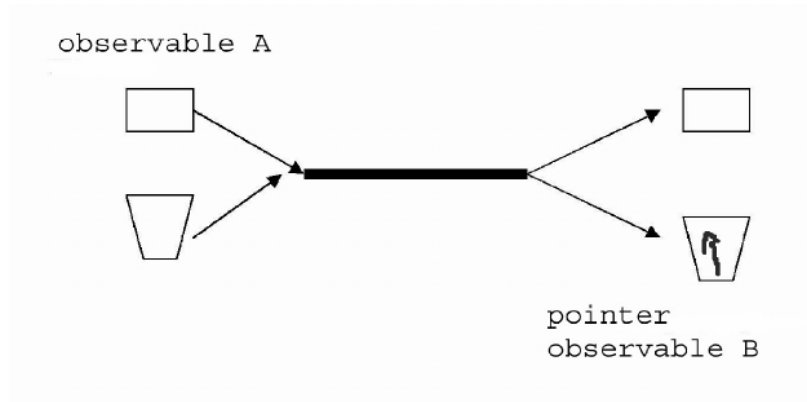




MODAL INTERPRETATION OF QM -- Copenhagen Variant of

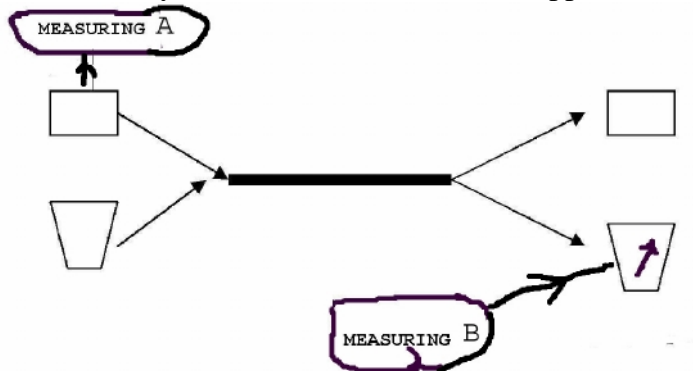
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0. Measurement and its dilemmas



The process depicted is a measurement only if the initial state of the object and the final state of the apparatus are related by $\text{Tr}(I_{|b(i)\rangle}) = (|a(i)\rangle, \cdot)$. This ‘transfer’ of probabilities is the main criterion used to define ‘Von Neumann measurement’

Intuitive story: if the object’s initial state is the the probability that the outcome is pointer reading $b(i)$ equals (by the Born rule) $(|a(i)\rangle, \cdot)$. But how to way to express that in QM? No apparent difference from: a further measurement on the apparatus will show pointer reading $b(i)$ with that probability. So the condition appears to be a consistency or coherence constraint, as opposed to a ‘truth condition’:



PROPOSAL I. The intuitive story is literally correct; and is equally correct if the entire set up is isolated (hermetically sealed in a spaceship e.g.)

PROPOSAL II. The outcome in question occurs iff B has value $b(i)$ at the end.

We'll symbolize that statement as $\langle B, b(i) \rangle$. More generally we'll symbolize 'm has value in E' as $\langle m, E \rangle$ -- where m is any measurable quantity.

It follows that attributions of states ('the object has state ' etc.) cannot be attributions of values to observables ('B has value b(i)' etc.). The state evolves deterministically [by reduction from the state of the whole]; the outcome is stochastic.

This is the basic idea of the modal interpretation. What probabilities and possibilities there are is determined by the state, so states give modal information.

1. The Modal Interpretation

(modified from Quantum Mechanics: An Empiricist View, Ch. 9)

Von Neumann read attributions of values to observables as attributing states:

1. Observable B has value b if and only if a B-measurement is certain to have outcome b.

The term 'state' is familiar from classical mechanics, thermodynamics, chemistry, engineering, and so on. Classically, we would assume that (a) a state is given if we know the value of all observables, and (b) if we knew the state, then we would know all there is to know about how the system will develop if left alone and how it will react if acted upon (where such action-upon includes measurement, of course).

