The Clustering of Extreme Movements:
Stock Prices and the Weather

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**Practitioner's Digest**

A striking feature of the United States stock market is the tendency of days with very large movements of stock prices to be clustered together. We define an extreme movement in stock prices as one that can be characterized as a three sigma event; that is, a daily movement in the broad stock-market index that is three or more standard deviations away from the average movement. We find that such extreme movements are typically preceded by, but not necessarily followed by, unusually large stock-price movements. Interestingly, a similar clustering of extreme observations of temperature in New York City can be observed.

A particularly robust finding in this paper is that extreme movements in stock prices are usually preceded by larger than average daily movements during the preceding three-day period. This suggests that investors might fashion a market timing strategy, switching from stocks to cash in advance of predicted extreme negative stock returns. In fact, we have been able to simulate market timing strategies that are successful in avoiding nearly eighty percent of the negative extreme move days, yielding a significantly lower volatility of returns. We find, however, that a variety of alternative strategies do not improve an investor's long-run average return over the return that would be earned by the buy-and-hold investor who simply stayed fully-invested in the stock market.


Key words: Volatility clustering, duration analysis, portfolio strategy

**The Clustering of Extreme Movements: Stock Prices and the Weather**

**1.      Introduction**

There is considerable empirical evidence suggesting that the random walk model for changes in stock prices can be rejected and that the distribution of stock returns is distinctly non-normal. Researchers have observed that the key deviation from normality stems from the existence of 'fat tails': there are a significantly higher number of extreme values—both negative and positive—than would be found in a normal distribution. This paper deals with the stock-market returns that reside in the fat tails.  Its main objective, however, is not to present further evidence on higher-than-normal frequency of days with large changes.  Instead, we focus on examining the duration between one extreme-move day and the next.  We use duration-model analysis to examine the factors that are associated with the onset of days with unusually large changes in the Dow Jones Industrial Index for the years 1900 through 2006.  We undertake a similar analysis in examining the duration between days with extreme movements in temperature using New York Central Park data for the years 1901 through 2006.  We find striking similarity in the patterns of extreme change in the Dow and in New York temperature. We also find that the existence of extreme-move days is at least partially predictable.  Finally, we ask whether the predictability of extreme-value changes in the stock market can be employed to develop a useful portfolio-timing strategy.

We make no attempt to propose a stochastic process for stock returns or temperature changes.  The modest goal of this paper would be accomplished if it triggers further inquiry into the question whether there exists a stochastic process common to many random phenomena, including stock market returns.

**2.      The Relevant Literature**

The idea that no useful regularities exist in security prices that would enable investors to earn "abnormal" profits is one that goes back for more than a century.  While Paul Samuelson (1965) is often associated with the general idea that stock prices change randomly, the random walk thesis dates back at least to the time of Bachelier (1900).  Bachelier's doctoral dissertation developed a theory of stochastic processes that was applied to prices of French government bonds. Bachelier found that price changes looked very much like a random walk process.  His work lay dormant for almost 60 years until it was discovered by Paul Samuelson.

Essentially Samuelson argued that economists should expect price changes to be random.  The profit seeking behavior of investors should eliminate any predictable movement in stock prices.  Samuelson defines "properly intercepted prices" as prices that at every date $t \leq T$ are based on all available information at $\Phi_t$, including all past price realizations for the security.  If the security has a single payoff $X_t$, assumed to be a random variable, then for all $t \leq T$:

$$P_t = E\ (X_t\ /\ \Phi_t)$$

Samuelson then proved that prices will fluctuate randomly since for all $t \leq T$:

$$P_t = E\ (P_{t+1}\ /\ \Phi_t),\ \text{or}\ E\ (\Delta P_{t+1}\ /\ \Phi_t) = 0.$$

If prices are properly anticipated, then all useful information contained in past price series will be incorporated into current prices.  Prices will follow a martingale and successive price changes will be uncorrelated.

Much of the subsequent empirical work on stock prices has found that the stock market does not meet the conditions for a random walk.  Lo and MacKinlay, in their book *A Non-Random Walk Down Wall Street*, are able to reject the random walk hypothesis.  They define $X_t$ as the log of $P_t$ and assume prices have the recursive relationship

$$X_t = \mu + X_{t-1} + \epsilon_t$$

where $\mu$ is an arbitrary drift parameter and $\epsilon_t$ is the random disturbance term.  The traditional random-walk hypothesis restricts the $\epsilon_t$'s to be independently and identically distributed normal random variables with a constant variance.  Lo and MacKinlay are able to reject the random-walk hypothesis for weekly stock returns

(of both indices and individual stocks) using a simple volatility-based specification test. Not only do they report significant positive serial correlation for weekly and monthly holding period returns, they further conclude that mean reverting models of Shiller and Perron (1985) and Fama and French (1987) cannot account for the departures of returns from random walk.

There is considerable evidence of several non-random patterns in the movement of stock prices. For example, Keim and Stambaugh (1985), Cross (1973), French (1980), Gibbons and Hess (1981), and Rogalski (1984) find evidence of day-of-the-week and weekend effects on stock returns, while Bremer and Sweeny (1991) find extremely large negative 10-day returns are followed on average by larger-than-expected positive rates of return over the following days. But none of this work appears to contradict the efficient-market hypothesis (EMH). Whatever predictable patterns that do exist do not appear to be large enough (or dependable enough) to permit an investor to make abnormal returns after paying transactions costs. Therefore, the evidence against the random walk hypothesis is not inconsistent with the practical implications of EMH.

