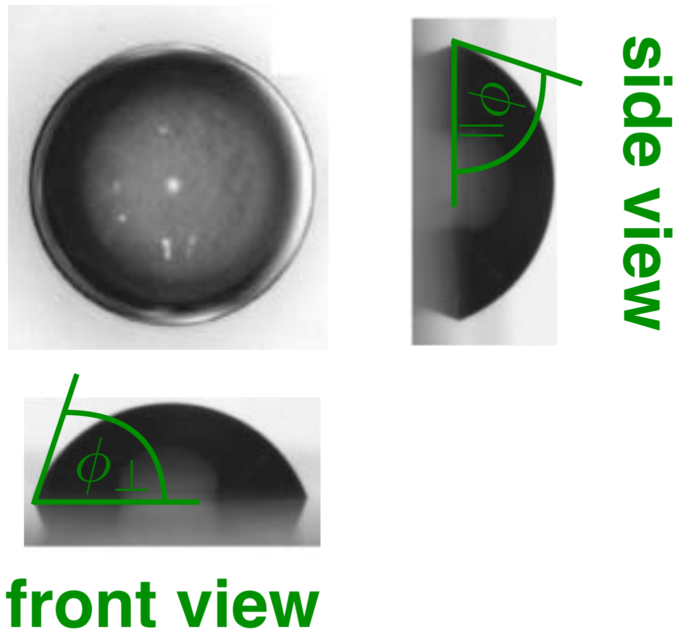


M. N. Costa et al.,
Nanotechnology **25**, 094006 (2014)

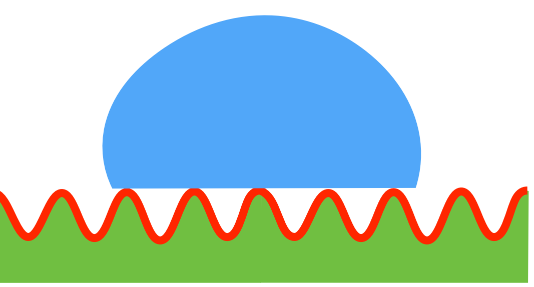
Tuning wetting angle via wrinkling

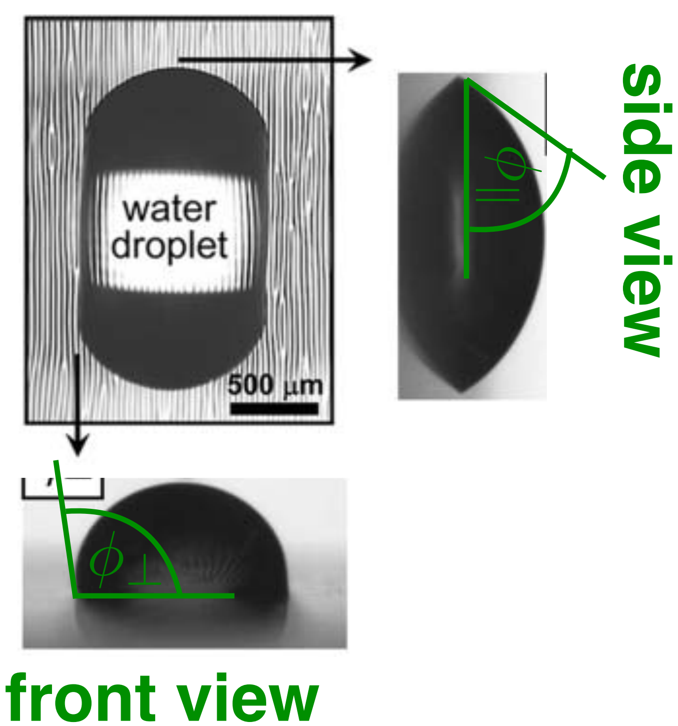
Water droplet on a flat surface

