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# THE LOGIC OF CONDITIONAL OBLIGATION

Various paradoxes in deontic logic have led to the introduction of concepts of conditional obligation. The aim of this paper is to develop a semantic theory of conditional obligation, a complete logical system pertaining thereto, and a translation into modal logic analogous to that provided by Anderson for normal deontic logics.

## I. ABSOLUTE OBLIGATIONS

Accepting the general obligation to bring about whatever ought to be the case, and the thesis that what ought to be the case is exactly what is the case in any ideal situation, deontic logicians have devised a minimal deontic logic variously called *D*, *DL* (Åqvist)[1], or *DM* (Fitch)[2]. In axiomatic form, it has (with *O* read as "it ought to be the case that").

| | |
|---|---|
| A1 | Axiom schemata for propositional calculus |
| A2 | $\vdash O(A) \supset \sim O(\sim A)$ |
| A3 | $\vdash O(A \supset B) \supset . \, O(A) \supset O(B)$ |
| R1 | If $\vdash A$ and $\vdash A \supset B$ then $\vdash B$ |
| R2 | If $\vdash A$ then $\vdash O(A)$ |

It is not surprising that this simple system allows the formalization of only a narrow fragment of discourse concering duties and obligations.

Of the various problems not handled, perhaps the most important is that of contrary-to-duty imperatives.[3] Granted that one ought not to steal, the obligation to make restitution if one does steal is one that arises when another obligation has been violated. But within deontic logic constructed along the lines indicated above, statements of such obligations are not adequately formulable.

For $O(\sim A)$ implies $O(A \supset B)$ no matter what $B$ is, and $A \supset O(B)$ contradicts the conjuction of $O(\sim A)$, $A$, and $O(\sim B)$.

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## II. CONDITIONAL OBLIGATIONS

In answer to problems of the kind mentioned above, von Wright proposed that the concept of an obligation to do something *given* certain conditions is not definable in terms of a concept of obligation *simpliciter*.[4] He proposed as minimal criterion for a logic of conditional obligations that absolute obligations should be definable as obligations conditional on tautologous conditions. That is, if we introduce the dyadic operator $O$, regarding $O(A/B)$ as stating that under conditions satisfying $B$, $A$ ought to be satisfied, the monadic operator of system $D$ should be definable by the equivalence $O(A) = O(A/B \supset B)$.
In the same article, von Wright suggested two axiom schemes for conditional obligations. He stated these in terms of what is permitted rather than what is obligatory. If we follow the usual course of defining "it is permitted that..." as "it is not obligatory that not..." these axiom schemes are

(1) $\vdash O(A/C) \supset \sim O(\sim A/C)$
(2) $\vdash O(A \vee B/C) \equiv O(A/C) \& O(B/C \& \sim A)$

In restating these axioms I have assumed not only the usual definition of permission but also the rule that if $A$ and $B$ are provably equivalent, then so are $O(A/C)$ and $O(B/C)$. From the article it is actually not clear what von Wright assumed concerning the logical apparatus beyond his axiom schemes.

If von Wright assumed the obvious and minimal generalization of A3 and R2, namely

R2' If $\vdash A \supset B$ then $\vdash O(A/C) \supset O(B/C)$

then we can deduce the following theorem

T $\vdash O(A/C) \supset O(A/C \& \sim B)$

(For $A \vdash B \vee A$, so $O(A/C) \vdash O(B \vee A/C)$. But by (2.) $O(B \vee A/C) \vdash O(A/C \& \sim B)$.). Now this consequence is made unacceptable by the problem of obligations overridden by new circumstances.

This problem was raised in an example that Powers constructed *à propos* a system due to Åqvist.[5] I quote a short formulation: Powers "gives the example of John Doe and Suzy Mae who violated a primary obligation. Due to the violation of this primary obligation a secondary obligation takes over, that of John marrying Suzy Mae. This is not all because John has violated another primary obligation by shooting Suzy Mae...so John cannot marry Suzy. Hence he does not have a secondary obligation to marry Suzy".[6] The main point is that, although given conditions $C$, it ought to be the case that $A$, conditions $C \& B$ may make it quite impossible for $A$ to be the case.



In a later work,[7] von Wright suggests as a 'natural' axiom scheme for conditional obligation (in effect):

$$(3) \quad \vdash O(A \& B/C \vee D) \supset O(A/C) \& O(B/C) \& O(A/D) \& O(B/D)$$

But assuming what von Wright calls a "rule of extensionality", that sentences provably equivalent in the propositional calculus may be substituted for each other everywhere, T can be deduced again. (For (3.) yields $\vdash O(A/(C \& B) \vee (C \& \sim B)) \supset O(A/C \& \sim B)$. and $(C \& B) \vee (C \& \sim B)$ is tautologously equivalent to $C$.). Thus it seems that von Wright's proposals for the logic of conditional obligation run afoul of the John and Suzy paradox, if R2' be accepted. (And the John and Suzy example is but one of a large family: questions should be answered truthfully, but not if a truthful answer will help to make a crime succeed; the 'everything else being equal' clause that tacitly accompanies statements of conditional obligation cannot be removed.) But R2' is not easily given up.[8]

To argue for the retention of R2', I shall outline an interpretation of $O(A/B)$ suggested by, but rather wider than, the interpretations considered informally by Powers. In the interpretation of statements of absolute obligation we use the following picture: a certain set of possible worlds is specified as ideal, and $O(A)$ is true in the actual world exactly if $A$ is true in all ideal worlds. We can regard $O(A)$ as playing a role in the evaluation of our world, ("Stealing ought not to happen, but it does; that is bad.") or in decision making ("our decisions realize various possible states for the world tomorrow; aim to produce an ideal state."). Staying with the second of these, we can liberalize the picture: with respect to tomorrow our choice is not merely to actualize an ideal state or a non-ideal state, but to actualize a more or less ideal state. That is, each possible outcome of our decisions today has a certain value. Now suppose that due to facts beyond our control or prior decisions, C will be the case tomorrow. Then we can only aim for higher values within the set of states that satisfy $C$. And so $O(A/C)$ presumably means something like "Given $C$, to maximize value requires that $A$".

I said "something like", for we have to consider various possibilities. Let us designate the set of attainable states satisfying a given sentence $S$ as $H(S)$. Then if there is a maximum among the values of states in $H(C)$, $O(A/C)$ is true just if that maximum lies in $H(A \;\&\; C)$. If there is no maximum, then we should compare $H(A \;\&\; C)$ with $H(\sim A \;\&\; C)$. If for every value in $H(A \;\&\; C)$ there is one at least as high in $H(\sim A \;\&\; C)$, $O(A/C)$ is *not* true. More concisely, $O(A/C)$ is true if some state or world in $H(A \;\&\; C)$ has a value higher than any to be found in $H(\sim A \;\&\; C)$. To put it still another way: $O(A/B)$ is true exactly if opting for $\sim A \;\&\; B$ precludes the attainment of some value which it is possible to attain if one opts for $A \;\&\; B$. But does this not ignore the problem of likelihood? Is gambling the most moral of pursuits if breaking the bank makes possible unrivalled philanthropy? I don't mean that of course. In assigning values to possible outcomes relative likelihood must be taken into account; this is an old theme of decision theory. And indeed, an old theme of morals: the gambler who loses his wages is culpable vis-à-vis his dependents even if all his winnings would have been spent to their benefit.