Another predictable pattern in the behavior of stock prices is the subject of this paper. Work by Osborne (1963), Alexander (1964), and Mandelbrot (1963) finds that the occurrence of transactions in a given stock is not independent of the past history of trades in that stock. There is a clustering of activity. Trading tends to come in "bursts." If recent trading activity is heavy, it is highly likely to continue to be heavy. Similarly, large price changes are likely to be followed by large changes in prices; as Mandelbrot (1963) writes, "…large changes tend to be followed by large changes—of either sign—and small changes by small changes…".

In this context, the ARCH model (Engel, (1982)) and its generalization to the GARCH (Bollerslev (1986)) specifications are relevant. These models set conditional variance equal to a constant plus a weighted average of past squared residuals. These models (as well as their numerous generalizations) have been proposed to explain two fundamental characteristics of stock returns: the presence of a surprising large numbers of extreme values and the fact that both the extremes and quiet periods are clustered in time. Despite the ubiquitous presence of GARCH models, however, many financial empiricists have observed that volatility responds asymmetrically to the nature of news—volatility tends to rise in response to "bad news" and to fall in response to "good news", contrary to the predictions of GARCH models (Nelson, 1991). GARCH models assume that only the magnitude and not the sign of unanticipated excess returns determines future volatility. Nelson (1991) recognized that large price declines forecast greater volatility than similarly large price increases. Since then numerous studies have found evidence of non-linearity, asymmetry, and long memory properties of volatility (Engel, (2004)).

Our paper complements this body of literature. To our knowledge ours is the only study that examines the temporal properties of 'fat tails'. We analyze the time duration between extreme daily movements in the Dow Jones Index. Consistent with Nelson's findings, we find that today's extreme return is indeed a powerful predictor of the next extreme value day. Additionally, days with extreme returns are preceded by large moves, both for negative and positive returns, although the relationship is asymmetric. Similarly, there is a temporal asymmetry—extreme value days are preceded by but are not followed by large moves days. Interestingly, we find a remarkably similar pattern in extreme changes in New York temperature (measured by the percent deviations from daily normal temperature). We undertake various tests to confirm the robustness of the results.

Finally, we explore whether the negative extreme value days are predictable enough to devise a useful asset allocation portfolio strategy. We ask whether an investor could unwind a long position in the stock market (as measured by the Dow Index) when the model predicts a high likelihood of a negative extreme-move day and temporarily hold cash. After a fixed period over which the portfolio remains unwound, the investor is assumed to re-initiate a long position in the Dow Index. We demonstrate that this portfolio strategy does not outperform a buy and hold investment in the Index when one allows for reasonable transaction costs. However, this asset allocation strategy is successful in avoiding nearly eighty percent of the extreme negative move days, yielding a significantly lower volatility of returns.

### 3. The Data and Descriptive Statistics

In this section, we describe the data used in the analysis of daily stock-market returns and N.Y. temperature changes. We also provide some simple statistics on the magnitude and frequency of the extreme-move days to motivate the duration analysis that follows in the next section.

We employed daily data on the Dow Jones Industrial Index for all trading days in the period between January 1, 1900 and December 31, 2006. Panel A of Table 1A provides some relevant statistics about the daily returns, measured in terms of logarithmic changes (i.e., $ln\left(I_t/I_{t-1}\right)$ where $I_t$ is the value of the Index on day $t$).

Our sample consists of 26,884 daily returns. For this sample, the average daily return is 0.0195%, and the standard deviation ($\sigma$) of daily returns is 1.131%, which translates to an annualized[1] mean of 7.02% and sigma of 21.46%. To provide a benchmark, we generated 26,884 pseudo returns by drawing random numbers from a normal distribution with the same mean and sigma as in the sample of returns of the Dow Jones Index.

**Table 1A: Dow Jones Daily Stock Returns Compared with Draws from a Normal Distribution**

**Panel A: Comparison of Frequencies of Days**

| Dow Jones Daily Returns: 1900-2006 | | | Random Draws from a Normal Distribution | | |
|---|---|---|---|---|---|
| # of Sigmas From the Mean | Number of Observations | Percentage of total | # of Sigmas From the Mean | Number of Observations | Percentage of total |
| Greater than 1 | 5,203 | 19.4% | Greater than 1 | 8,568 | 31.9% |
| Greater than 2 | 1,217 | 4.5% | Greater than 2 | 1,208 | 4.5% |
| Greater than 3 | 427 | 1.6% | Greater than 3 | 70 | 0.3% |
| Greater than 4 | 183 | 0.7% | Greater than 4 | 1 | 0.0% |
| Greater than 5 | 82 | 0.3% | Greater than 5 | 0 | 0.0% |
| Greater than 6 | 52 | 0.2% | Greater than 6 | 0 | 0.0% |
| Sub-Total | 7,164 | | Sub-Total | 9,847 | |
| Total Trading Days | 26,884 | | Total Observations | 26,884 | |

**Panel B: Time Interval Between 3-Sigma Plus Days**

| Dow Jones Daily Returns: 1900-2006 | | | Random Draws from a Normal Distribution | | |
|---|---|---|---|---|---|
| Time Interval in Days | Number of Observations | Percentage of total | Time Interval in Days | Number of Observations | Percentage of total |
| 1 | 76 | 17.8% | 1 | 0 | 0.0% |
| 2 | 57 | 13.4% | 2 | 0 | 0.0% |
| 3 to 5 | 96 | 22.5% | 3 to 5 | 0 | 0.0% |
| 6 to 25 | 93 | 21.8% | 6 to 25 | 4 | 5.8% |
| Subtotal | 322 | 75.6% | Subtotal | 4 | 5.8% |
| Greater than 25 | 104 | | Greater than 25 | 65 | |
| Total | 426 | | Total | 69 | |

Note: A "3-Sigma Plus day" is one where the daily absolute return of Dow Jones Average is greater than or equal to 3-sigmas from the mean daily returns.