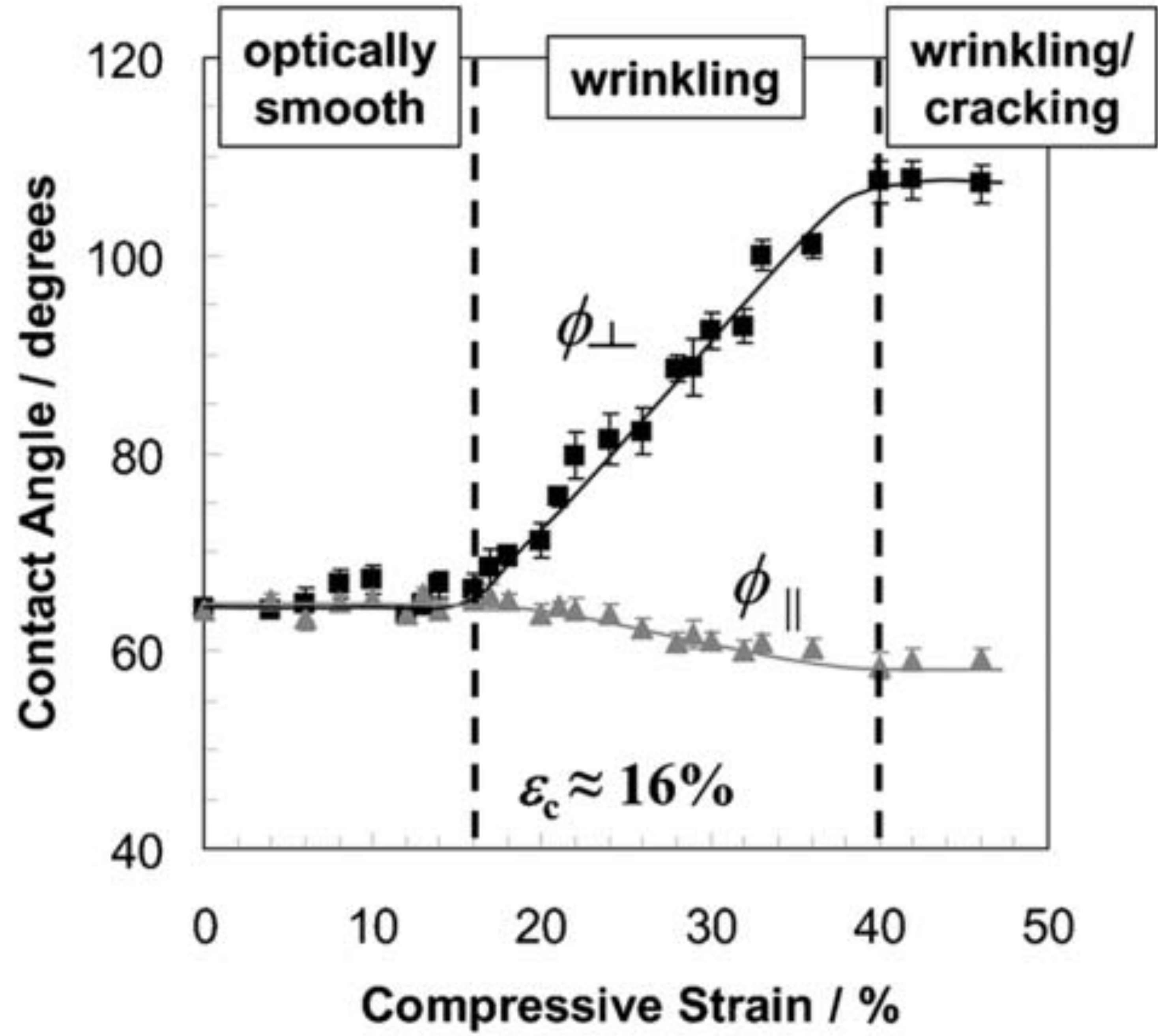




Water droplet on a wrinkled surface (wrinkling increases contact angle)



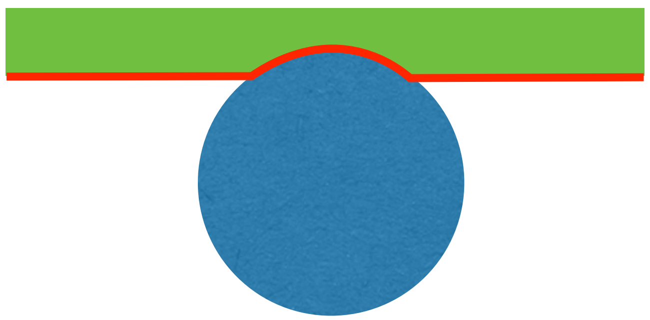




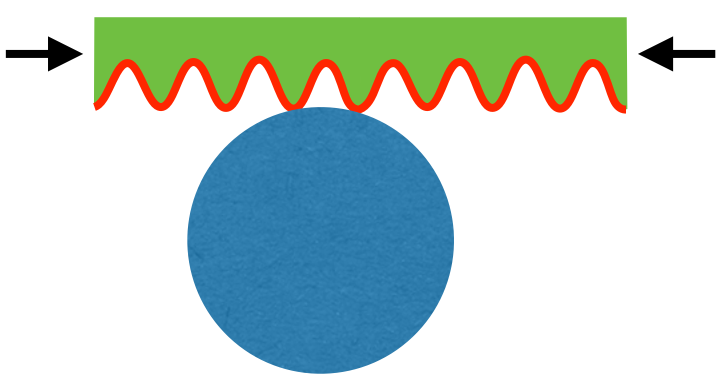
J. Y. Chung et al., Soft Matter **3**, 1163 (2007)