Before examining the John and Suzy paradox anew, let us scrutinize the relation between 'ought' and 'better', as Åqvist did in connection with absolute obligations.[9] The set of outcomes that satisfy $A$ is *better* than the set that satisfy $B$ if some element of the former has a value higher than any fount in the latter; symbolically, $B(A/B)$. But then $O(A/B)$ is, by our account, exactly equivalent to $B(A \;\&\; B/\sim A \;\&\; B)$. Now the von Wright theorem that runs afoul of this paradox is $O(A/B) \supset O(A/B \;\&\; C)$. This is then exactly equivalent to $B(A \;\&\; B/\sim A \;\&\; B) \supset B(A \;\&\; B \;\&\; C/\sim A \;\&\; \&\; B \;\&\; C)$. But that is easily refuted. For example, if $H(A \;\&\; B \;\&\; C)$ is empty, it cannot be better than anything. Or, more mundanely, if somewhere in $H(A \;\&\; B)$ we see a high value, that value might nevertheless not lie in $H(A \;\&\; B \;\&\; C)$. (There is an easy procedure for checking this: draw a Venn diagram and write number variables in the compartments.)[10]

## III. CRITERIA FOR A LOGIC OF CONDITIONAL OBLIGATIONS

The criterion proposed by von Wright is that in the logic of conditional obligations, the monadic operator $O$ defined by $O(A) \equiv O(A/B \supset B)$ should satisfy system $D$.[11]



I wish to strengthen this criterion: we should be able to demonstrate that if a sentence $B$ is added as an axiom, then in the extended system the monadic operator $O^B$ defined by $O^B(A) \equiv O(A/B)$ should satisfy system $D$. After all, in that extended system, $B$ has the status of $B \supset B$.

As a further criterion, I propose that if something is a necessary condition of discharging an obligation then it is itself an obligation, given the same conditions. This is clearly the rule discussed in the preceding section.

Together these criteria leave much undetermined, since they say nothing about how different conditions are related. Before going on to that problem, let us state the system $CD$ – as we shall call the logic of conditional obligations – to the extent that it can now be determined.

| | |
|---|---|
| AC 1 | Axiom schemata for propositional logic. |
| AC 2 | $\vdash O(A/C) \supset \sim O(\sim A/C)$ |
| AC 3 | $\vdash O(A \supset B/C) \supset . O(A/C) \supset O(B/C)$ |
| RC 1 | if $\vdash A$ and $\vdash A \supset B$, then $\vdash B$ |
| RC 2 | if $\vdash A \supset B$ then $\vdash O(A/C) \supset O(B/C)$ |
| RC 3 | if $\vdash A$ then $\vdash O(A/A)$ |

Obviously, without rule RC 3 the assumption that $\vdash B$ leads only to $\vdash O(B/B) \supset O(A \supset A/B)$; and to satisfy our criteria, we must be able to prove $O^B(A \supset A)$ on the assumption that $\vdash B$. All of AC 1–3 and RC 1–2 are directly demanded by our criteria, and RC 3 is a minimal addition to guarantee that the criteria are entirely satisfied.

There is a rudimentary semantic criterion that yields another axiom and rule. The intuitive meaning of "given $A$" is such that, if a sentence ends with it, then any possibility that does not satisfy $A$ is irrelevant to the evaluation of that sentence. Thus, the evaluation of $O(A/B)$ cannot depend on $H(A)$ as such but at most on $H(A \;\&\; B)$. Succinctly: there must be a relation $R$ such that $O(A/B)$ is true exactly if $H(A \;\&\; B)$ bears $R$ to $H(B)$. This criterion seems to me to be largely independent of our special interpretation, and does not require acceptance of the axiological slant of our current approach. But it entails directly the necessity for the following additions to the logical system.

| | |
|---|---|
| AC 4 | $\vdash O(B/A) \supset O(B \;\&\; A/A)$ |
| RC 4 | If $\vdash C \equiv D$ then $\vdash O(A/C) \equiv O(A/D)$ |

We require, finally, a set of axioms that do reflect that axiological slant. Recall the relation *better*: we said that $O(A/B)$ is true exactly if $B(A \& B/\sim A \& B)$ is. With very little ingenuity (which I shall take pains to display when I formalize the semantic account) it can be shown that $B(A/B)$ is in turn equivalent to $O(\sim B/A \vee B)$. So we add a definition to this effect, and then axioms which will ensure that 'better' has all its intuitively rightful properties (such as transitivity):

*Definition* '$B(A/B)$' for '$O(\sim B/A \vee B)$'

AC 5 $\vdash B(A/B) \supset [B(B/C) \supset B(A/C)]$

AC 6 $\vdash \sim B(A/B) \supset [B(A/C) \supset B(B/C)]$

AC 7 $\vdash \sim B(A/B) \supset [B(C/B) \supset B(C/A)]$

For future reference, we list some theorems.

T 1 $\vdash O(A \vee \sim B/B) \equiv O(A/B)$

For $O(A/B) \supset O(A \vee \sim B/B)$ by RC 2; $O(A \vee \sim B/B)$ implies $O[(A \vee \sim B) \& B/B]$ by AC 4 and hence $O(A/B)$ by RC 2.

T 2 $\vdash O(A/C) \& O(B/C) . \supset O(A \& B/C)$

For $O(B/C) \supset O(A \supset A \& B/C)$ by RC 2, which together with $O(A/C)$ implies $O(A \& B/C)$ by AC 3.

T 3 $\vdash \sim O(\sim A/A)$

For suppose $O(\sim A/A)$; then $O(A \& \sim A/A)$ by AC 4. But then $O(A/A)$ by RC 2 and hence $\sim O(\sim A/A)$ by AC 2.

T 4 $\vdash O(A/B) \equiv B(A \& B/\sim A \& B)$

For $B(A \& B/\sim A \& B) \equiv O(A \vee \sim B/B) \equiv O(A/B)$ by T 1.

## IV. PRACTICAL ACTION AND THE PARADOXES

Suppose that one considers what is to be done, with an eye on the moral values of the possible outcome of one's actions. Then if one knows that the actual outcome must satisfy $C$, and that $O(B/C)$ is true, ought one to follow a course of action leading to an outcome that satisfies $B$? The answer is "no, not necessarily"; for example one may know as well that courses of action leading to outcomes satisfying $B$ are not possible. This is clearly the lesson of the John and Suzy paradox.



We have a problem here analogous to the problem of detachment for conditional probabilities. And I propose that the former be solved analogous to the solution proposed by Carnap for the latter: by separating the principles for the application of the calculus from the principles of the calculus, and imposing a 'total evidence' requirement. Thus suppose that what we know will be the case tomorrow regardless of our actions can be summed up exactly in statement $A$. Then, if $O(B/A)$ is true, it is to be accepted that we ought to follow a course of action that leads to an outcome satisfying $B$ – or at least that we ought to try. Our calculus makes this maxim consistent, since $O(B/A) \& O(\sim B/A)$ cannot be deduced. The maxim is not helpful to one whose knowledge cannot be finitely axiomatized in his own language, but such a person would in any case be well advised to switch to a language with greater resources.

