Erratum for "Are Rindler Quanta Real"

Hans Halvorson, 19 May 2007

Proposition 3 (page 461) supposedly shows that for a "conservative about states" (i.e. the physical states lie in a single folium), physical equivalence corresponds to quasiequivalence. I still believe that the proof is mathematically valid. However, I also believe that the result shows that the definition of physical equivalence (on page 430) was formulated incautiously. (Thanks to David Baker for pointing out this issue.) Consider the following example:

Let A be the Weyl algebra, let ω be a pure state of A, and let γ be an automorphism of A such that ω and $\omega \circ \gamma$ are disjoint. (To see that such ω and γ exist: one can choose ω to be the Minkowski vacuum state for the free Bose field, and $\gamma(W(f)) = e^{iL(f)}W(f)$ where $L(f)$ is a linear functional that is unbounded relative to the Hilbert space norm on the one-particle space.) Let (H_π, π) be the GNS representation of A induced by the state ω, and let (H_ϕ, ϕ) be the GNS representation of A induced by the state $\omega \circ \gamma$. Then (H_π, π) and (H_ϕ, ϕ) are disjoint. Let N_π be the set of normal (i.e. ultraweakly continuous) states of $\pi(A)$, and let N_ϕ be the set of normal states of $\phi(A)$.

We claim now that the pair $(\pi(A), N_\pi)$ can be considered to describe the "same physics" as the pair $(\phi(A), N_\phi)$. That is, there are bijections $\alpha : \phi(A) \to \pi(A)$ and $\beta : N_\phi \to N_\pi$ such that

$$\beta(\rho)(\alpha(a)) = \rho(a), \qquad (\forall a \in A).$$

Define the map $\alpha : \phi(A) \to \pi(A)$ by setting

$$\alpha(\phi(a)) = \pi(\gamma^{-1}(a)),$$

for all $a \in A$. (Note that α does not satisfy Eqn. 21 in the definition of physical equivalence. I say: so much the worse for that definition — it was wrong headed to require that the two theories share the *same* set of primitives.) Since γ^{-1} is an automorphism, α is a bijection.

By assumption, A is simple, and so π has an inverse. Thus we can define a map β from N_ϕ to the state space of $\pi(A)$ by setting

$$\beta(\rho) = \rho \circ \phi \circ \gamma \circ \pi^{-1},$$

for all ρ in N_ϕ. That is,

$$\beta(\rho)(\pi(a)) = \rho(\phi(\gamma(a))).$$

Then

$$\beta(\rho)(\alpha(\phi(a))) = (\rho \circ \phi \circ \gamma \circ \pi^{-1})(\pi(\gamma^{-1}(a))) = (\rho \circ \phi \circ \gamma)(\gamma^{-1}(a)) = \rho(\phi(a)).$$

We claim also that $\beta(\rho) \in N_\pi$, which is equivalent to the claim that the state $\beta(\rho) \circ \pi$ of A is unitarily equivalent to $\omega \circ \gamma$. Indeed, $\beta(\rho) \circ \pi = \rho \circ \phi \circ \gamma$. But $\rho \circ \phi$ is unitarily equivalent to ω, and automorphisms preserve unitary equivalence. That is, $\beta(\rho) \circ \pi = \rho \circ \phi \circ \gamma$ is unitarily equivalent to $\omega \circ \gamma$.

The original definition of physical equivalence assumes that the "two observers" already have established a translation scheme between their primitives — i.e. $\pi(a)$ corresponds to $\phi(a)$. But this requirement is overly restrictive, because once the $\pi(a) \to \phi(a)$ correspondence is fixed, the translation scheme between states is also fixed (i.e. the two theories are inter-translatable only if they have the "same" states relative to the fixed correspondence $\pi(a) \to \phi(a)$, and hence the same states relative to A).