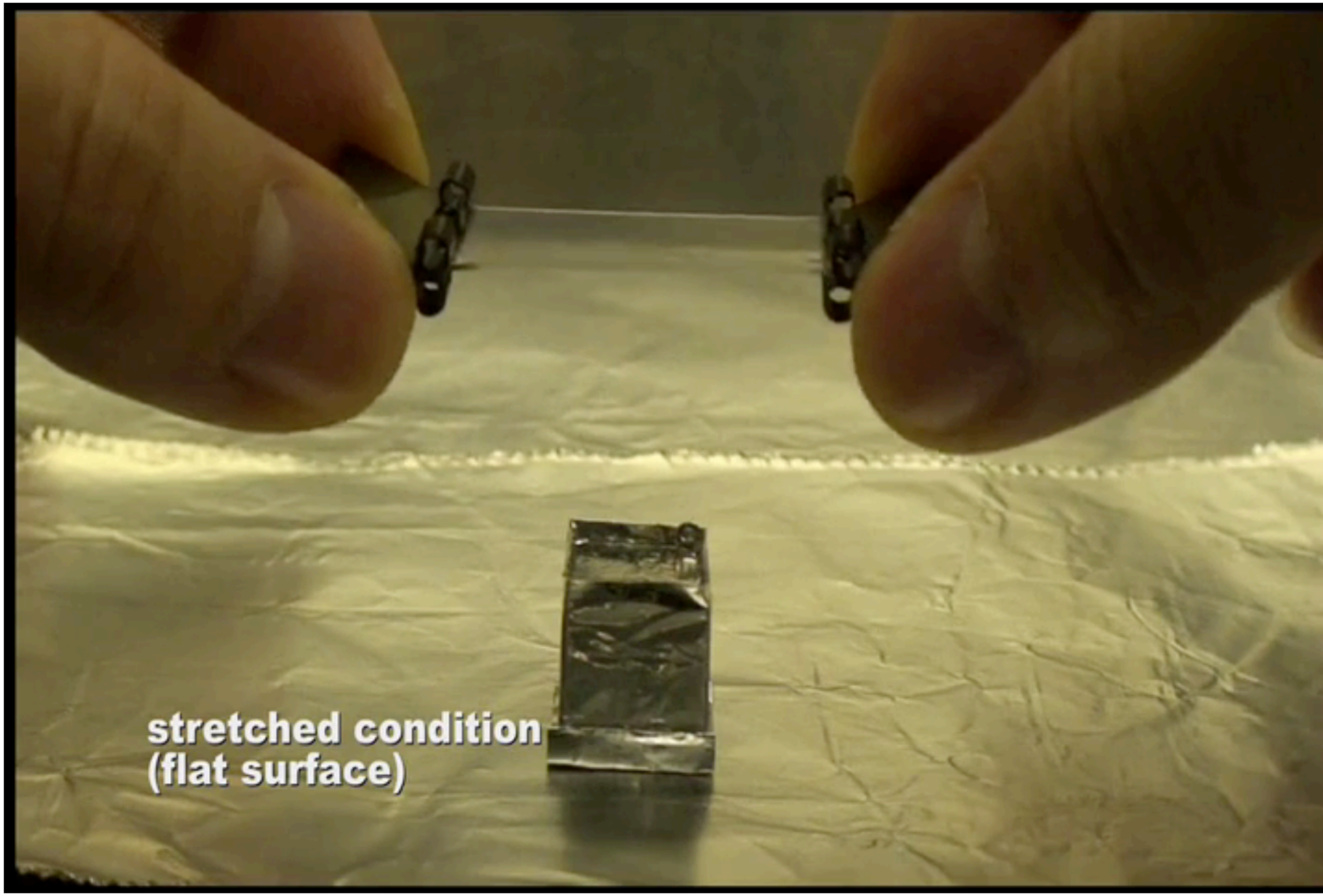
Tuning adhesion via wrinkling

Flat compliant surface has enhanced adhesion (larger contact area)



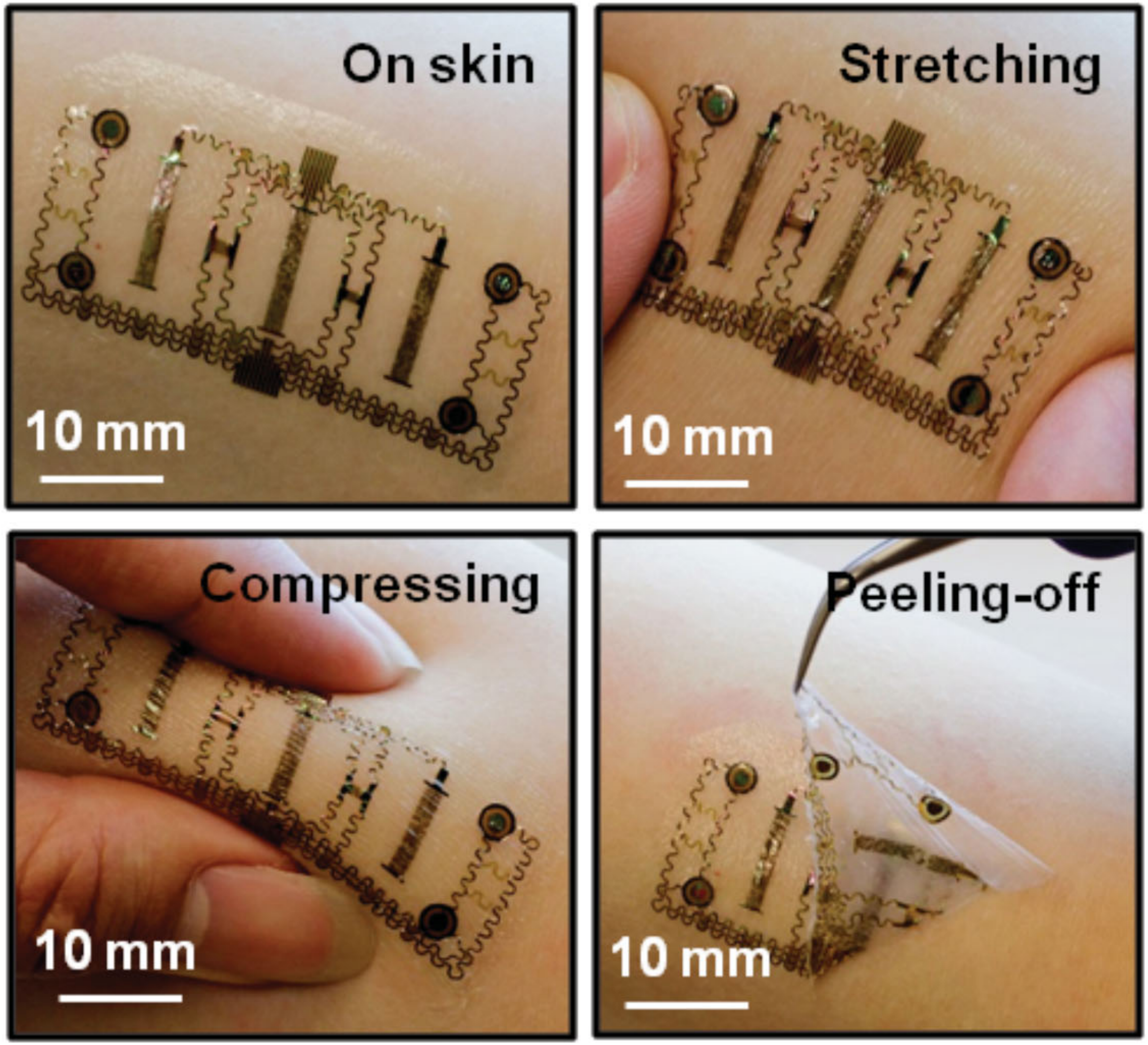
Wrinkling reduces adhesion (smaller contact area)