Thus we distinguish between practical judgments (injunctions or mandates) and theoretical judgments of obligation. The question what is to be done may be answered by the practical judgment that $X$ ought to be done. But this practical judgment does not state an unconditional obligation; it is warranted or justified by a theoretical judgment that it ought to be that $X$ be done, given the known conditions. This theoretical judgment alone can be expressed in our language. And every theoretical judgment carries a *ceteris paribus* rider. "Thou shalt not kill" either states an unconditional obligation or is a practical judgment warranted by one. The statement of unconditional obligation can be expressed by "It is (morally) better not to kill than it is to kill (*ceteris paribus*)". This leaves it open that under certain special conditions (defense of one's virtue, say) it is morally justified to kill. Unconditional statements of obligation, like Aristotle's universal statements, are normally subject to exceptions.

As Åqvist already pointed out, the Good Samaritan paradox is a special case of the problem of contrary-to-duty imperatives.[12] Given that a man has been robbed, we are obligated to help a man who has been robbed; but simpliciter, it is better that there be no man who has been robbed. But I would like to consider briefly the use of this paradox by Castañeda to critize a certain principle (that looks somewhat like our RC 2).[13] Suppose that Robert is the man whom Benjamin robs, and that Arthur is obligated to bandage Robert. Then is ought to be that Arthur bandage the man Benjamin robs. But that Arthur bandages the man Benjamin robs implies that Benjamin robs some one. Hence it ought to be that Benjamin rob someone. This inference must clearly be rejected.

Prima facie, what is at fault is that $D$ has both A 3 and R 2, which together yield that if $\vdash A \supset B$ then $\vdash O(A) \supset O(B)$. And if this is indeed the exact location of the fallacy, then our RC 2 would also be impugned. But I think that rather, Castañeda's example shows again that certain inference patterns cannot be adequately represented in $D$ (to which, and like calculi, his criticism was addressed). Let us formalize the argument, using the obvious symbols:

(1) $r = (\imath x)(Rbx)$
(2) $O(Bar)$
(3) $O(Ba(\imath x)(Rbx))$
(4) $\vdash Ba(\imath x)(Rbx) \supset (Ex)(Rbx)$
(5) $O((Ex)Rbx)$.

The move from (1) and (2) to (3) by substitutivity of identity is not warranted in $D$, in which (1), $Bar$, and $Ba(\imath x)(Rbx)$ are atomic statements. But the following principle may be assumed,

(0) $\vdash r = (\imath x)(Rbx)\ \&\ Bar. \supset Ba(\imath x)(Rbx)$

Hence the principle formulable in $D$ which is rejected is that which leads from (0), (1), and (2) to (3), namely

(6) if $\vdash A\ \&\ B \supset C$ then $\vdash A\ \&\ O(B). \supset O(C)$

And (6), of course is rejected in $D$.

However, this does not do justice to the example, for there is certainly a sense in which we may conclude from the facts of the case that Arthur is obligated to bandage the man whom Benjamin robs. It seems to me that this sense is exactly this: If Arthus is obligated to bandage Robert given that Robert is the man whom Benjamin robs, then Arthus is obligated to bandage the man whom Benjamin robs given that Robert is the man that Benjamin robs. This inference is sanctioned by the principle,

(7) if $\vdash A\ \&B. \supset C$ then $\vdash O(A/B) \supset O(A\ \&\ C/B)$

which is correct, and accepted in our calculus (by AC 4 and RC 2). Leaving off the condition at any point destroys the validity of the inference, however; Arthur may, for example, have a primary obligation to help Robert, but not have that obligation given that helping Robert will advance the cause of the Antichrist; and he may have an obligation to help a man who was robbed given that someone was robbed, but not *simpliciter*. Thus this example shows very clearly not the incorrectness of RC 2, but the inadequacy of the means of expression in $D$.



## V. A SEMANTIC ACCOUNT OF CONDITIONAL OBLIGATION

In Section II, I already sketched an interpretation of conditional obligations. There I made the interpretation rather general by not assuming that among the values assigned to the physically accessible states there was a maximum. In formal semantics we prefer generality, of course, and I shall now further liberalize our notions by not assuming that each possible world is assigned one value, but rather that it is assigned a set of values. These values I will assume to be ordered linearly. The first generalization probably affects the logic (in that the insistence on a finite set of values would probably lose us compactness); I think that the second generalization does not. Finally I shall not assume that the ordering of the values and/or the assignment of values to worlds remains the same; in terms of Powers' thought-experiments, tomorrow the pay-off machine may have been reprogrammed. (If that is not allowed, we would need an extra axiom eg. the $S_4$ – like $O(B) \supset O(O(B))$.)

Thus we define a *C-model structure* (briefly, $C$-ms) as a quadruple $M = \langle K, V, R, f\rangle$ where

(1) $K$ and $V$ are non-empty sets
(2) $R$ is a function with domain $K$ and such that for each $\alpha$ in $K$, $R_\alpha = R(\alpha)$ is an asymmetric, transitive and connected relation on a non-empty subset of $V$, that is:
(a) If $R_\alpha(u, w,)$ then not $R_\alpha(w, u)$
(b) If $R_\alpha(u, w)$ and $R_\alpha(w, z)$ then $R_\alpha(u, z)$
(c) If $u \neq w$ then $R_\alpha(u, w)$ or $R_\alpha(w, u)$ for all $u, w, z$ in the field of $R_\alpha$ (hereafter, $V_\alpha$).
(3) $f$ is a function with domain $K$ such that for each $\alpha$ in $K$, $f_\alpha = f(\alpha)$ is a mapping of $K$ into subsets of $V_\alpha$, and such that $\bigcup\{f_\alpha(\beta) : \beta \in K\}$ is not empty.

We may read "$R_\alpha(v, u)$" as "$v$ is greater than $u$ for $\alpha$" and "$f_\alpha(\beta)$" as "the set of values of $\beta$ with respect to $\alpha$."

These model structures can now be used to define truth conditions in a language with statements of conditional obligations. The language LC has as syntax

(1) an infinite set of sentential parameters
(2) the logical signs $\sim, \supset, O, /, ), ($
(3) a set of sentences defined by
  (a) sentential parameters are sentences
  (b) if $A$, $B$ are sentences so are $\sim A$, $(A \supset B)$, $O(A/B)$.

Other connectives are defined in the usual way.

The semantics of LC is given by defining its *admissible valuations* to be exactly the mappings $v_\alpha$ such that $\alpha$ is a member of a set $K$ and $v$ is a valuation on a $C$-ms, $M = \langle K, V, R, f\rangle$, this latter notion being defined by:

(4) A *valuation on* a $C$-ms $\langle K, V, R, f\rangle$ is a function $v$ defined on $K$ such that for each $\alpha$ in $K$, $v(\alpha) = v_\alpha$ satisfies:
  (a) $v_\alpha$ maps the sentences of LC into $\{T, F\}$
  (b) $v_\alpha(\sim A) = T$ iff $v_\alpha(A) = F$
  (c) $v_\alpha(A \supset B) = T$ iff $v_\alpha(A) = F$ or $v_\alpha(B) = T$
  (d) $v_\alpha(O(A/B)) = T$ iff there is an element $\beta$ in $K$ and an element $w$ such that $v_\beta(A \,\&\, B) = T$, $w \in f_\alpha(\beta)$ and $R_\alpha(w, u)$ for every element $u$ belonging to $f_\alpha(\gamma)$ for every $\gamma$ such that $v_\gamma(\sim A \,\&\, B) = T$.