If it is assumed that observables simply have values, there to be seen if we look, and also that the temporal evolution is determined entirely by what those values are, then there is no need to keep (a) and (b) very separate. But the times have changed. So we must be cautious, and distinguish two concepts of state, one for each role:

Value state: fully specified by stating which observables have values and what they are

Dynamic state: fully specified by stating how the system will develop if isolated, and how if acted upon in any definite, given fashion

Another way to present this same conceptual distinction was to distinguish between two sorts of propositions:

Value-attributing proposition: e.g. $\langle m, E \rangle$ says that observable m actually has a value in E

State-attributing proposition: e.g. $[m, E]$ says that the state is such that a measurement of m must have an outcome in E

However, we do think of measurement outcomes as relevant to the question what state the system was in. This suggests salvaging from von Neumann's account at least the Half Eigenstate-Eigenvalue Link: that $[m, E]$ implies $\langle m, E \rangle$: if the state was such that m must show a value in E if questioned, then it does actually have a value in E.

But if $[m, E]$ were not true — if the state were not such that m must have a value in E — then it would still be possible that m does actually have a value in E anyway. This is a true transition from the possible to the actual.

Statistical inference.

More generally, from information about what values observables did have, we can — by looking for a statistical fit — infer backward to the state. In general, such inference, being statistical, will require much more than a single measurement. It will be possible to conclude, in this way, that a certain source or preparation procedure produces systems in state W — or to attribute state W to the systems in an ensemble of which we have inspected a sample — but not to determine that a single given system is in this state, otherwise.

An objection?

A state can always be represented by a statistical operator — if only a projection operator, projecting along a specific vector — and that is a Hermitean operator. But Hermitean operators are meant to represent observables. Does it not follow then that being in a state is the same as, or in some strong sense equivalent to, a certain observable's having a certain value?

This generalizes too much on the case of pure states -- 'is in pure state x ' = ' I_x has value 1' -- but already does not work for mixed states:

To be in mixed state W is not the same as for observable W to have value 1. Consider:

$$W_1 = \frac{1}{2}I_x + \frac{1}{2}I_y$$

$$W_2 = \frac{1}{4}I_x + \frac{3}{4}I_y \quad \text{where } x \perp y.$$

Then the yes/no observables which have value 1 with certainty in these states are the same, namely those represented by a projection operator $I_{[x,y]}$. But the two mixed states are not the same.

A state is uniquely specified if and only if we give the expectation values of all observables in that state — equivalently, the probabilities of all measurement outcomes. Only for the pure states does that reduce to spelling out the certainties.

2. The Modal Account Developed

We know how states are described and represented in quantum mechanics — what about events such as outcomes? They are described by statements of form

Observable B pertaining to system X (actually) has value b

and the Born interpretation rule sometimes tells us the probability of this event actually occurring (this statement actually being true) at the end of an appropriate measurement.

At the end of a von Neumann measurement of non-degenerate observable A on system X by apparatus Y with pointer-observable B , we have the following description of total and reduced states:

- (a) X is in state $W_X = \sum c_i^2 I_{|a(i)\rangle}$
- (b) Y is in state $W_Y = \sum c_i^2 I_{|b(i)\rangle}$
- (c) $X + Y$ is in state $\Psi_t = \sum c_i |a_i\rangle \otimes |b_i\rangle$

Now the Born Rule, given our proposed modal alternative to von Neumann's interpretation rule, leads here to

(d) For some index k , pointer-observable B has value b_k ; and the probability that this index k is the index i , equals c_i^2 .

It is clear that (d) only pertains to the end of an A -measurement and the statements attributing values to B . Could this be a consequence of a larger principle which connects states more generally with values of observables? In general, which value-attributions are true?

The response to this question can be very conservative or very liberal. I take it that the Copenhagen interpretation—really, a roughly correlated set of attitudes expressed by members of the Copenhagen school, and not a precise interpretation—introduced great conservatism in this respect. Copenhagen scientists appeared to doubt or deny that observables even have values, unless their state forces us to say so. I shall accordingly refer to the following very cautious answer as the Copenhagen variant of the modal interpretation (briefly: CVMI). It is the variant I prefer.

The CVMI Principles

This interpretation says that, if system X has dynamic state W at t , then the state-attributions $[M, E]$ which are true are those such that $\text{Tr}(WI_E^M) = 1$. About the value-attributions, it says that they cannot be deduced from the dynamic state, but are constrained in three ways:

- (i) EE HALF LINK: If $[M, E]$ is true then so is the value-attribution $\langle M, E \rangle$: observable M has value in E :
- (ii) JOINT DISPLAYABILITY: All the true value-attributions could have Born probability 1 together:
- (iii) MAXIMALITY: The set of true value-attributions is maximal with respect to feature (ii).