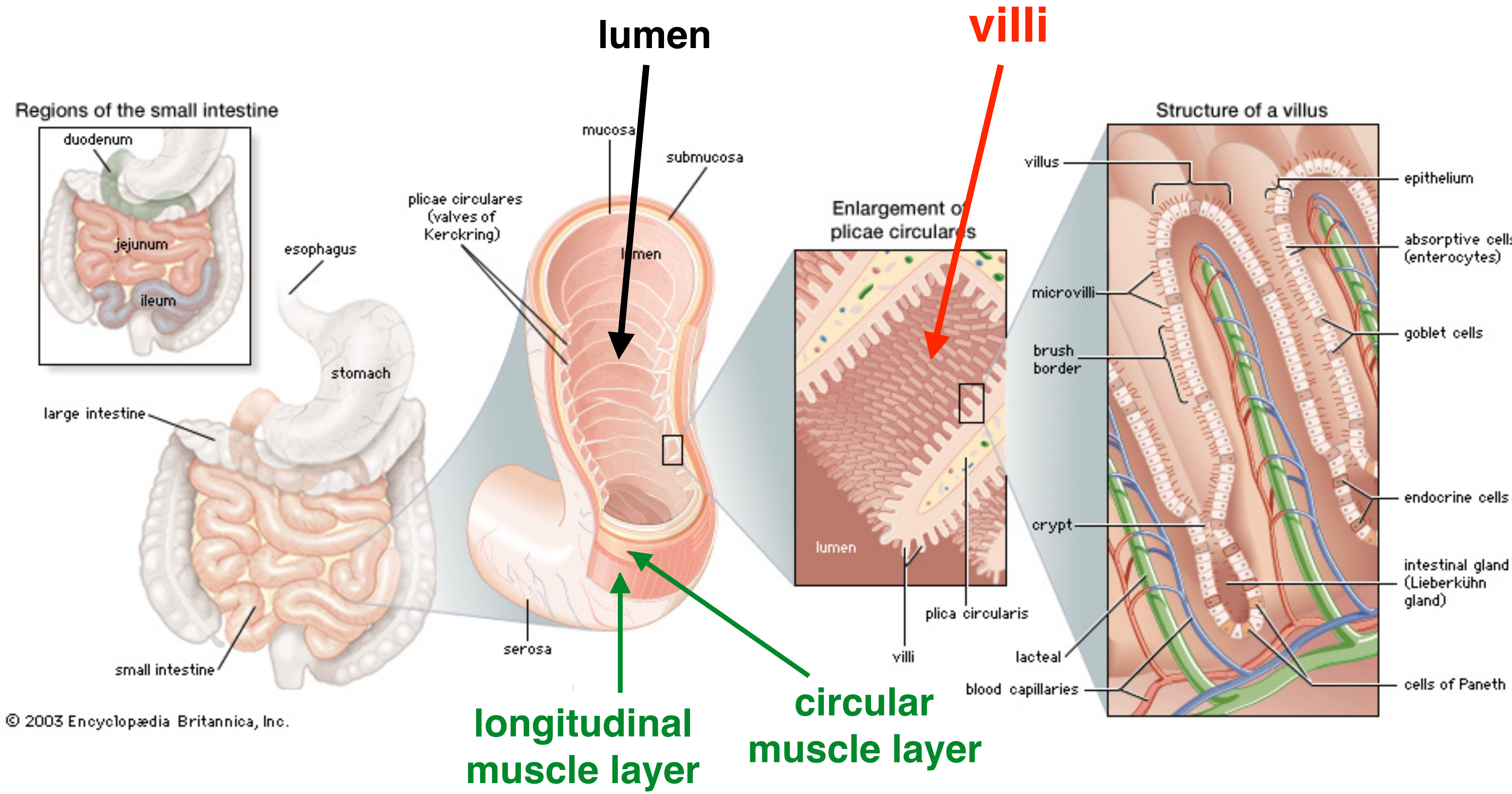
P.-C. Lin et al., Soft Matter **4**, 1830 (2008)

Wrinkled structures can be used for flexible electronics



B. Xu et al., Adv. Mater. **28**, 4462 (2016)

How are villi formed in guts?



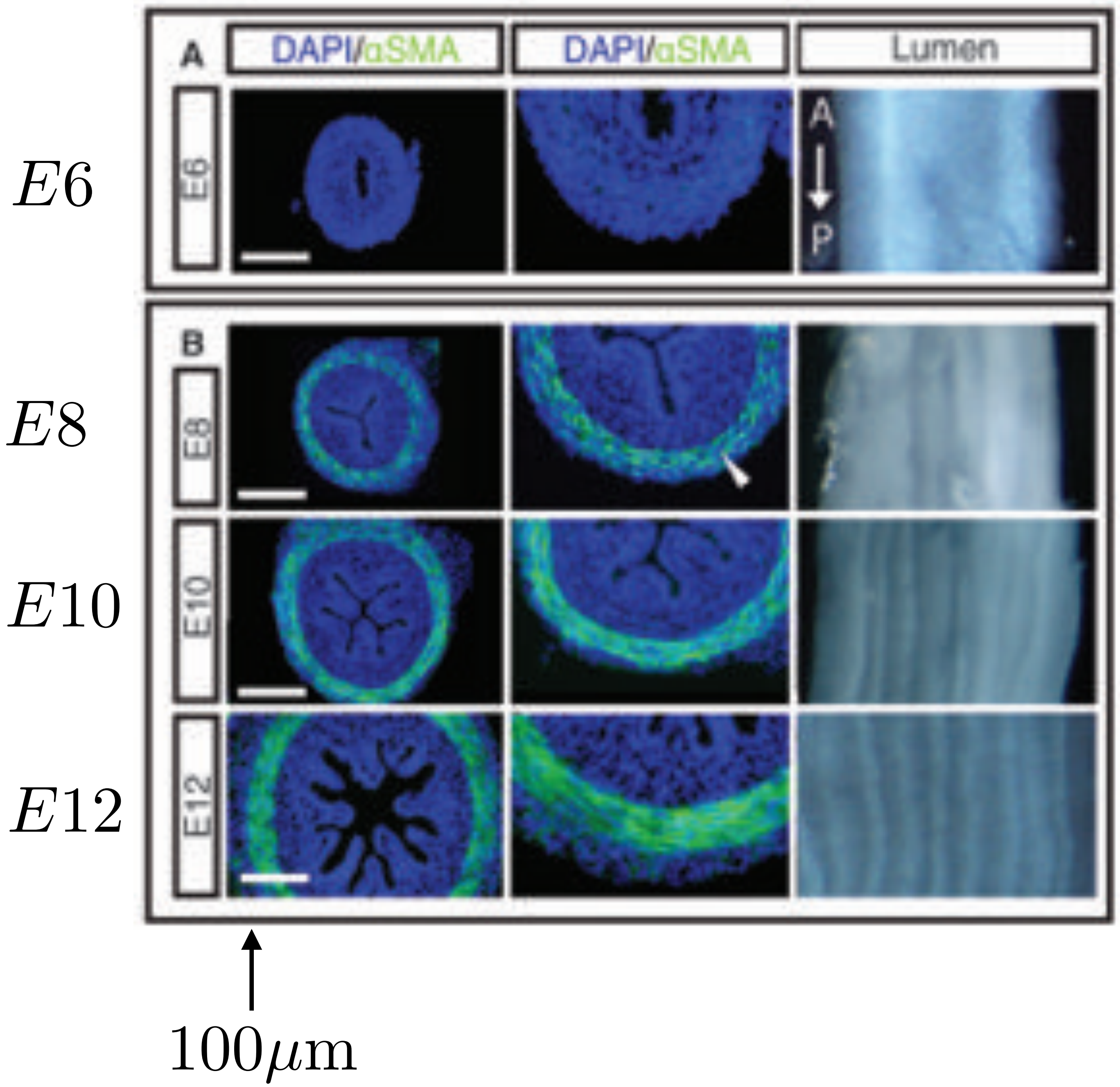
Villi increase internal surface area of intestine for faster absorption of digested nutrients.