Since (d) is somewhat complex, we will rephrase it. Let $\beta R_\alpha \gamma$ mean that $f_\alpha(\beta)$ has a member $u$ such that $R_\alpha(u, w)$ for each $w$ in $f_\alpha(\gamma)$. Secondly, let $K_v(A) = \{\delta \in K : v_\delta(A) = T\}$. Thirdly, let us say that $K_v(A)\, R_\alpha K_v(B)$ exactly if $K_v(A)$ has a member $\beta$ such that $\beta R_\alpha \gamma$ for each $\gamma$ in $K_v(B)$. Omitting the subscript '$v$' when the context prevents ambiguity, we can now reformulate (4) (d) as:

(4)(d') $v_\alpha(O(A/B) = T$ iff
$K(A \,\&\, B)\, R_\alpha K(\sim A \,\&\, B)$.

It is helpful to note that if $A \Vdash B$ then $K_v(A) \subseteq K_v(B)$, and that $K_v(A \,\&\, B) = K_v(A) \cap K_v(B)$, $K_v(A \vee B) = K_v(A) \cup K_v(B)$, and $K_v(\sim A) = K - K_v(A)$.



As corollaries to this we observe that if $\Vdash A$ then $K_v(A) = K$ and if $A \equiv B$ is a truth functional tautology then $K_v(A) = K_v(B)$.

Let us now examine the axioms and rules of system CD in the light of this semantic account. Two common expressions used in formal semantics are defined hereby:

> $A$ is *valid* in LC ($\Vdash A$) exactly if every admissible valuation of LC satisfies $A$ (i.e. assigns $T$ to $A$). $X$ *semantically entails* $A$ in LC ($X \Vdash A$) exactly if every admissible valuation of LC which satisfies $X$ (i.e. satisfies every member of set $X$) also satisfies $A$.

Are all theorems of CD valid (in LC)? That is, are all axioms valid and do all rules preserve this property?

That AC1 and RC1 are all right is clear. When a given valuation $v_\alpha$ is the only one under discussion, let us say that value *is in A* when it belongs to $\bigcup\{f_\alpha(\beta) : \beta \in K_v(A)\}$. For AC2, suppose there is a value in $A \,\&\, C$ that is higher any in $\sim A \,\&\, C$; then the converse cannot hold. Hence if $O(A/C)$ is true, $O(\sim A/C)$ is not. For AC3 we may disregard the conditionalization, since it is the same throughout (just assume that $K_v(C) = K$). Suppose then there is a value $w$ in $A \supset B$ higher than any in $A \,\&\, \sim B$, and a value $u$ in $A$ higher than any in $\sim A$. Now $w$ must lie either in $\sim A$ or in $B$. If it lies in $\sim A$, then $u$ is higher than $w$. So we have a value in $\sim A$ higher than any in $A \,\&\, \sim B$, but a value in $A$ higher than any in $\sim A$ or in $A \,\&\, \sim B$. The latter value, $w$, must therefore lie in $A \,\&\, B$. If a value $z$ lies in $\sim B$ it lies in $A \,\&\, \sim B$ or in $\sim A$. Hence $w$ in $A \,\&\, B$, and hence in $B$, is higher than any value in $\sim B$. On the other hand, if $w$ does not lie in $\sim A$, it lies in $B$, and hence in $A \,\&\, B$. If $w$ is higher than or equal to $u$, then it is higher than any value in $A \,\&\, \sim B$ or in $\sim A$, and hence higher than any value in $\sim B$. If $u$ is higher than $w$, it is higher than any value in $A \,\&\, \sim B$, and hence lies in $A \,\&\, B$, so there is a value in $B$ higher than any in $A \,\&\, \sim B$ or in $\sim A$. As we see, all possibilities substantiate AC3.

For RC2, we note that if $\Vdash A \supset B$, then $K_v(A) \subseteq K_v(B)$, so $K_v(A \,\&\, C) \subseteq K_v(B \,\&\, C)$, while $K_v(\sim B \,\&\, C) \subseteq K_v(\sim A \,\&\, C)$. Thus a value in $A \,\&\, C$ higher than any in $\sim A \,\&\, C$ will at once be a value in $B \,\&\, C$ higher than any in $\sim B \,\&\, C$. For RC3 we invoke the special condition on $f$ that $\bigcup\{f_\alpha(\beta) : \beta \in K\}$ is not empty; since this is exactly the set of values in $A$ if $\Vdash A$, we clearly have $K_v(A \,\&\, A)\, R_\alpha K_v(\sim A \,\&\, A)$ in that case. RC4 is substantiated at once by the consideration that $K_v(C)=K_v(D)$ if $\Vdash C\equiv D$.

Of AC4–AC7 I will explicitly discuss only AC5. Recall that $B(A/B)$ was defined as $O({\sim}B/A\vee B)$; is that a good definition? We want to have $v_\alpha(B(A/B))=T$ exactly if $K_v(A)R_\alpha K_v(B)$. But the latter condition says that some value in $A$ is higher than any in $B$; such a value *can only* lie in $A\ \&\ {\sim}B$. Hence $K_v(A)R_\alpha K_v(B)$ exactly if $K_v({\sim}B\ \&\ A)R_\alpha K_v(B)$. And that is the case exactly if $v_\alpha(O({\sim}B/A\vee B))=T$. Now we see therefore that AC5 is valid exactly if $R_\alpha$ is transitive; and it is.

## VI. COMPLETENESS OF THE SYSTEM CD

The discussion of CD at the end of the preceding section showed, in effect, that CD is *sound* with respect to LC: if $A$ can be deduced from premises $X$ via system CD then $X\Vdash A$ in LC. Now I want to show that CD is (strongly) *complete* with respect to LC: if $X\Vdash A$ in LC, then $A$ can be deduced from $X$ via CD. As a preliminary, an indifference relation is defined, and a number of theorems proved concerning CD.

'$S(A/B)$' for '${\sim}B(A/B)\ \&\ {\sim}B(B/A)$'

T5 $\vdash B(A/B)\supset{\sim}B(B/A)$

For $B(A/B)\equiv O({\sim}B/A\vee B)$. Assuming both $B(A/B)$ and $B(B/A)$ we have $O({\sim}B/A\vee B)\ \&\ O({\sim}A/A\vee B)$, hence $O({\sim}A\ \&\ {\sim}B/A\vee B)$, hence $O({\sim}(A\vee B)/A\vee B)$ by T2. But ${\sim}O({\sim}(A\vee B)/A\vee B)$ by T3.

T6 $\vdash B(A/B)\supset[B(B/C)\supset B(A/C)]$

This follows at once from AC5.

T7 $\vdash S(A/B)\supset[B(A/C)\supset B(B/C)]$

T8 $\vdash S(A/B)\supset[B(C/A)\supset B(C/B)]$

These follow from AC6 and AC7 respectively.

T9 $\vdash S(A/A)$

This follows from ${\sim}B(A/A)\equiv{\sim}O({\sim}A/A)$, and T3.

The following two theorems are tautologies.

T10 $\vdash S(A/B)\supset S(B/A)$

T11 $\vdash S(A/B)\vee B(A/B)\vee B(B/A)$

T12 $\vdash S(A/B)\supset[S(B/C)\supset S(A/C)]$



For suppose $S(A/B)$ and either $B(A/C)$ or $B(C/A)$. Then either $B(B/C)$ or $B(C/B)$ follows by T7–8.