Let us use the following terminology: W makes $[M, E]$ true exactly if $\text{Tr}(WI_E^M) = 1$. What can we deduce now?

- We already have a 'bookkeeping device' to identify the set of true value-attributions. Clauses (ii) and (iii) tell us what it is. Call this set S .
- Here (ii) tells us there must be dynamic state W such that $\langle M, E \rangle$ is in S only if W makes $[M, E]$ true.
- Adding (iii), we see that W is pure, that $\langle M, E \rangle$ is in S if and only if W makes $[M, E]$ true, and that W is unique.
- Finally, (i) tells us that W is possible relative to W , in a sense that corresponds in the mathematical representation to non-orthogonality.

That pure dynamic state W is the bookkeeping device which identifies the true value-attributions correctly. Hence it can be used to represent the value state. That does not mean that value states are dynamic states, but only that each admits the same sort of mathematical representation.

We can sum all this up in a single Postulate, which describes our family of models for physical situations governed by quantum mechanics:

(e) Given that system X is in state W at time t , then for all observables M pertaining to X :

(e1) a state-attribution $[M, E]$ is true if and only if W makes it true;

(e2) there is a certain pure state x which is possible relative to W , and the value-attribution $\langle M, E \rangle$ is true if and only if x makes $[M, E]$ true.

The first consequence we deduce from this postulate is that

(i)-(iii) hold, as required.

The second consequence concerns the identification of observables. My terminology and notation 'Hermitean operator M represents observable m ' allow for the possibility that two distinct observables can be represented by the same operator. If that could happen, principle (e) above would have to be read as applying to all observables represented by given operator M . But so read, it implies that, if two observables are represented by M , it is not only their measurement outcome probabilities that are always the same, but also their values. This is because of the 'if and only if' in (e2). Hence there is then no difference at all between these observables; they are the same:

Identity of Observables: If observables m and m' are represented by the same Hermitean operator, then $m = m'$.

It is not a tautology! It does give us the advantage that we may use the same name now for an observable and its representing operator.

Third consequence: in a certain sense, it is as if the ignorance interpretation of mixtures were correct. For if system X is in mixed state W , then the actual values of observables pertaining to X are exactly those it would have had if it had been in a pure state in the image space of W . But we don't know which pure state.

The fourth consequence is the rejection of the Classical Principle, that each observable always has one of its possible sharp values. We can derivatively attribute 'unsharp' values. To observable M there correspond a large set of observables I_E^M , as we know, having eigenvalues 1 and 0 only. Now we have the equation for the Born probability: $P_W^M(E) = 1$ if and only if $\text{Tr}(WI_E^M) = 1$. Therefore we have the following result concerning state-attributing propositions:

$$[M, E] = [I_E^M, 1]$$

The Copenhagen variant of the modal interpretation entails now, because of its adoption of (e2)—or, equivalently, of (i)-(iii)—that, similarly,

$$\langle ME \rangle = \langle I_E^M, 1 \rangle$$

That also means that $\langle M, E \rangle$ is not the classical disjunction of the value attributions

$\langle M, r \rangle : r \in E$. Indeed, we should note:¹

If $\langle I_E^M, 1 \rangle$ is true if and only if Borel set $E_0 \subseteq E$, then E_0 is also the smallest Borel set such that $\langle M, E_0 \rangle$ is true

and we should say that M has unsharp value E_0 .

We can therefore distinguish, for value-attributing propositions, the principle of Excluded Middle ($\langle M, R \rangle$ is true for every observable M), which is correct, and that of Bivalence (either $\langle M, E \rangle$ is true or $\langle M, R - E \rangle$ is true), which is false.

3. What Happens in a Measurement?

Officially, quantum mechanics allows for only one way to assign probabilities—via Born's Rule. We can interpret this to extend to all processes, even on the microscopic level in the ionosphere, which meet the minimal requirements for a physical correlate of measurement. Can we go a little further?

To spell out in detail what happens in measurement, and how the Born probabilities as interpreted in (d) are in accordance with the large principle (e), we must become a little more precise.