Panel A of Table 1A shows that in about nearly eighty percent of the trading days – 80.6% to be precise – the Dow's daily change was less than one sigma away from its mean daily return. In the random draws from the normal distribution only 68.1% of the observations were less than one sigma away from the mean. However, the contrast between the two distributions (Dow returns and draws from a normal distribution) is most pronounced for days where the returns are three or more sigmas away from the mean. Extreme value

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[1] Throughout the paper, when annualizing daily returns we have assumed a 360-day calendar year.

days are far more prevalent for the Dow than for the normal distribution.  These findings are consistent with the evidence of 'fat tails' noted in numerous prior research studies.

In Panel B of Table 1A, we examine the daily time interval between one extreme-move day to the next. Here we define an 'extreme-move' day as one where the absolute value of daily return is more than three sigmas away from the mean (which we will call a '3σ+ day').  Interestingly, of the 426 time intervals between the 3σ+ days, 322 had intervals less than or equal to 25 days; and more than 31% of the 3σ+ days occurred within two days of each other!  This clustering of extreme-move days for the Dow is very different from what is found from random draws from a normal distribution.  If the stochastic process underlying the Dow's return were normal, one would have observed less than 6% of extreme-move days to have a time interval of less than or equal to 25 days; by contrast, for the Dow this percentage is 76%.
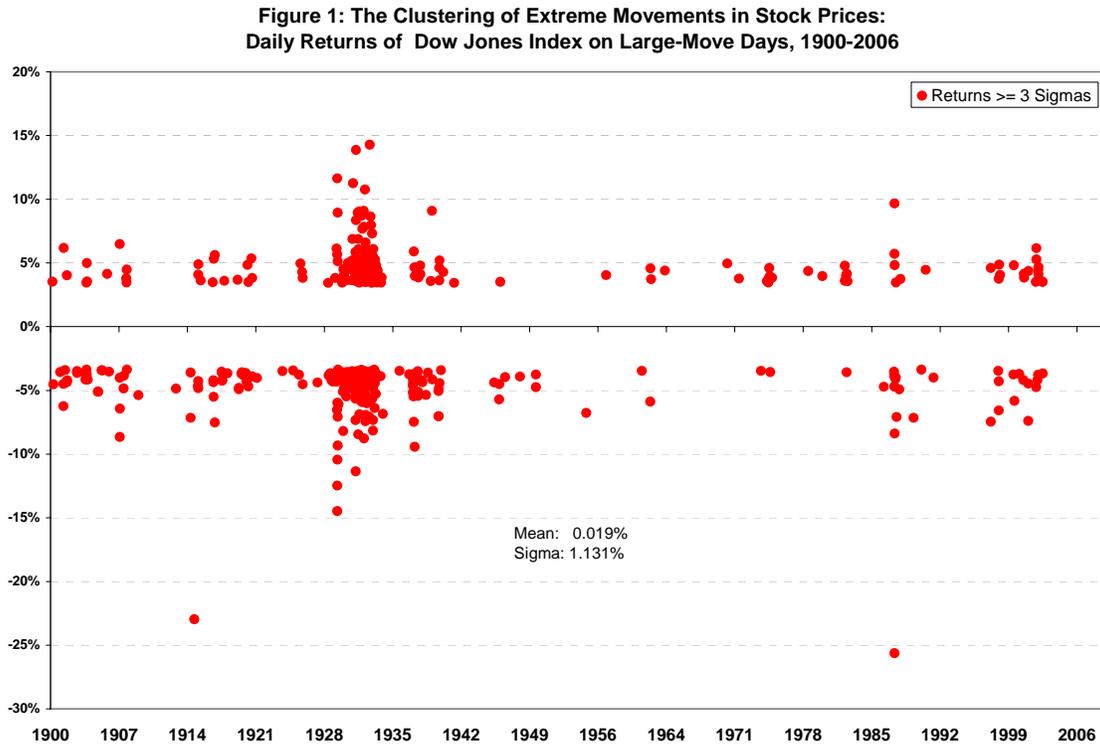
We further explore the phenomenon of the clustering of extreme-move days for the Dow in Panel A of Table 1B.  The average duration in days (i.e. the time interval) between one 3σ+ day and the next for the entire period, 1990 – 2006, was 178 days (the median was 116 days).  Excluding the 1930s, this average rises to 194 days (the median is 126 days).  Of the 427 3σ+ days, more than half occurred in 1930s.  Indeed, in the 1930s, 9% of all trading days were characterized by returns greater than three sigmas from the mean.  By contrast, the 1950s were years of calm: less than ¼ of 1 percent of trading days were ones with extreme moves.