T13 If $\vdash A\supset B$ then $\vdash{\sim}B(A/B)$

For $B(A/B)\equiv O({\sim}B/A\vee B)\equiv O({\sim}B/B)$ if $A\vdash B$ (for then $\vdash B\equiv A\vee B$; RC4). But $\vdash{\sim}O({\sim}B/B)$ by T3.

As corollary we have:

T14 If $\vdash A\equiv B$ then $\vdash S(A/B)$

T15 $\vdash B(B/{\sim}(A\supset A))\vee S(B/{\sim}(A\supset A))$

Immediate from T13 and T11.

T16 $\vdash O(B/B)\equiv B(B/{\sim}(A\supset A))$

$B(B/{\sim}(A\supset A))\equiv B(B/B\ \&\ {\sim}B)\equiv O(B\supset B/B)$. But $O(B/B)\supset O(B\supset B/B)$ by RC2.

Conversely, $O(B\supset B/B)$ implies $O(B/B)$ by AC4.

T17 $\vdash O(A/B)\supset O(A\ \&\ B/A\ \&\ B)$

Suppose ${\sim}O(A\ \&\ B/A\ \&\ B)$. Then by T15 and T16 we have $S(A\ \&\ B/{\sim}{\sim}(A\supset A))$. Now $S(A\ \&\ B/{\sim}(A\supset A))$ implies ${\sim}B(A\ \&\ B/{\sim}A\ \&\ B)$ by T15 and T7. But $O(A/B)\equiv B(A\ \&\ B/{\sim}A\ \&\ B)$ by T4.

T18 $\vdash{\sim}B(D/D\ \&\ C)\vee{\sim}B(D/D\ \&\ {\sim}C)$

Suppose $B(D/D\ \&\ C)\ \&\ B(D/D\ \&\ {\sim}C)$. Then $O({\sim}(D\ \&\ C)/D)\ \&\ O({\sim}(D\ \&\ C)/D)$. By T2 we arrive at $O({\sim}D/D)$ which is impossible by T3.

T19 $\vdash B(A\ \&\ B/{\sim}(A\supset A))\ \&\ S({\sim}A\ \&\ B/{\sim}(A\supset A)).\supset$
$\supset O(A/B)$

Assume the antecedent. By T8, $B(A\ \&\ B/{\sim}A\ \&\ B)$. Hence $O(A/B)$.

We can now turn to the completeness proof proper. Every set of sentences that is consistent with respect to CD can be extended into a maximal set, by Tukey's lemma and the fact that the deducibility relation is finitary. Hence it suffices to show that all maximal consistent sets can be satisfied. We shall make up a single $C$-ms to show this: $M=\langle\sum, \mathrm{V}, R, f\rangle$ where $\sum$ is the family of maximal consistent sets, $V=\bigcup\{V_\alpha:\alpha\in\sum\}$, and $V_\alpha$, $R_\alpha$, and $f$ are as we shall now define them.

Let $\alpha$ be in $\sum$. For any sentence $A$, we define $[A]_\alpha=\{B:S(A/B)\in\alpha\}$, and define $V_\alpha = \{[A]_\alpha$: $A$ is a sentence of LC$\}$. In addition, we define $R_\alpha = \{<[A]_\alpha, [B]_\alpha>: B(A/B) \in \alpha\}$, and for $\beta$ in $\sum$, define $f_\alpha(\beta) = \{[A]_\alpha : A \in \beta$ and for all $B$ in $\beta$, $\sim B(B/A) \in \alpha\}$. We shall omit the subscripted Greek letters sometimes when confusion is prevented by context.

With respect to these definitions, we note first that the relation $S(A/B) \in \alpha$ between $A$ and $B$ is an equivalence relation (T9, 10, 12) and that if $S(A/B) \in \alpha$, then if $B(A/C) \in \alpha$, so is $B(B/C)$, and if $B(C/A) \in \alpha$, so is $B(C/B)$, by T7–8. Hence we deduce readily that $R_\alpha$ is transitive (T6), asymmetric (T5), and connected (T11), and has $[\sim(A \supset A)]_\alpha$ as lowest element (T15). By a 'lowest' element of a collection, we mean one which does not bear $R_\alpha$ to any (other) member of that collection. In fact, we may call $[\sim(A \supset A)]_\alpha$ *the* lowest element. For we have either $B(B/\sim A \supset A))$ or $S(B/\sim(A \supset A))$: in the former case $[B]$ is higher, and in the latter case $[B] = [\sim(A \supset A)]$.

The relation $R_\alpha$ is therefore as the definition of $C$-ms requires. The lemma which follows and which will also be of use later on, shows that the requirement that $\bigcup\{f_\alpha(\beta) : \beta \in K\}$ be non-empty is also fulfilled, since $O(B \supset B/B \supset B) \in \alpha$ by RC3.

For a given sentence $B$, we are going to define a set $T_\alpha(B)$, which will be a maximal consistent set under suitable conditions. We define $T_\alpha(B)$ to be the deductive closure of the set $\{D_1, D_2, \ldots,\}$ where $C_1, C_2 \ldots$ are exactly the sentences of LC and

$$D_1 = B$$
$$D_{i+1} = D_i \;\&\; C_i^* \text{ where } C_i^* \text{ is } C_i \text{ if}$$
$$\sim B(D_i/D_i \;\&\; C_i) \in \alpha, \text{ and } C_i^* \text{ is } \sim C_i \text{ otherwise.}$$

Clearly, $T_\alpha(B)$ is maximal if consistent.

*Lemma* If $O(B/B) \in \alpha$, then $T_\alpha(B) \in \sum$ and $f_\alpha(T_\alpha(B)) = \{[B]_\alpha\}$

*Proof.* First, if $O(B/B) \in \alpha$, then $B(B/\sim(A \supset A)) \in \alpha$, by T16. Secondly, either $\sim B(D_i/D_i \;\&\; C_i)$ or $\sim B(D_i/D_i \;\&\; \sim C_i)$ by T18; hence $\sim B(D_i//D_{i+1})$. We can now conclude that for J $= 1, 2, 3, \ldots$, $[D_i] R_\alpha [\sim(A \supset A)]$, hence $T_\alpha(B)$ is consistent (see T14). We want to show that for all $C$ in $T_\alpha(B)$, $\sim B(C/B) \in \alpha$. Well, if $C$ is in $T_\alpha(B)$, then it logically implied by $D_i$ for some index $i$. But then $\sim B(C/D) \in \alpha$ by T13, and since in addition $\sim B(D_i/B) \in \alpha$, because $\vdash D_i \supset B$ (see Theorem T13); we conclude



$\sim B(C/B) \in \alpha$ by AC6. So $[B]_\alpha$ is in $f_\alpha(T_\alpha(B))$; in addition, if $[C]_\alpha$ also belongs, we must have $\sim B(B/C)$ and $\sim B(C/B)$, hence $[B]_\alpha = [C]_\alpha$. This ends the proof.

*Theorem* There is a valuation $v$ on $M = \langle \sum, V, R, f \rangle$ such that $v_\alpha$ satisfies $\alpha$, for each $\alpha$ in $\sum$.