The situation of system X at given time t is characterized according to (e) by two states: its dynamic state W and its value state x . What does this look like when X is a compound system?² We need to characterize the situation for it and also for its components. So if $Z = X + Y$, we have both a dynamic state and a value state for each of Z, X, Y . I shall here discuss only the case in which Z has a pure dynamical state Ψ . Let us designate the mixed states assigned to X and Y , by 'reduction of the density-matrix' as $\#^\Psi$ and $\Psi^\#$. Then the situation is this:

(f) $(X+Y), X,$ and Y have dynamic states $\Psi, \#^\Psi, \Psi^\#$ respectively

(g) $(X+Y), X,$ and Y have as value states Ψ, x, y respectively, where x is possible relative to $\#^\Psi$ and y possible relative to $\Psi^\#$.

We cannot specify what x and y are: there are a number of possibilities. We only know that they must be possible relative to $\#^\Psi$ and $\Psi^\#$. This means mathematically that they are vectors in the image spaces of $\#^\Psi$ and $\Psi^\#$.

How about a probability distribution on these possibilities? The non-unique decomposability of mixtures stands in the way of a general rule for probability assignments which is at once simple and consistent. But the Born Rule is meant to assign probabilities coherently if the situation comes at the end of a measurement. The rule presupposes that measurement is a process which is so structured that it singles out certain observables, and tells us the probabilities for their possible values—without the inconsistencies that plague the ignorance interpretation. That means of course that the observables singled out are mutually compatible -- there is a 'privileged basis' then.

¹ If M and M' are incompatible observables, which have no eigenvectors in common, and $\langle M, s \rangle$ is true, then $\langle M, r \rangle$ is not true for any value r . Yet $\langle M, R \rangle$ is still true, because it just means $\langle I_R, 1 \rangle$ is true, and that is a tautology (R the set of all real numbers.)

² A full treatment of compound systems requires adding a further constraint on the value states. See QM book Ch. 9 section 8, and next footnote

The major recognized classes of measurement in the literature satisfy this 'metacriterion'. When that metacriterion is met, reasoning of the following form:

1. Process PP was a measurement of observable A with B designated as pointer-observable, and also an A-measurement with B designated as pointer-observables, and . . .

2. The probability that B had value a at the end equals P_a , for $a = a_1, a_2, \dots$; the probability that B had value a at the end equals p_a , for $a = a_1, a_2, \dots$; . . . in which statement 2 follows from 1 by Born's Rule, never leads to an incoherent probability assignment.

Well, how could it anyway? Easily enough, if we respect the functional relations among observables. That is the message of the 'no hidden variables' theorems. If the above sort of reasoning is continued with, i.e.

3. $B = f(B)$ and $B = g(B)$ and . . . So the probability that B had a value in E equals the probability that B had a value in $f^{-1}(E)$, the probability that B had a value in F equals the probability that B had a value in $g^{-1}(F)$, and . . .

then those theorems tell us that 1-3 will lead to incoherence (i.e. to inconsistency with the classical probability calculus) unless all those observables are mutually compatible.

The metacriterion thus imposed on measurement implies that, if a process measures various observables on an object system jointly, then it ends with the apparatus in a mixture of joint eigenstates of the pointer-observables. It is to these, in the first instance, that the Born Rule assigns probabilities. As usual, the paradigm illustration is the von Neumann measurement of an observable with non-degenerate spectrum. For this we can consistently add to our previous principles:

(h) If the situation described in (f) and (g) is at the end of an A-measurement with B designated as pointer-observable, and with $\Psi = \sum c_i |a_i\rangle \otimes |b_i\rangle$, then the probability is $|c_k|^2$ that $y = |b_k\rangle$

This has Born's Rule translated into our present representation of this physical situation, as corollary, since $y = |b_k\rangle$ exactly if B has value b_k . It is consistent with the preceding because x and y do, as they must, lie in the image spaces of $\# \Psi$ and $\Psi \#$. But of course, a von Neumann measurement has a special feature. It does not just allow statistical inference backward to the initial state of the object system. In addition, it effects a perfect correlation between measurement and pointer-observables. Hence we can go one step further, and add:

(h) If the situation described in (f) and (g) is at the end of an A-measurement with B designated as pointer-observable, and with $\Psi = \sum c_i |a_i\rangle \otimes |b_i\rangle$, then the probability equals $|c_k|^2$ that both $y = |b_k\rangle$ and $x = |a_k\rangle$.