Lumen patterns in chick embryo

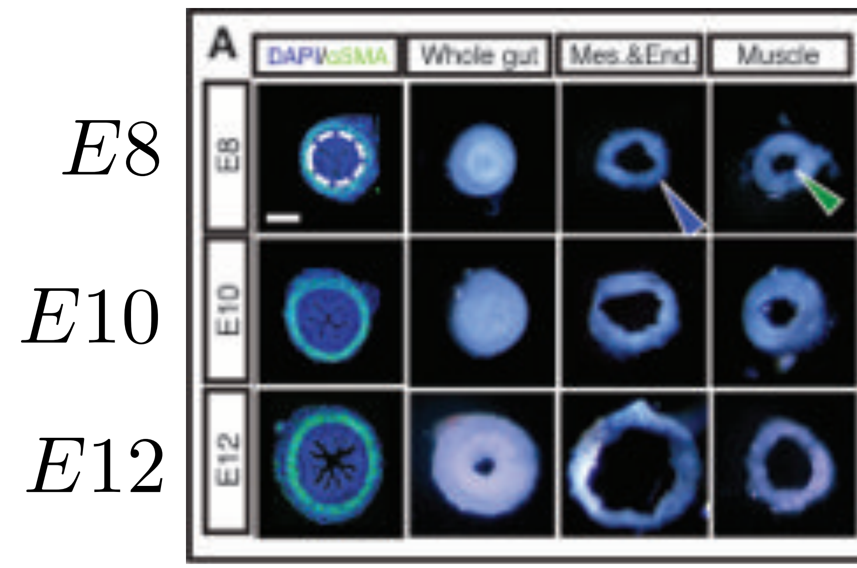
DAPI marks cell nuclei

aSMA marks smooth muscle actin

E…: age of chick embryo in days

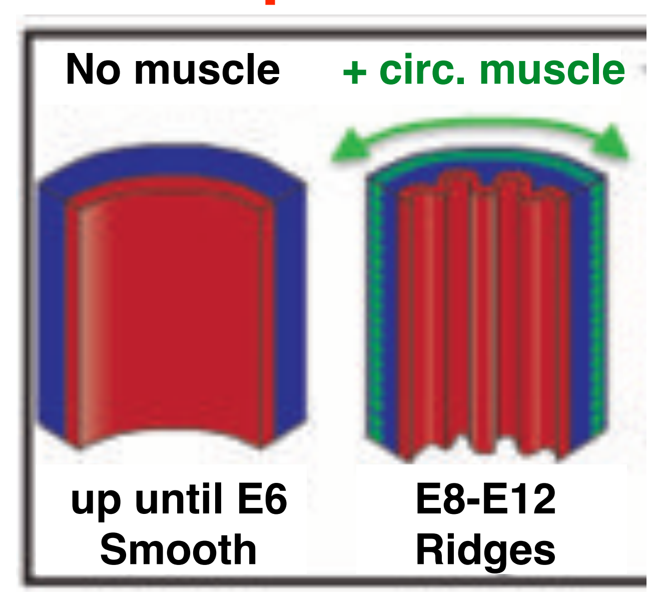


Stiff muscles grow slower than softer mesenchyme and endoderm layers



radial compression due to differential growth produces striped wrinkles