*Proof.* We define $v$ by $v_\alpha(A) = T$ if $A \in \alpha$, and $v_\alpha(A) = F$ otherwise. That $v$ satisfies clauses (a)–(c) in the definition of a valuation on a $C$-ms is obvious. To show that clause (d) is satisfied, we consider two cases.

*Case 1.* $O(A/B)$ is in $\alpha$. Then $O((A \;\&\; B)/A \;\&\; B)) \in \alpha$ by T17. Hence $f_\alpha(T_\alpha(A \;\&\; B)) = \{[A \;\&\; B]\}$. Now suppose that $\sim A \;\&\; B \in \beta$, and $f_\alpha(\beta)$ contains $[C]$. Then $[C]$ can be no higher than $[\sim A \;\&\; B]$. But since $O(A/B) \in \alpha$, so is $B(A \;\&\; B/\sim A \;\&\; B)$, by T4. Thus $[A \;\&\; B]$ is higher than $[\sim A \;\&\; B]$ and hence higher than $[C]$. So there is an element in $A \;\&\; B$ which is higher than any in $\sim A \;\&\; B$, in our earlier terminology.

*Case 2.* $O(A/B) \in \alpha$. Consider $(\sim A \;\&\; B)$: either $O(\sim A \;\&\; B/\sim A \;\&\; B)$ is in $\alpha$ or it is not. If it is in $\alpha$, then $f_\alpha(T_\alpha(\sim A \;\&\; B)) = \{[\sim A \;\&\; B]\}$, which contains a value no lower than $[A \;\&\; B]$, since $\sim O(A/B) \in \alpha$, by T4, and hence no lower than any value to be found in $f_\alpha(\beta)$ when $\beta$ contains $A \;\&\; B$. Suppose that $O(\sim A \;\&\; B/\sim A \;\&\; B)$ is not in $\alpha$. In that case $S(\sim A \;\&\; B/\sim(A \supset A)) \in \alpha$ by T15–16; we claim that similarly $S(A \;\&\; B/\sim(A \supset A)) \in \alpha$, so that if $\beta$ in $\sum$ contains $A \;\&\; B$ or $\sim A \;\&\; B$, then $f_\alpha(\beta) = \{[\sim(A \supset A)]_\gamma$. But our claim follows from T15 and T19, given that $O(A/B)$ is not in $\alpha$. This ends the proof.

## VII. THE ANDERSON MODIFICATION MODIFIED

A. R. Anderson introduced a device, since known as the *Anderson modification*, which was designed to reduce deontic logic to alethic modal logic.[14] We choose a normal system of modal logic, add a constant $S$ (generally read as "All hell breaks loose."), the axiom $\vdash \Diamond \sim S$, and define the monadic operator $O$ by $O(A) \equiv \Box(\sim A \supset S)$. Then all the laws of $D$ are provable. (In addition, the semantics of modal and deontic logics developed since then shows that no non-theorems of $D$ will be provable in $M$: we take $S$ to be false exactly in the ideal possible worlds, true in the non-ideal ones.)

As a reduction in the technical sense, in which a system is classified as alethic or deontic by the syntactic form of its theorems, the Anderson modification is a successful and highly useful device. Anderson also introduced the thesis that this is a key to the correct interpretation of deontic concepts: there is something (which is bad) and which happens exactly if any obligation is violated. We do not have to look far to find what that something is, of course: what happens in all and only those cases in which an obligation is violated is that an obligation is violated. Critics have urged that this clearly does not explicate deontic concepts in non-deontic terms.[15] If we were to find, say, that physical laws are such that in any physically possible world, some obligation is violated if and only if someone proves that deontic logic is reducible to alethic logic, we could take $S$ to state *that*: But it would still have to be admitted that the *meaning* of deontic terms cannot be given in terms of physical necessity and logical activity.

Be that as it may, Anderson's translation into modal logic shows very clearly the shallow character of deontic distinctions that can be expressed in $D$. Some moral violations lead to the Deluge, some to Hell, some to Purgatory, some to prison, some to gout, and some to gubernatorial disapproval; but in $D$ they are classified alike as leading to something bad. In addition, one and the same course of action may earn time in prison and merit in heaven, and a choice may be between the devil and the deep blue sea; no such distinctions are possible in $D$.

For CD, the Anderson modification is not possible. For example, we cannot find one sentence true in every member of $K_v(A \ \& \ B)$ and false in every member of $K_v({\sim} A \ \& \ B)$ whenever $O(A/B)$ is true; this would make the conjunction of $O(A/B)$ and $O(A \ \& \ C/B)$ impossible when $K_v(A)$ is not the same as $K_v(C)$. However, we can introduce a device similar to Anderson's. Let us suppose that when a world (situation, state of affairs) has value, this is it because in it there exists something that has that value (with or without further qualifications, such as that there be nothing in that same world with lesser value). Then our interpretation, when used to answer someone who asks "What ought I to do?", is: if $K$ is the set of possible outcomes you can achieve, then $O(A/B)$ exactly if there is something $X$ such that $\Diamond(A \ \& \ B \ \& \ x$ exist) is true, and for all y such that $\Diamond({\sim} A \ \& \ B \ \& \ y$ exists) is true, $x$ has greater value than $y$ (with or without qualifications, such as that $y$ have some value).

The supposition made in the preceding paragraph is philosophically



either dubious or unenlightening, while technically beyond reproach. For example, if the supposition is not true under any non-trivial interpretation, we can simply agree to count the value(s) of a world among its inhabitants, using perhaps a special predicate to express the distinction between them and normal residents (animate, inanimate, or abstract). Suitable reinterpretation (and perhaps relativization of the quantifiers) would help to keep the language's resources as great as before. The only problem I can see is that I am making it impossible for a valuable world to be empty. I am powerless to deal with that case unless an empty world is devoid of moral value. Barring recent progress in moral theory as yet unknown to me, I cannot find that admission very damaging.

As a convenient system of quantificational modal logic I shall choose $Q_1M$, a system devised by Thomason.[17] The adjustments I make are to ignore names and definite descriptions, and to stipulate that there be at least one monadic and one dyadic predicate. To be precise, the new syntax will contain all the sentence parameters of LC, and the logical signs $\Box$, $\sim$, $\supset$, ), (, $=$, for each integer $n \rangle 1$ a set of predicates (of degree $n$), these sets being non-empty for $n=1$ and $n=2$, an infinite set of variables, and the set of sentences is defined by induction as usual. We assume a well-ordering of the expressions, and designate the first monadic predicate as $E!$, the first dyadic predicate as $R$.

The axioms and rules for $Q_1M$ are those for quantificational logic with identity, and in addition

AM 1 $\vdash \Box A \supset A$
AM 2 $\vdash \Box(A \supset B) \supset . \Box A \supset \Box B$
AM 3 $\vdash (x) \Box A \equiv \Box (x) A$
AM 4 $\vdash (x)(y)(x=y \supset \Box x=y)$
RM 1 If $\vdash A$ then $\vdash \Box A$

To form the extended system $Q_1M^+$ we add

AM 5 $\vdash (x)(y)(Rxy \supset {\sim} Ryx)$
AM 6 $\vdash (x)(y)(z)(Rxy \supset : Ryz \supset Rxz$
AM 7 $\vdash (x)(y)(x \neq y \supset Rxy \vee Ryx)$
AM 8 $\vdash (Ex)(\Diamond E!x)$
Def. '$O(A/B)$' for '$(Ex)(\Diamond(A \ \& \ B \ \& \ E!x) \ \& \ (y)$
$(\Diamond({\sim} A \ \& \ B \ \& \ Ey!) \supset Rxy))$
where $x$ and $y$ are distinct variables.