The extension of (h) to (h') is important, because it leads immediately to the result that a pointer-reading result b_k implies with certainty that observable A had value a_k at that point:³

- (i) If the situation described in (g) and (h) is the end of an A -measurement with $\Phi = \sum_i c_i |a_i\rangle \otimes |b_i\rangle$, then the probability that value-attribution $\langle A, a_k \rangle$ is true, given that value-attribution $\langle B, b_k \rangle$ is true, equals 1.

We are tempted to exclaim: it is as if the Projection Postulate were correct. For at the end of the measurement of A on system X , it is indeed true that A has the actual value which is the measurement outcome. But, of course, the Projection Postulate is not really correct: there has been a transition from possible to actual value, so what it entailed about values of observables is correct, but that is all. There has been no acausal state transition.

4. Puzzle: How Far Does Holism Go?

There are three puzzling features in the CVMI. The first is the possibility of observables taking unsharp values, the failure of Bivalence. The second concerns what happens when no measurement is going on, and I shall discuss that in the next section. The third is the way in which the holism of quantum states extends also to observables and their values.

Recall Identity of Observables: if observables are represented by the same Hermitean operator, they are one and the same observable. Given the relations the theory imposes between states, observables, and measurement probabilities we can restate this:

1. Identity of Observables (second version): If the probabilities for measurement outcomes for observables A and B are the same for every dynamic state of any system to which both A and B pertain, then $A = B$.

Turning now to a reflection on holism: what does statement 1 entail for compound systems? [Note well: complete specifications of the CVMI for compound systems are not included here]

The reduced state $\#W$ assigned to compound X of system $X + Y$ in state W is determined by definition through the equation⁴

$\#W$ is the state such that $\text{Tr}(A\#W) = \text{Tr}((A \otimes I)W)$ for all observables A pertaining to X .

What about A and $A \otimes I$? We tend to think of them as 'essentially' the same, since it seems to be choice whether to model X 's behaviour by itself or as part of a larger system.

³ The proof of this is simple. The probabilities of the combinations ($x = |a_i\rangle$ and $y = |b_j\rangle$) sum to 1, therefore the probability of any combination ($x = |a_i\rangle$ and $y = |b_j\rangle$) with $i \neq j$ must equal 0. Notice also, however, that a possibility which has probability 0 may still really occur. Probability 0 does not imply impossibility. The addition of (h) and (h') to our interpretation assigns probabilities; it does nothing else.

⁴ Let H_X and H_Y be the Hilbert spaces for X and Y respectively, so $H_X \otimes H_Y$ is the Hilbert space for $(X + Y)$. The quantification 'for all A ' is therefore over all the Hermitean operators on H_X , and I is the identity operator, in H_Y .

But Identity of Observables does not apply to A and $A \otimes I$ at all, since they do not pertain to the same system. There is certainly a very intimate relation between them, in the Born probabilities:

The probabilities of measurement outcomes for A and for $A \otimes I$, conditional on their measurement on system X and system $X + Y$ respectively, are the same.

But that is not the same equivalence relationship as discussed in Identity of Observables. The importance of this point appears in the more general context of how joint probabilities for A and B , pertaining to X and Y , are related to those for $A \otimes B$. The first thing to notice is that there is in general not a one-to-one correspondence between the outcomes:

$$(A \otimes B)(|a\rangle \otimes |b\rangle) = ab(|a\rangle \otimes |b\rangle)$$

for ab does not in general stand in one-to-one correspondence to the pair $\langle a, b \rangle$.

Nor do the predictions of measurement outcomes taken separately relate very closely. The probabilities of outcomes a and b , for measurements on A and B on the two components of $X + Y$ in state $\sum_{ab} c_{ab} |a\rangle \otimes |b\rangle$, do not tell us what the probability of outcome ab for a measurement of $A \otimes B$ is-not even if the correspondence $\langle a, b \rangle \rightarrow ab$ is unique (i.e. not even if $ab = a'b'$ only if $a = a'$ and $b = b'$). For the cross-reference in the coefficients c_{ab} determines the correlation, which cannot be read off from the two marginal probabilities. Quite independent of the details of the modal interpretation, we must therefore beware of fudging the distinctions between such observables as A, B on the one hand, and $A \otimes B, A \otimes I$, etc., on the other.

To see the holism of states reappear as holism of values, consider the question:

2. Is there an observable $A \& B$, definable by the equivalence: $A \& B$ has value a_{km} if and only if A has value a_k and B has value b_m ?