**Table 1B:  Extreme- Move Days for the Dow Jones and New York Temperature : Number of Days by Decade**

| | Panel A | | | | Panel B | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | Dow Jones Daily Returns :  1900-2006 | | | | New York  Daily Average Temp's  % Deviation from Normal: 1901-2006 | | |
| Decade | Number of 3-Sigma Plus days | % of Trading Days | Average Duration Between 3-Sigma Plus Days | Decade | Number of 3-Sigma Plus days | % of Calendar Days | Average Duration Between 3-Sigma Plus Days |
| 1900 s | 37 | 1.47% | 62 | 1900 s | 11 | 0.34% | 292 |
| 1910 s | 28 | 1.16% | 93 | 1910 s | 30 | 0.82% | 121 |
| 1920 s | 43 | 1.72% | 59 | 1920 s | 13 | 0.36% | 279 |
| 1930 s | 225 | 8.94% | 11 | 1930 s | 27 | 0.74% | 124 |
| 1940 s | 17 | 0.67% | 135 | 1940 s | 15 | 0.41% | 264 |
| 1950 s | 4 | 0.16% | 567 | 1950 s | 13 | 0.36% | 228 |
| 1960 s | 5 | 0.20% | 309 | 1960 s | 20 | 0.55% | 181 |
| 1970 s | 11 | 0.43% | 343 | 1970 s | 23 | 0.63% | 189 |
| 1980 s | 24 | 0.94% | 116 | 1980 s | 21 | 0.57% | 175 |
| 1990 s | 11 | 0.44% | 207 | 1990 s | 30 | 0.83% | 108 |
| 2000 s | 22 | 1.25% | 51 | 2000 s | 17 | 0.66% | 171 |
| **Total** | **427** | | | **Total** | **220** | | |
| **Average** | | **1.58%** | **178** | **Average** | | **0.57%** | **194** |
| **Average w/o 1930s** | | **0.84%** | **194** | | | | |

Note:  A "3-Sigma Plus day" is one where the daily absolute return of Dow or the absolute deviation from normal temperature is greater than or equal to 3-sigmas.

The clustering of 3σ+ days over the decades is illustrated in Figure 1.  The figure suggests several interesting observations.  First, there seems to be a considerable clustering of extreme-change days.  We note that the greatest clustering took place during the late 1920s and early 1930s, reflecting the boom and bust of the stock market and the depression that followed. Another clustering is apparent around the recession (depression) of economic activity in the late thirties and the uncertainty leading up to World War II.  The most recent clustering occurred during the late 1990s and early 2000s associated with the high-tech internet bubble, the busting of the bubble, and the recession of the early 2000s.  Note the existence of a few outliers of extremely large one-day declines in the stock market, of which the crash of October 19, 1987 stands out.  Finally, note that the picture of large positive and negative changes is generally symmetric, with the exception of the aforementioned highly unusual steep declines, and that there is no evidence of increasing volatility over time. Indeed, there have been large gaps of several years during which no three-sigma events occurred.

**Figure 1: The Clustering of Extreme Movements in Stock Prices:
Daily Returns of Dow Jones Index on Large-Move Days, 1900-2006**



We also gathered data on the daily temperature in New York's Central Park[2] for the years 1901 to 2006. We then defined the temperature's daily 'change' as the percent deviation from the "normal" temperature for that day. In calculating the daily normal temperature, we used the 106-year average for any given day. Our sample consists of 38,655 observations of daily percent deviations from normal temperature. The standard deviation ($\sigma$) for the sample of daily temperature changes is 1.46%. We then defined an "extreme-move" day as one where the percent deviation in temperature exceeded three sigmas away from the mean. As we did for the stock market, we call these 3$\sigma$+ days.

Panel B of Table 1B contains data on the occurrence of and the duration between 3$\sigma$+ days for New York temperatures. Comparisons of Panels A and B reveal some interesting differences and similarities. Unlike the Dow, extreme-move days for N.Y. temperature are more evenly distributed across the decades. For the years 1901-2006, there were a total of 220 extreme movement days in temperature. While temperature change extremes occurred only about half as often as extreme movements for the Dow, such movements are far more comparable if the 1930s are excluded for the Dow. For N.Y. temperature, the average duration in days between one 3$\sigma$+ day to the next is 194 days, which is higher than the 178 days figure for the Dow. However, for the Dow, if one excludes the 1930s, the average duration between 3$\sigma$+ days becomes 194, exactly the same as the average duration between extreme value days for the N.Y. temperature.

**4. Duration Analysis**

In this section, we explore the factors that may explain the onset of an extreme-move day. In particular, we examine whether extreme-value days – both for the Dow and the N.Y. temperature – are preceded and followed by larger-than-average changes. We begin with a simple regression analysis of the time between extreme value days and then undertake a more formal duration analysis using the Cox Proportional model.

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[2] We procured this data set from Weather Source.

The key explanatory variables we consider are: (i) the average (of the absolute value of returns for the Dow and the percent deviations from normal for temperature) daily move in the three-day interval immediately *preceding* a 3σ+ day; (ii) the average daily move in the three-day interval immediately *after* a 3σ+ day; and (iii) the percentage change, in absolute terms, on a 3σ+ day.[3]

The important statistics (mean and standard deviation) for these three variables are presented on Table 2. As a benchmark for comparison we also present the statistics for the absolute value of returns and the daily deviation for the entire sample of 26,884 observations for the Dow and 38,655 for the N.Y. temperature. The first row of this table shows that the mean absolute value of the Dow's daily returns is 0.743% for the entire sample and the standard deviation is 0.853%. These figures differ from the ones presented in Table 1A because here we report the mean and standard deviation of the *absolute* value of the returns.

The next three rows of Table 2 contain summary statistics on the three explanatory variables. The mean and standard deviation for these three variables are based on 427 observations for Dow and 220 observations for N.Y. temperature, which corresponds to the number of 3σ+ days for each.

Comparison of the figures in the first row to the ones in rows two and three of Table 2 reveals that the average daily movement in the three-day interval preceding and following a 3σ+ day is markedly different from the typical daily movement. For example, the average daily return in the three-days before a large-move days is 2.11%, which is almost three times larger than the average daily return in the sample. The same holds true for the average daily return during the three-day period after a 3σ+day. The mean absolute return on all 3σ+ days is 4.96%, almost seven times larger than the average for all days. This difference is to be expected because of the 427 days with 3σ+ returns, there are a sizeable number of days with returns as much as six or seven sigmas away from the mean.