endoderm
mesenchyme
muscle



A. Shyer et al., Science **342**, 212 (2013)

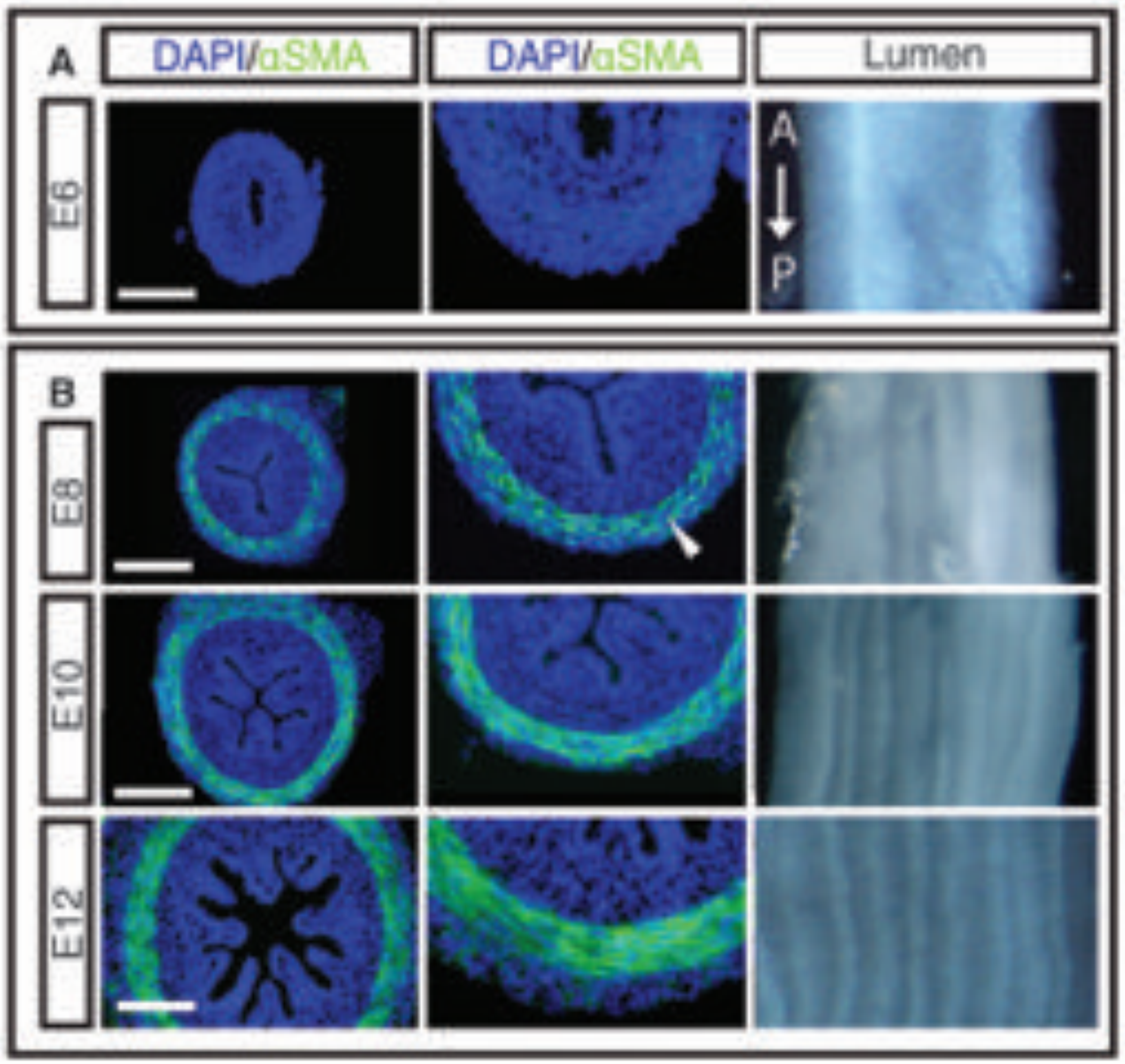
Lumen patterns in chick embryo

E6

E8

E10

E12



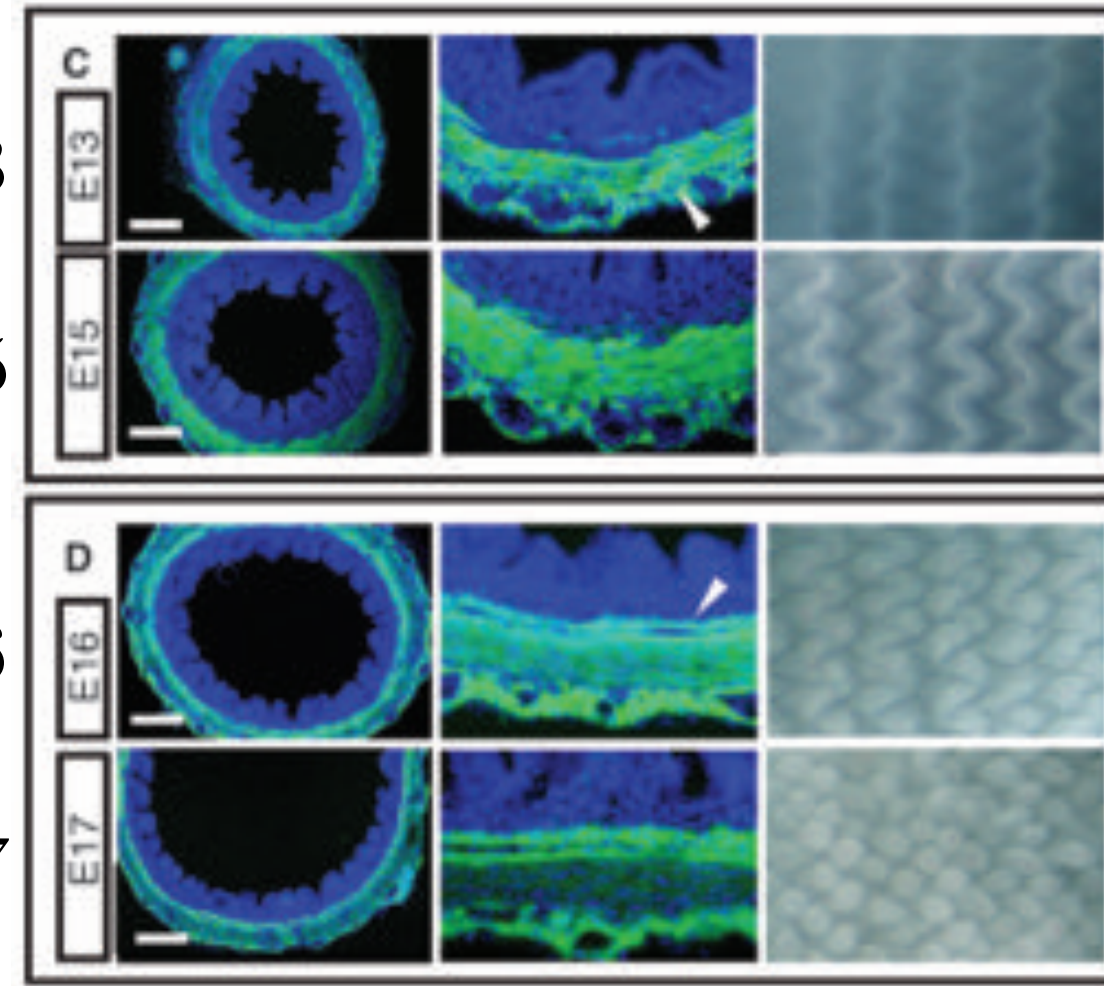
Formation of longitudinal muscles at E13 produces longitudinal compression