Here $k, m \rightarrow a_{km}$ must be a unique correspondence; probative is the case when A and B pertain to different systems X and Y , so that $A \otimes B$ pertains to $X + Y$. There we have

3. The probability of an outcome in E for a measurement of $A \otimes B$ equals the probability of outcomes a_k and b_m such that a_{km} is in E , for joint measurement of A and of B .

Putting 1 and 3 together, we must conclude that, if there is such an observable as $A \& B$ then $A \& B = A \otimes B$.

This shows that there is no such observable at all!⁵ Question 2 must therefore be answered: in general, no! Of course there is no similar obstacle to the principle that $A \otimes B$ has value a_{km} if and only if $A \otimes I$ has value a_k and $I \otimes B$ has value b_m ; for in our example, none of the three has a sharp value at all.

⁵ For in the case in which $X + Y$ is in correlated pure state $\Psi = \sum_i c_i |a_i\rangle \otimes |b_i\rangle$, for example, it is quite possible that A and B have sharp values. But since $X + Y$ has a pure dynamic state, its value state is the same. Hence if several c_i^2 are between 0 and 1 then $A \otimes B$ has no sharp value.

How could A and $A \otimes I$ have different values? When we translate our puzzlement into a truly empirical question, it disappears. To see this, suppose that A and $A \otimes I$ are both measured on systems X and $(X + Y)$ respectively. This entails that they are part of a large system $(X + Y + Z + W + V + \dots)$. It may be that Z measures A on X and W measures $A \otimes I$ in $(X + Y)$, or perhaps Y itself measures A on X . The question is: will the outcomes of the measurements agree? To get a probability for that, we must assume that a further measurement is being made, by V say, on that state of $(X + Y + Z + W)$ to see if there is such agreement. My assertion is that the theory predicts that this last measurement will have outcome yes with probability 1.

To keep the discussion simple, suppose that X is a particle in state

$$x = (1/\sqrt{2})(|+\rangle_A + |-\rangle_A)$$

that Y is an A -measurement apparatus with groundstate y and pointer observable B :

$$\begin{aligned} |+\rangle_A \otimes y &\rightarrow |+\rangle_A \otimes |+\rangle_B \\ |-\rangle_A \otimes y &\rightarrow |-\rangle_A \otimes |-\rangle_B \end{aligned}$$

and that Z is a measurement apparatus with groundstate z and pointer-observable C . The eigenvalues of C are 1 to register agreement and 0 to register disagreement:

$$|i\rangle_A \otimes |j\rangle_B \otimes z \rightarrow |i\rangle_A \otimes |j\rangle_B \otimes |\delta_{ij}\rangle_C$$

and of course in all cases the evolution is linear. Now we see what happened:

$$\begin{aligned} x \otimes y \otimes z &\rightarrow (1/\sqrt{2})[|+\rangle_A \otimes |+\rangle_B + |-\rangle_A \otimes |-\rangle_B] \otimes z \\ &= (1/\sqrt{2})[|+\rangle_A \otimes |+\rangle_B \otimes z + |-\rangle_A \otimes |-\rangle_B \otimes z] \\ &\rightarrow (1/\sqrt{2})[|+\rangle_A \otimes |+\rangle_B \otimes |1\rangle_C \\ &\quad + |-\rangle_A \otimes |-\rangle_B \otimes |1\rangle_C] \\ &= (1/\sqrt{2})[|+\rangle_A \otimes |+\rangle_B + |-\rangle_A \otimes |-\rangle_B] \otimes |1\rangle_C \end{aligned}$$

in other words, the answer will be yes, agreement with probability 1.

The strange holism which allows A and $A \otimes I$ to take different values in our models therefore 'shows up' only outside measurement contexts—in other words, it has probability 0 of ever really showing up. That is why we can in practice ignore the difference between A and $A \otimes I$.

5. Puzzle: Is There Chaos Behind the Regularities?

Quantum mechanics gives us probabilities only for measurement outcomes, conditional on the hypothesis that a measurement is made. We are still left with the question: what can happen when no measurement is being made?

The answer on our present interpretation is: anything is possible [within the bounds of the distinctive Copenhagen principles above].

Things can happen in actuality which have zero probability of ever appearing as measurement outcomes. Something which has Born probability 1 under some circumstances need not have Born probability 1 under any circumstances realized in any one particular system or process. A particular dynamic state may have correlations in it which are not to be found in all dynamic states of which the system is capable.