**Table 2: Descriptive Statistics For Extreme-Move Days for Stock Prices and NY Temperature**

|  | Dow Jones Daily Returns: 1900-2006 | | New York Daily Average Temp's Deviation from Normal: 1901-2006 | |
|  | Mean | Standard Deviation | Mean | Standard Deviation |
| --- | --- | --- | --- | --- |
| Absolute Value of Daily Return/Daily Change | 0.743% | 0.853% | 1.138% | 0.909% |
|  | (N = 26,884) | | (N = 38,655) | |
| Average Daily Move 3-days Before a 3 Sigma-Plus Day | 2.105% | 1.741% | 2.563% | 1.107% |
| Average Daily Move 3-days After a 3 Sigma-Plus Day | 2.122% | 1.602% | 2.441% | 1.102% |
| Average Daily Move on the Day of a 3-Sigma Plus Day | 4.952% | 2.166% | 4.984% | 0.556% |
| Number of observations | 427 | | 220 | |

Turning now to the summary statistics on the explanatory variables for the N.Y. temperature, we find a remarkable similarity with the Dow's moves. The mean daily deviations in the three days before and after a 3σ+ day is considerably larger (approximately 2 ½ times) than the average daily deviations from normal temperature for all days. Interestingly, the means of the variable 'daily move on the day of a 3σ+ day', are virtually identical for the N.Y. temperature and the Dow; 4.98% and 4.95%, respectively.

The summary statistics on Table 2 suggest that these variables will have some explanatory power to predict the onset of extreme-move days. However, before undertaking a formal duration analysis, it may be useful to examine the time interval between 3σ+ days using a simple least squares regression. The results of two least squares analysis, one for the Dow and the other for N.Y. temperature, are presented in Table 3. The

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[3] We examine not only a three-day interval *after* a 3σ+ day, but also *before* to explore whether a large move day can be predicted by "pre-shocks"; by definition "after-shocks" cannot have any predictive power. Later in this paper (see Section 5), we explicitly explore this predictive relationship by examining an asset allocation strategy.

dependent variable in each regression is *ln (T),* where *T* denotes the time interval in days from one extreme movement day to the next.  The explanatory variables are the ones discussed in the preceding paragraphs.

**Table 3:  Least Squares Analysis of the Time Interval Between 3 Sigma-Plus Days**

| | Dow Jones Daily Returns | | New York  Daily Average Temp's Deviation from Normal | |
|---|---|---|---|---|
| | Estimated Coefficient | T-stat | Estimated Coefficient | T-stat |
| Three-Days Before | -54.87 | -12.20 *** | -123.73 | -10.04 *** |
| Three-Days After | -8.21 | -1.64 | 5.21 | 0.41 |
| The Day of | -6.37 | -1.77 | -68.14 | -2.65 *** |
| Intercept | 3.72 | 19.54 *** | 9.84 | 8.22 *** |
| Number of obs | 427 | | 220 | |
| Adjusted R-Squared | 0.327 | | 0.362 | |

Note:  The dependent variable in each regression is the log of the duration in days between 3-sigma-plus days.

**Definitions:**

For the Dow, the sigma is based on the standard deviation of daily log returns.
For New York Temp, the sigma is based on the standard deviation of daily % deviation from normal (=106-year average) temperature.

The 3-Days Before (after): for the Dow, it is the average absolute return in the three days prior (after) to a 3-sigma-plus day; for NY temp, it is the average absolute % deviation three days prior (after).

The Day of:  For the Dow it is the absolute value of return, for NY Temp it is the absolute % deviation on a 3-sigma-plus day.

*** denotes statistically significant at 99% level of confidence.

Qualitatively, the results for the Dow and the N.Y. temperature are very similar.  In both cases, the coefficient of the variable 'average daily move 3-days before a $3\sigma+$ day' is negative and highly significant (with t-statistics exceeding 10), whereas the coefficient of the variable 'average daily move 3-days after' is not.  Thus, we find that extreme value days are preceded by, but not necessarily followed by, changes that are relatively large.

The coefficient of the variable 'daily move on a $3\sigma+$ day' is negative for both the Dow and the N.Y. temperature, but not significant at the 95% level for Dow.  The negative coefficient of this variable suggests that the magnitude of the change on a $3\sigma+$ day is itself negatively correlated with the time interval between extreme-move days.

### *Duration Analysis Using the Cox Proportional Hazard Model*

In Table 4A, we present the results of the duration analysis using the Cox Proportional Hazard model. We use the semi-parametric Cox model rather than a parametric model such as the Weibull or logistic models because the Cox model does not impose a particular shape to the hazard function.

**Table 4A:  Estimation of Time Between Extreme-Move Days Using the Cox Duration Model**

| | Dow Jones: 1900-2006 | | | | NY Temp: 1901-2006 | |
| | | | **Excluding the 1930s** | | | |
| | **Coeff-Est** | **Z-stat** | **Coeff-Est** | **Z-stat** | **Coeff-Est** | **Z-stat** |
| **All 3-Sigma Plus Days** | | | | | | |
| 3-Days Before | 32.74 | 13.25 *** | 29.80 | 8.11 *** | 45.60 | 6.69 *** |
| 3-Days After | 4.49 | 1.42 | 1.21 | 0.27 | -2.03 | -0.31 |
| That Day | 6.26 | 2.51 ** | 7.27 | 2.09 ** | 43.98 | 3.38 *** |
| Number of Obs | | 427 | 202 | | 220 | |
| **Only Positive 3-Sigma Plus Days** | | | | | | |
| 3-Days Before | 89.38 | 9.97 *** | 115.94 | 6.10 *** | 36.46 | 3.76 *** |
| 3-Days After | 23.53 | 3.07 *** | 15.98 | 1.13 | 9.48 | 1.06 |
| That Day | 10.72 | 2.50 ** | 21.57 | 2.16 ** | 47.19 | 2.08 ** |
| Number of Obs | | 178 | 80 | | 115 | |
| **Only Negative 3-Sigma Plus Days** | | | | | | |
| 3-Days Before | 46.35 | 9.84 *** | 41.13 | 6.18 *** | 45.75 | 4.48 *** |
| 3-Days After | 3.77 | 0.89 | 2.64 | 0.48 | -6.34 | -0.59 |
| That Day | 9.07 | 3.45 *** | 10.98 | 3.25 *** | 39.78 | 2.31 ** |
| Number of Obs | | 249 | 122 | | 105 | |