E13

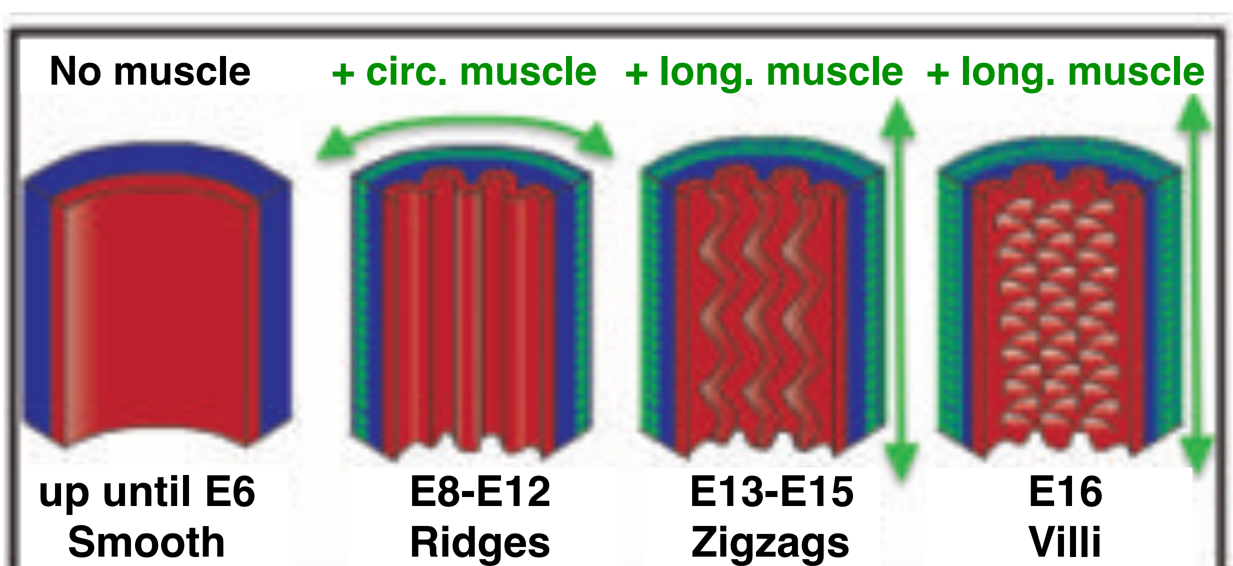
E15

E16

E17



endoderm
mesenchyme
muscle



A. Shyer et al., Science **342**, 212 (2013)

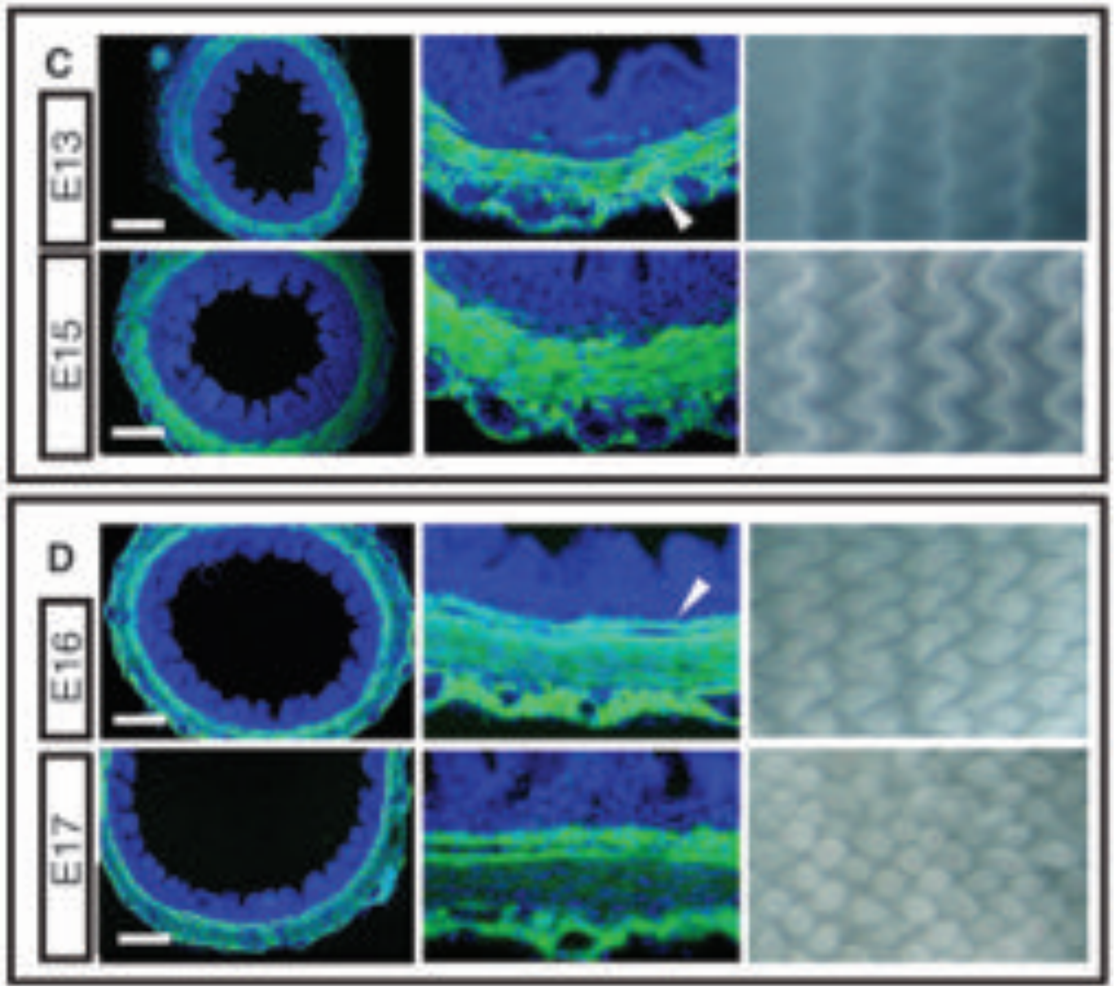
Lumen patterns in chick embryo

$E13$

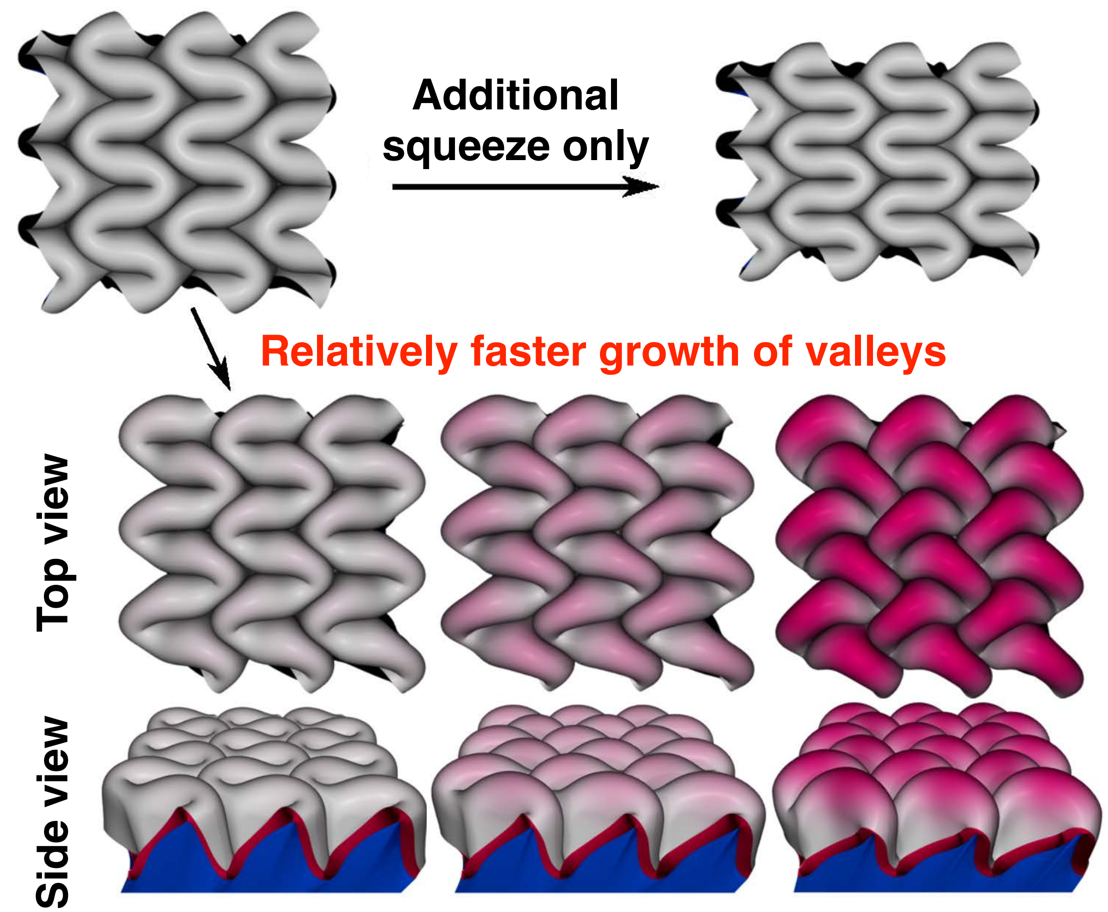
$E15$

$E16$

$E17$