The probabilities provided are conditional on measurement; what is more, the probabilities evolve—embodied in the quantum-mechanical dynamic states—without 'feedback' from the actual values.⁶

If we study the temporal evolution of a wave function of a compound system in quantum mechanics, certain episodes qualify as the measurement of some observable on one part of the system by another part. There the Born Rule gives us probabilities for events. The total history of the system then consists of two parts:

- (a) the aforementioned, temporally evolving, quantum-mechanical state, and
- (b) the sequence of actual outcome events.

The latter could logically speaking be any sequence of possible measured values of the observables in question—but, of course, we have an induced probability measure on those sequences. So far we have a parallel.

Nevertheless, we may here suspect a problem for the CVMI. What if a die or its tossing device had been so constructed that each time a face came up it was more likely to come up again later? Then probability-evolution and outcome-sequence would be interdependent. Should we not think analogously that in quantum physics the process of interaction designated as measurement changes the probabilities? What, for instance, if I

⁶ I think we should compare this carefully with what could happen in a classical statistical theory. Here is a classical example of evolving probability. I toss a die repeatedly; the die is fair to begin, but it is so constructed that with each toss it becomes more biased in a certain way.

Let $p[n, i]$ be the probability that face 'i' comes up in trial n.

Suppose $p[1, i] = \frac{1}{6}$ for $i = 1, \dots, 6$, but

$$p[(n+1), i] = ip[n, i] / T(n)$$

where $T(n)$ is the needed normalization factor (so that the sum of probabilities remains equal to 1). Now we have two descriptions of this world: first, the deterministic development of that probability distribution, and second, the actual series of toss outcomes. Logically speaking, the latter can be any sequence of integers between 1 and 6. Note that this class of possible outcome sequences has an induced probability measure—roughly speaking, the higher-numbered faces become continually more likely to turn up. The 'mechanics' however resides solely in the first part of the description, the temporal evolution of the probabilities.

am the measurement apparatus, and I decide beforehand that if I detect a red light I shall shoot the piano player? Does it not follow then the occurrence of an actual value (and not just the total quantum-mechanical state) affects the probability of what happens next?

There is a fallacy here. The interaction affects the measured system if we think of it by itself, apart from the total isolated system of which it is part. But if we have before us the description of the total evolution of the state of the entire system, then the description of some internal episode as a measurement adds nothing—it is simply a classification.

If we want to reflect on, for example, von Neumann's graphic 'immediate repetition' demand, we do it as follows. We ask: what about the total evolution of a system in which a certain episode qualifies as an immediate succession of two measurements of the same observable, plus a measurement which checks on whether the two outcomes agreed? What are the probabilistic predictions for outcomes of that? To get the answer, we look again to the wave function of a total system in which that happens, and the Born Rule allows us to deduce: we can expect the outcome the two agreed! with probabilistic certainty.⁷

It is exactly at this point that our intuitions tend to declare war on our deductions. 'We don't want to know', they insist, 'what probabilities are derived for the outcome of a third measurement, if performed—we want to know purely and simply whether, if the first two measurements occur, their outcomes do agree!'

But this question has a presupposition: namely that quantum mechanics, if it is to be complete and accurate, should give us such information. It is assumed that the theory must give us more than the probabilities of measurement outcomes. This assumption does not come from physics. If it had some independent justification of its own, it would be a desideratum which might be imposed as a criterion of adequacy for interpretation. But it does not.

Should we not expect a very chaotic life if the measurement outcomes' probabilities are set by the evolving state, while the outcomes do not provide a 'feedback' input to that evolution? E.g. in the case of Schroedinger's Cat, the CVMI is logically compatible with the cat being sometimes alive and sometimes dead, at times between the triggering of the device and the opening of the box.

This is only a more complicated version of earlier points about repeated measurement. Here we have a correlation over time; we could ask a similar question about a simultaneous correlation. In Aspect's experiment, each photon has a 50 per cent probability of passing if the filters' orientation is parallel: why should there be a correlation observed? The answer is the same to both questions: given the total state of the system, the uncorrelated outcome combinations are given a zero or negligible probability.

⁷ Leeds and Healey, 1996 argues that the CVMI does not do justice to immediately repeated non-disturbing measurements. I show that the objection is mistaken because they ignore the constraints on value states in compound systems, in "Modal interpretation of repeated measurement: reply to Leeds and Healey", *Philosophy of Science* 64 (1997), 669-676.