Note:  See Table 3 for definition of the variables

*** denotes statistically significant at 99% level of confidence
** denotes statistically significant at 95% level of confidence

In comparing the estimated coefficients of the explanatory variables in the least squares regression and the Cox model, one should note that the signs of the coefficients are expected to be positive rather than negative. Positive estimated coefficients imply that these variables shorten the duration, i.e., 'hasten' the arrival of the next $3\sigma$+ day.  For example, consistent with the regression findings, the estimated coefficient of the variable 'average daily move 3-days before a $3\sigma$+ day' is positive and significant at the 99% confidence level in the Cox model both for the Dow and the N.Y. temperature.  Similarly, the estimated coefficient of the variable 'daily move on a $3\sigma$+ day' is also positive and statistically significant, both for the Dow and the N.Y. temperature. However, the coefficient of the variable, 'average daily move 3 days after a $3\sigma$+ day' is not statistically significant for both the Dow and the N.Y. temperature.

For the Dow, we re-estimated the Cox model by excluding the 1930s (which leaves us with 202 $3\sigma$+ days).  The results stay virtually unchanged: the signs and significance of the estimated coefficients are identical to those using the entire sample period 1990-2006.

To explore Nelson's (1991) observation that there is an asymmetry in the market's response to "good news" and "bad news", we next considered the two subsets – positive and negative $3\sigma$+ days – separately.  In examining these subsets, we define the explanatory variables slightly differently than before.  For example, for the set of only positive $3\sigma$+ days, the variable 'average daily move 3-days before a $3\sigma$+ day' now measures the average of only the positive returns or temperature changes in the three-day intervals.  The converse applies to the explanatory variables for the analysis of negative $3\sigma$+ days: the average of only the negative moves in the three-day intervals is measured.

The results are consistent across sub-samples. The coefficients of the variables 'average daily moves three days before' and 'the day of' for both negative and positive $3\sigma+$ days, remain statistically significant for the Dow (with and without the 1930s included) and for the N.Y. temperature. Similarly, the coefficient of the variable 'three day after' is insignificant in all cases except for positive $3\sigma+$ days for the Dow. However, even in this case, the coefficient estimate becomes insignificant when the 1930s are excluded from the estimation.

Although the estimation results for positive and negative $3\sigma+$ days are qualitatively similar in term of the signs and significance of the estimated coefficients, there are some differences, particularly for the Dow. The coefficient of the variable 'average daily move 3-days before' is substantially larger for positive $3\sigma+$ days than the negative ones for the Dow. This result holds true even when one excludes the 1930s. This implies that the positive returns in the three-day period before a positive $3\sigma+$day 'hasten' the arrival of the next extreme, positive-move day much more than does the negative returns preceding a negative $3\sigma+$ day.

However, a similar asymmetry is not observed for positive and negative large-movement days in the N.Y. temperature. In fact, the coefficient of the variable 'average daily move three days before' for the positive $3\sigma+$ days is slightly smaller than the coefficient of this variable for negative extreme-move days.

Despite these minor dissimilarities, the nine sets of duration model results contained in Table 4A are remarkably consistent. For both the Dow Jones and the N.Y. temperature, larger-than-average moves three days prior and on the day of a $3\sigma+$ day 'hasten' the arrival of the next extreme-move day.


*Validation of the Robustness of Results*

We have undertaken a large number of variations of the Cox duration model to verify that our results are robust. For example, (a) we have considered the average moves during different windows of time (e.g., three-day, five-day, seven-day windows) before and after a $3\sigma+$day; (b) we have excluded various sub-periods, including the years 1928-35 for the Dow; (c) we have excluded observations where the difference between two consecutive trading days exceed four days as was the case during the World Wars or 9-11-2001 for the Dow; (d) we have excluded all observations with moves greater than or equal to 5-sigmas to examine the impact of outliers; (e) we have included a variety of dummy variables for decades, for months of the year, for periods of recession, etc. While we do not report the results of all these robustness checks here, they are available on request. They confirm that the key findings of this paper are remarkably robust. In Table 4b, however, we provide a few examples these robustness checks.

As shown in this table, we re-estimated the duration model for the Dow with two sub-periods: 1900-1950 and 1951-2006. The daily $\sigma$ for the first sub-period is a fair bit higher since it includes the 1930s: 1.34% versus 0.91%. Despite this difference, the explanatory power of the model and the qualitative results across the two sub-periods are very similar. The coefficient of the variable 'average daily move 3-days before' continues to be positive and statistically significant at the 99% level of confidence.

In this table, we also report results when a dummy variable for recession quarters is included in the model. The 'recession' dummy variable takes a value of one for all trading days in the quarters that the National Bureau of Economic Research[4] has identified as being recessionary. The recession dummy is not statistically significant in either sub-period, 1900-50 or 1951-06; and the sign and significance of the 'three days before' variable's coefficient estimate remain unchanged.

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[4]The NBER study can accessed electronically at www.nber.org/cycels.html.