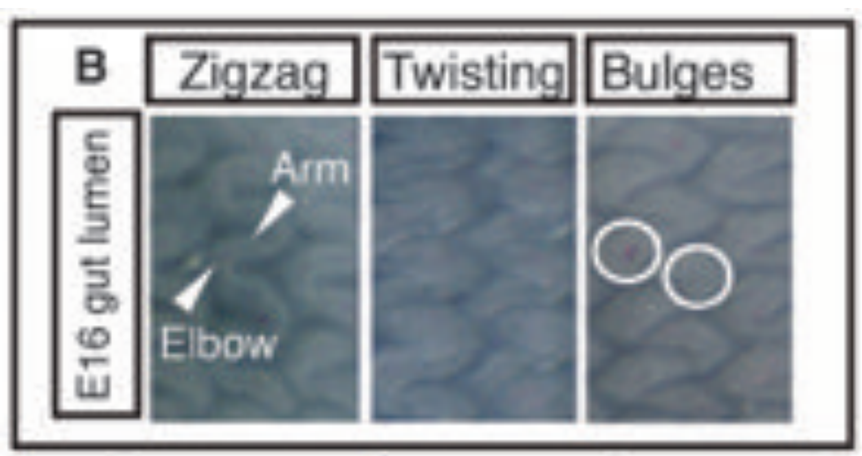


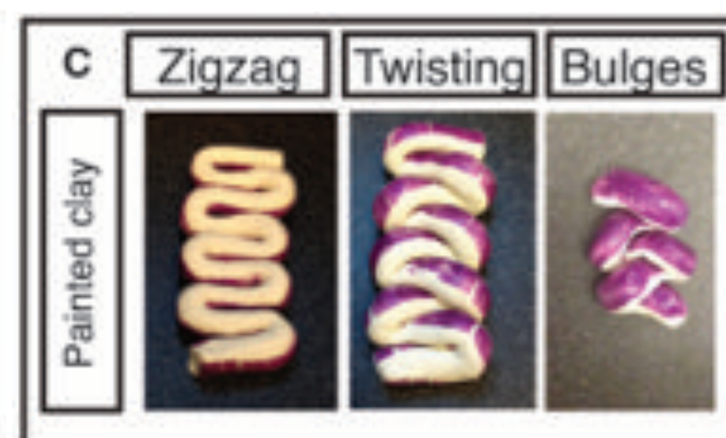
Villi start forming at E16 because of the faster growth in valleys



Zigzag Twisting Bulges

The same mechanism for villi formation also works in other organisms!





A. Shyer et al., Science **342**, 212 (2013)