We shall read "$Rxy$" as "$x$ is higher (in value) than $y$", and "$E!x$" as "$x$ exists"; clearly we should read "$(x)A$" as "for all possibles $x$, $A$", since $(x)(E!x)$ cannot be deduced. A sentence is a theorem in $Q_1M^+$ exactly if it can be derived in $Q_1M$ from premises of the form given in AM 5-8, hence a reduction of CD to $Q_1M^+$ is also a reduction to $Q_1M$. We prove the reduction semantically, by adjusting the models of $Q_1M$ so that they satisfy $Q_1M^+$.

A *Q-ms* is a triple $<K, \Pi, D>$ where $K$ and $D$ are non-empty sets and $\Pi$ a reflexive dyadic relation on $K$. A *valuation* $v$ on $<K, \Pi, D>$ is a function which maps the variables into $D$ and assigns to each member $\alpha$ of $K$ a mapping $v_\alpha$ fulfilling the conditions

(a) $v_\alpha(x) = v(x)$ for any variable $x$
(b) $v_\alpha(A) \in \{T, F\}$ for any sentence parameter $A$
(c) $v_\alpha(P) \subseteq D^n$ for any $n$-ary predicate parameter $P$
(d) $v_\alpha(R)$ is asymmetric, transitive, and connected in $D$
(e) $\bigcup\{v_\beta(E!) : \alpha\Pi\beta\}$ is not empty

The truth-values of the sentences that are not sentence parameters are then given inductively:

(1) $v_\alpha(x = y) = T$ iff $v_\alpha(x) = v_\alpha(y)$
(2) $v_\alpha(Px_1 \ldots x_n) = T$ iff $<v_\alpha(x_1), \ldots, v_\alpha(x_n)> \in v_\alpha(P)$
(3) $v_\alpha(\sim A) = T$ iff $v_\alpha(A) = F$
(4) $v_\alpha(A \supset B) = T$ iff $v_\alpha(A) = F$ or $v_\alpha(B) = T$
(5) $v_\alpha((x)A) = T$ iff $v^d/x_\alpha(A) = T$ for each $d$ in $D$ (where $v^d/x$ is exactly like $v$ except for assigning $d$ to $x$)
(6) $v_\alpha(\Box A) = T$ if $v_\beta(A) = T$ for each $\beta$ in such that $\alpha\Pi\beta$.
(7) $v_\alpha(A) = F$ if $v_\alpha(A) \neq T$.

An admissible valuation of this syntax is a mapping $v_\alpha$ ahere $\alpha$ is a member of $K$ and $v$ a valuation on $<K, \Pi, D>$ for some $Q$-ms $<K, \Pi, D>$. The completeness theorem for $Q_1M^+$ with respect to $LC^+$ so formed is a corollary to Thomason's completeness proof for $Q_1M$.

Suppose now that $M = <K, \Pi, D>$ is a $Q$-ms and $v$ a valuation on $M$. We define the structure $M^1 = <K, V, R, f>$ as follows: $V = D$, $R_\alpha = v_\alpha(R)$, $f_\alpha(\beta) = \Lambda$ if not $\alpha\Pi\beta$, otherwise $f_\alpha(\beta) = \{d \in D : d \in v_\beta(E!)\}$. We note that $V_\alpha = V$ for all $\alpha$, $R_\alpha$ is antisymmetric, transitive, and connected in $V_\alpha$, and $\bigcup \{f_\alpha(\beta) : \beta \in K\}$ is not empty. Hence $M^1$ is a $C$-ms. If we look only at sentences formed from sentence parameters and $\sim$, &, $O$, then $v$ obviously fulfills conditions (a) - (c) in the definition of a valuation on a $C$-ms. To show that it also satisfies (d), we need only show that $\bigcup\{f_\alpha(\beta) : v_\beta(A) = T\}$, i.e. the set of values in members of $K_v(A)$, is just $\{d \in D : v^d/x_\alpha(\Diamond A \ \& \ E!x))\}$. Well $d$ is in the first set exactly if $v^d/x_\beta(E!x) = T$ for some $\beta$ such that $\alpha\Pi\beta$ and $v_\beta(A) = T$. But that is the case exactly if $v^d/x_\alpha(\Diamond(A \ \& \ E!x)) = T$.



Thus admissible valuations of $LC^+$ are also admissible valuations of LC. To prove the converse, suppose that $M^1 = <K, V, R, f>$ is a $C$-ms, and $\alpha$ a specific member of $K$. We define the structure $M = <K, \Pi, D>$ as follows: $\Pi = K^2$, $D = V_\alpha$. Clearly $M$ is a $Q$-ms. Now let $v^1$ be a valuation of $M^1$; we define a valuation $v$ on $M$, with the hope of showing that $v^1_\alpha$ and $v_\alpha$ are the same as far as the sentences of LC are concerned.

For any $\beta$, and any sentence parameter $A$, let $v_\beta(A) = v^1_\beta(A)$. Let $v_\beta(E!) = f_\alpha(\beta)$, and let $v_\beta(R) = R_\alpha$. What $v$ does for variables and predicates other than $E!$ and $R$ is immaterial: we assume a suitable choice is made. Then $v$ is a valuation on $M$, since $R_\alpha$ has the requisite properties and $\bigcup\{f_\alpha(\beta) : \beta \in K\} = \bigcup\{v_\beta(E!) : \alpha\Pi\beta\}$ is not empty. We must show that for the LC sentences, $v^1_\alpha(O(A/B)) = v_\alpha(O(A/B))$. For this we note first that $\bigcup \{f_\alpha(\beta) : v_\beta(A \ \& \ B) = T\} = \{d \in D : d \in v_\beta(E!x \ \& \ A \ \& \ B)$ for some $\beta$ such that $\alpha\Pi\beta\} = \{d \in D : v^d/x_\alpha(\Diamond(A \ \& \ B \ \& \ E!x)) = T\}$; $\bigcup\{f_\alpha(\gamma) : v_\gamma(\sim A \ \& \ B) = T\} = \{e \in D : v^e/y_\alpha(\Diamond(\sim A \ \& \ B \ \& \ E!y)) = T\}$. Thus $v_\alpha((E!x)(\Diamond(A \ \& \ B \ \& \ E!x) \ \& \ (y)(\Diamond(\sim A \ \& \ B \ \& \ E!y) \supset Rxy)) = T$ exactly if there is a member $d$ of the first set that bears $R_\alpha$ to each member of the second set. So every admissible valuation of LC can be extended into an admissible valuation of $LC^+$.

## VIII. SCEPTICAL POSTSCRIPT

At several points in the preceding sections I argued that there are moral distinctions that simply cannot be expressed adequately in the language of absolute obligations. I hold exactly the same view concerning the language of conditional obligations just constructed, even though I think it is an improvement with respect to the problems considered so far.