**Table 4B: Results of Cox Duration Model Estimation: Pre and Post 1950s**

| | Dow Jones: 1900-1950 (Sigma=1.334%) | | | |
|---|---|---|---|---|
| | **Coeff-Est** | **Z-stat** | **Coeff-Est** | **Z-stat** |
| 3-Days Before | 38.18 | 10.11 *** | 37.53 | 9.83 *** |
| 3-Days After | -2.60 | -0.57 | -3.25 | -0.71 |
| That Day | 7.50 | 2.19 ** | 7.41 | 2.18 ** |
| Recession | | | 0.19 | 1.26 |
| Number of Obs | 223 | | 223 | |

| | Dow Jones: 1951-2006 (Sigma=0.909%) | | | |
|---|---|---|---|---|
| 3-Days Before | 28.07 | 6.40 *** | 28.08 | 6.39 *** |
| 3-Days After | 6.92 | 1.10 | 6.91 | 1.10 |
| That Day | 0.59 | 0.11 | 0.61 | 0.11 |
| Recession | | | 0.01 | 0.05 |
| Number of Obs | 154 | | 154 | |

Note: See Table 3 for definition of the variables

*** denotes statistically significant at 99% level of confidence
** denotes statistically significant at 95% level of confidence

The remarkable consistency of the results – particularly the finding that extreme-move days are preceded by but not necessarily followed by periods of higher volatility – across two seemingly unrelated random phenomena, stock-market returns and temperature changes, seem to suggest the existence of a common stochastic process. We do not propose such a process here. We believe our findings simply lend further support to the arguments of financial economists and mathematicians who have contended that many complex random processes, including stock-market returns, cannot be explained by simplistic processes such as the random walk. The challenge, however, lies in uncovering and formulating the stochastic processes that can be shown to possess a reliable degree of predictability.[5]

## 5. Predictability and the Returns of a Portfolio Strategy

In this section, we explore the predictive power of our duration-model results and examine whether they can be used to produce a useful market timing strategy. In particular, we examine whether our model can be used to predict an extreme, negative-move day in the stock market where the investor could shift a portfolios' asset allocation from stocks to cash. Of the three explanatory variables used in the duration analysis, only one, 'the average daily move three days before a 3σ+ day,' has the potential to be used as a predictor, because the value of other explanatory variables is not known until *after* a large-move has already occurred.

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[5] In this context, the writings Benoit Mandelbrot, who has long argued for a fractal explanation of various disparate and diverse stochastic phenomena, deserve special attention. In his latest book, "The Misbehavior of Markets: A Fractal View of Risk, Ruin, and Reward" he elaborates on his fractal view of various random phenomena, including the behavior of the financial markets.

Our proposed asset allocation strategies are based on our empirical finding that negative extreme-move days are typically preceded by days when equity returns are negative and unusually large in absolute value. [6] Based on this finding, we use the following rules to construct asset-allocation strategies:

(a)   From the beginning of our sample period, the portfolio is 100 percent invested in the Dow index.  That is, the portfolio's daily return is assumed to be the same as the Dow's.

(b)    If a day's return is negative, and the average absolute value of the preceding three day's negative returns exceeds a set threshold, then the portfolio is unwound and reinvested in cash.

(c)   The portfolio stays unwound for a set period of time over which the assets earn a rate of return equivalent to the risk-free rate.

(d)   After the set period of time, the investor is assumed to re-initiate a long position in the Dow index.

(e)   The portfolio incurs a fixed transaction cost, both for unwinding and re-initiating the portfolio.


We define the 'set threshold' in (b) above, as the average of the absolute values of daily returns of all non-3σ days with a negative return in the entire sample.  We call this threshold the "trigger condition to unwind".  We have considered various periods of time over which the portfolio remains unwound, including 2, 3, 5, 7 and 10 days.  On each day in this interval during which the portfolio remains unwound it is assumed to earn the daily risk-free rate of 0.0103%. This is based on the average annual Treasury bill rate of 3.7% for the years 1926 through 2006.[7]

As a measure for transaction cost, we have used 5 basis points each way – that is, the portfolio incurs a cost of 0.05% on the day it is unwound, and another 0.05% when it is re-initiated. We have used two sample periods in the portfolio analysis:  the entire sample of the 107 years, and the sub-sample comprising the years 1936-2006.  The results of our analysis are presented on Table 5.

**Table 5: Performance Statistics for Alternative Trading Strategies Based on Extreme-Move Days**

| | All data (1900-2006) | | | | | 1936-2006 Only | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Annual Mean Return (%) | Annual Sigma (%) | Sharpe Ratio | % Days | % of Negative 3-Sigma-plus days avoided | Annual Mean Return (%) | Annual Sigma (%) | Sharpe Ratio | % Days | % of Negative 3-Sigma-plus days avoided |
| **Actual** | 7.01 | 21.46 | 0.15 | | | 9.45 | 17.76 | 0.32 | | |
| **Hypothetical Portfolio** | | | | | | | | | | |
| Unwound period 2 days | 6.59 | 16.85 | 0.17 | 12% | 54% | 7.42 | 14.57 | 0.26 | 12% | 47% |
| Unwound period 3 days | 7.63 | 16.05 | 0.25 | 16% | 65% | 7.04 | 14.03 | 0.24 | 16% | 55% |
| Unwound period 5 days | 8.62 | 14.66 | 0.34 | 22% | 75% | 7.53 | 12.94 | 0.30 | 23% | 67% |
| Unwound period 7 days | 8.42 | 13.61 | 0.35 | 28% | 80% | 7.45 | 11.98 | 0.31 | 29% | 74% |
| Unwound period 10 days | 8.28 | 12.53 | 0.37 | 34% | 86% | 7.07 | 11.03 | 0.31 | 36% | 79% |

Note: "% Days" is the percent of the trading days during which the portfolio remains invested in cash as opposed to being invested in Dow.


When the entire 107-year-period is considered, the portfolio is found to outperform the Dow's actual annual average returns when the unwound period is at least two days. The percentage of all trading days during which the portfolio stays invested in cash varies from 12% to 34%, depending on the length of the unwound period.

Two findings warrant special attention:  First, the portfolio strategy yields a considerably lower volatility of returns than the actual Dow.  While the annualized standard deviation for the actual Dow index is 21.46%, the portfolio's annual σ is almost 400 basis points lower at 16.85%.  Not unexpectedly, the portfolio's σ diminishes considerably when the portfolio stays unwound for a longer period of time.  In this context, it is informative to consider the Sharpe ratio (where we divide the difference between the mean return and the risk-

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[6]To illustrate the point, consider one of the worst negative move days in Dow's history, October 19th 1987, when the Dow fell by 27% (in log returns).  This day was preceded by fairly large negative move days: in the three days prior to this day, the Dow fell on average by 3.7% per day and cumulatively by nearly 11%. Yet, in the three day-period following October 19th 1987 the Dow's moves were relatively uneventful.

[7] Ibbotson Associates, 2007 Yearbook, Chicago, IL.

free annual rate of 3.7% by the sigma of the return). In all cases, the portfolio yields a higher Sharpe ratio than the actual Dow's returns. In fact, for the unwound period of ten days, the Sharpe ratio for the portfolio (0.37) is more than twice as large as the actual Dow's ratio (0.15).

Secondly, our portfolio strategy, with a rather simple trigger condition for unwinding, seems to be fairly effective in avoiding negative $3\sigma+$ days. Even with an unwound period of only two days, the strategy avoids 54% of the negative $3\sigma+$ days. When the unwound period increases to 10 days, this percentage jumps to 86%, implying that the trigger condition is fairly accurate in predicting the next negative extreme-move day.[8]

In Table 5, we also report the results of the portfolio analysis using data only for the period 1936-2006. For this period, the Dow's actual mean annualized return is 9.45% (as opposed to 7.01% for the entire period of 1900-2006) and annualized sigma is 17.76% (as opposed to 21.46% for 1900-2006). Now, however, the portfolio does not outperform the Dow. In fact, at its best – when the unwound period is 5 days – the mean return is 7.53%, about 200 basis points worse than Dow's actual performance. It is not surprising that the asset-allocation strategy has a more negative effect during the past 70 years. The stock market has enjoyed a particularly strong long-term uptrend following the great depression and the buy-and-hold investor has generally benefited from being totally invested in all periods. But the portfolio's volatility ($\sigma$) continues to be substantially smaller than Dow's actual volatility. Even with a two-day unwound period, the portfolio's annualized $\sigma$ is 14.57% more than 250 basis points lower than Dow's. The portfolio's Sharpe ratio is roughly comparable to the actual Dow's when the unwound period is 5 days or longer.

The percentage of trading days affected by the portfolio strategy is virtually identical regardless of which period is considered, all 107 years or the last 70 years. However, that is not the case for the number of negative $3\sigma+$ days avoided. Consistent with the finding of lower returns for the sub-period 1936-2006, we find that the percentage of avoided negative $3\sigma+$ days is slightly smaller as well. With a two-day unwound period, this percentage is 47%, which rises to 79% when a ten-day unwound period is applied.

While there is strong empirical evidence of a relationship between an extreme-move day and its preceding three-day's average return, we find that this relationship is not predicable enough to devise a dependable asset-allocation (market-timing) strategy to outperform the market. However, if the objective of the trading strategy is to reduce the portfolio's volatility, then the strategy seems to be reasonably effective: from 50% to 80% of the negative extreme-move days are avoided by adopting the strategy suggested by our duration model results.

**6.      Concluding Comments**

There has been considerable empirical work confirming that stock returns are not normally distributed. Moreover, we know that stock prices do not follow a strict random-walk process and may, to some extent, be predicable. The paper has examined the temporal properties of the "fat tails" of the stock-return distributions. We found that there has been a clustering of days when the stock market has been subject to "extreme movements," which we define as daily stock returns that are three or more standard deviations away from the mean return. Moreover, a similar clustering of extreme movements is shown to exist in the daily time series of temperatures recorded in New York City.

Utilizing a duration model we found that extreme movements can, to some extent, be predicted on the basis of unusually large movements in preceding periods. One particularly robust finding is that extreme movements in stock prices are usually preceded by large average daily movements during the preceding three trading days. A similar finding was reported for daily N.Y. City temperatures.

The strong results suggest that it may be possible to fashion an asset-allocation strategy, whereby the investor would switch from stocks to cash in advance of predicted extreme negative returns in the stock market. We found, however, that it was not possible to use the predictability that exists to improve an investor's long-

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[8] The reason that returns are not improved considerably even though so many negative days are avoided is that many extreme positive days are eliminated as well. As is well known, a substantial part of the generous returns earned by long-run equity investors comes from the infrequent number of days when the market rallies sharply.

run return over the return that would be earned by the buy-and-hold investor who simply stayed fully invested in the stock market. The market-timing strategy that we simulated was, however, useful in reducing the volatility of returns. Because the timing strategy does permit the investor to avoid between half and more than three quarters of the trading days in which the market suffers substantial declines, the suggested strategy does substantially lower the volatility of the portfolio's returns over time.

We have shown that a duration model analysis can shed considerable light on the factors that are associated with unusually large changes in the stock market and the weather. The task for the future is to better understand the underlying stochastic processes common to many random phenomena.

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