First, in constructing CD I decided to accept von Wright's criterion, that $O(A/B \supset B)$ should follow that logic of absolute obligation which is now standard in deontic logic. This made it necessary to assume that values are ordered linearly. This means in turn that $O(A/C)$ and $O(\sim A/C)$ can not both be true. However, we often find ourselves in a situation in which we have at least *prima facie* conflicting obligations. Hence it would be more apt to say that we have here a logic of obligations that remain after obligational conflicts are resolved.

If we hold that 'ought implies can', at least in the logical sense of 'can' and theorem T2 holds, then it follows that there are no moral conflicts incapable of resolution, that is, no possible situation in which we really (and not just *prima facie*) have obligations that cannot possible all be fulfilled. This is the point of view of, for example, Castañeda: "it is the function of the ethical 'ought' and the *ethos* it governs to solve the conflicts of duties... "[18]. I would not deny that *ideally* this is the case, but I do not believe that it is true of actual morality. If a legal system leaves problems unsolved, or has laws that conflict in a given unforeseen situation, the judicial system amends the law by precedent and reinterpretation. But there is no judical system for morality, and new moral rules do not come into existence by fiat or plebiscite.

A second point of criticism concern the formula $O(B/B)$.[19] This is almost always true; it is true if some value attaches to some possible world (attainable outcome) in which $B$ is true. That means then that the violation of a (primary) obligation is a (secondary) obligation relative to the assumption that the obligation is in fact violated. 'Rightly understood' of course, it is true; if we have put ourselves in a situation in which a certain ideal can no longer be attained, then doing the best one can will involve not attaining that ideal. No use crying over spilt milk. But clearly there are also many moral evaluations and value judgments concerning such a situation which our schema leaves out of account altogether. The moral questions that can be asked go far beyond the simple "What ought to be done now?" Perhaps the addition of tense-logical machinery will alleviate some of the shortcomings, by allowing answers also to "What ought to have been done?"[20]

Finally, in moral discourse obligation is qualified and relativized in many ways. In the article cited above, Castañeda argued that "ought" should be subscripted, to allow the expression of obligations due to etiquette, laws, professional standards, and so on. These are non-ethical obligations. But ethical obligations might be divided into sub-categories too. There are duties to one-self, and duties to others; obligations incurred by promises and obligations incurred by acquiescence; obligations devolving upon one due to one's rank or due to one's happening to be in a certain place, and so on. Each of these can certainly be subdivided into sub-sub-categories: duties to others may comprise conflicting duties toward Peter and toward Paul; conflicting obligations might be incurred, perhaps quite unintentionally, by promises to Mary and to Martha, and so on.



This seems to me to be a case of 'variable polyadicity', and the use of subscripts can hardly be adequate to handle it, though it provides a first approximation. One avenue to approach might be to introduce prepositions and other adverbial devices into $LC^+$, and to consider the effects of adverbial modifications of the predicate R there. But the success of any such attempt would require the previous success of a systematic analysis of rights, duties, values, and obligations.

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## NOTES

* The research for this paper was supported by Canada Council grant 69-0650. The author wishes to acknowledge gratefully his debt to A. al-Hibri, 'A Critical Survey in Deontic Logic'. In forthcoming work on counterfactuals, David Lewis discusses the relation of the system CD and language LC here constructed to this minimal logic of counterfactuals CO and to the work of Bengt Hanson, 'An Analysis of Some Deontic Logics'.

1 L. Åqvist, 'Good Samaritans, Contrary-to-Duty Imperatives, and Epistemic Obligations'.

2 F. Fitch, 'Natural Deduction Rules for Obligation'.

3 While the paradox was already presented by Bradley, it is related to deontic logic by R. M. Chisholm, in 'Contrary-to-Duty Imperatives and Deontic Logic'.

4 G. H. von Wright, 'A Note on Deontic Logic and Derived Obligation'.

5 L. Powers, 'Some Deontic Logicians'.

6 al-Hibri, *op. cit.*, p. 32.

7 G. H. von Wright, 'An Essay in Deontic Logic and the General Theory of Action', especially pp. 25 and 35.

8 In Section IV, I shall discuss an argument by Castañeda against a similar principle.

9 L. Åqvist, 'Deontic Logic Based on a Logic of 'Better''.

10 The concept of conditional permission called 'natural' by von Wright (*op. cit.*, p. 35) which apparently fits N. Rescher, 'An Axiom System for Deontic Logic' has as theorem $O(A/B \vee C) \equiv O(A/B) \vee O(A/C)$, if "$O($" is defined as "$\sim P) \sim$" as usual. But on our interpretation even the weaker $O(A/B) \supset O(A/B \vee C)$ does not hold. For to steal and make restitution is better than to steal and not make restitution, but to have a cup of tea instead is better yet.

11 von Wright, *op. cit.* p. 30.

12 *op. cit.*, pp. 371–3.

13 H.-N. Castañeda, 'Acts, the Logic of Obligation, and Deontic Calculi', especially pp. 13–4.

[14] A. R. Anderson, 'A Reduction of Deontic Logic to Alethic Modal Logic'.
[15] See for example J. Berg, 'A Note on Deontic Logic' and P. H. Nowell-Smith and E. J. Lemmon, 'Escapism: the Logical Basis of Ethics'.
[16] This makes the point (cf. Lemmon and Nowell-Smith, *op. cit.*, p. 291) that violations are not always followed by appropriate sanctions simply irrelevant.
[17] See R. H. Thomason, 'Some Completeness Results for Modal Predicate Calculi', and 'Modal Logic and Metaphysics'.
[18] H. N. Castañeda, 'A Theory of Morality', p. 345.
[19] For this paragraph I am indebted to the critical discussion of Åqvist's system $DL_w$ in al-Hibri, *op. cit.*, pp. 36–7.
[20] Cf. R. H. Thomason, 'Deontic Logic as Founded on Tense Logic'.

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HARRY BEATTY

# ON EVALUATING DEONTIC LOGICS

*Comments on van Fraassen's Paper*

Professor van Fraassen, in his paper 'The Logic of Conditional Obligation', argues that certain problems, raised for the minimal deontic logic *D* by contrary-to-duty imperatives, by the Good Samaritan paradox, and by Powers' John and Suzy paradox, can be handled by his conditional logic of obligation *CD*. In my comments I shall for the most part neither attack nor support this claim: rather, I shall point out some reasons why I find it difficult to evaluate.

I shall begin with some general reasons why I have problems evaluating arguments for and against deontic systems. Typically such arguments turn on appeals to features of natural language captured or not captured by the deontic system in question. Take for example the specific argument constructed by Åqvist against *D* based on contrary-to-duty imperatives. He argues (Åqvist, 1967, p. 364) that the following set of sentences is intuitively consistent and independent (i.e. none of the four can be inferred from the other three), but that the sentences cannot be formalized in *D* so as to be consistent and independent.

(I) It ought to be that Smith refrains from robbing Jones.
(II) Smith robs Jones.
(III) If Smith robs Jones, he ought to be punished for robbery.
(IV) It ought to be that if Smith refrains from robbing Jones he is not punished for robbery.

For example, if we paraphrase (I)–(IV) into *D* as

(1) $O(\sim A)$
(2) $A$
(3) $A \supset O(B)$
(4) $O(\sim A \supset \sim B)$

respectively, we can derive a contradiction. For $O(B)$ follows from (2) and (3), while $O(\sim B)$ follows from (1) and (4), since *D* allows distribution of '$O$' across '$\supset$'. Similar problems arise for other likely paraphrases.

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