AST 553. Plasma Waves and Instabilities

Syllabus

Introduction

❖ Basic concepts

- Waves in plasmas as oscillations of electromagnetic field
- Maxwell's equations, Gauss's laws as initial conditions
- Model of a linear medium, conductivity operator
- Conductivity in the spatial representation
- Ohm's law, spatial and temporal dispersion
- Susceptibility operator, dielectric operator
- Maxwell's equations for linear media
- Frequency operator, wavevector operator
- Dispersion operators for electrostatic and electromagnetic waves

❖ Waves in homogeneous linear media

- Model of a homogeneous linear medium
- Spectral representation of the conductivity operator
- Convolution theorem, physical meaning of $\boldsymbol{\sigma}(\omega, \boldsymbol{k})$
- Kramers–Kronig relations
- Spectral representation of the susceptibility operator
- Dielectric tensor, dispersion tensor, refractive index
- Applicability of the electrostatic approximation
- Dispersion relation, dispersion branches, scalar and vector waves
- Wave polarization, polarization vectors
- Quasimonochromatic waves
- Phase velocity and group velocity
- Dispersive and dispersionless waves
- Schrödinger equation for the wave envelope

❖ Nonmagnetized cold plasma as a linear medium

- Linearized fluid equations
- Conductivity, susceptibility, and dielectric operators
- Conductivity in the coordinate and spectral representation

- Dispersion tensor, dispersion branches
- Applicability, infinitesimally weak collisions

❖ **Waves in homogeneous nonmagnetized cold plasma**

- Static magnetic-field mode
- Electron Langmuir oscillations
- Transverse electromagnetic waves

❖ **Basic wave transformations**

- WKB approximation
- Field structure near cutoff, Airy model
- Reinstating the WKB approximation near cutoffs
- Oblique incidence on a cutoff: TE wave
- Oblique incidence on a cutoff: TM wave

General theory of quasimonochromatic waves

❖ **Asymptotic expansion of a dispersion operator**

- Geometrical-optics (GO) parameter
- Wavevector and wavevector operator in spacetime
- Waves as vectors in Hilbert space
- Wigner–Weyl transform (WWT) and its basic properties
- Envelope dispersion operator
- Envelope equation
- Dispersion operator: symbol vs. dispersion tensor

❖ **Equations of geometrical optics**

- GO ordering, approximate envelope equation
- Scalar equation for the complex amplitude
- Local dispersion relation, local polarization
- Consistency relations
- Ray equations, quantum interpretation
- Wave-action density $\mathcal{I}$ and flux density $\boldsymbol{\mathcal{J}}$
- Connection between the group velocity, $\mathcal{I}$, and $\boldsymbol{\mathcal{J}}$
- Equation for the action density, $\mathcal{I}$ on a ray
- Spin Hall effect of light

❖ **Wave action, energy, and momentum**

- Action conservation, action as an adiabatic invariant
- $\mathcal{I}$ and $\boldsymbol{\mathcal{J}}$ for electromagnetic waves
- Wave energy, transport equation
- Wave energy flux
- Absorbed power, Ohmic heating
- Wave momentum, transport equation
- α-channeling as momentum conservation

Waves in plasmas: fluid theory

❖ Waves in cold magnetized plasma

- CCS magnetized plasma: symbol of $\hat{\boldsymbol{\epsilon}}$ vs. dielectric tensor
- Expression for $\boldsymbol{\epsilon}$ in Stix's notation (S, D, R, L, P)
- High-frequency and low-frequency limits of $\boldsymbol{\epsilon}$
- General dispersion relation
  - Biquadratic equation for the refraction index N
  - Equation for $\tan\theta$
  - Cutoffs and resonances
  - Lower-hybrid and upper-hybrid frequencies
  - Electrostatic dispersion relation
- Parallel propagation
  - Cutoffs, resonances, polarization
  - L and R waves, whistlers
- Perpendicular propagation
  - O wave: dispersion, polarization, cutoff
  - X wave: dispersion, cutoffs
- Propagation at a general angle, level repulsion
- Characteristic frequencies in magnetically confined fusion plasmas

❖ Waves in warm fluid plasma

- Fluid equations for warm plasma
- Waves in warm nonmagnetized plasma
  - Perpendicular and parallel susceptibility
  - Warm species, Langmuir waves
  - Hot species: Debye shielding
  - Ion acoustic waves, ion plasma waves
- Waves in warm magnetized plasma (qualitatively)

Waves in plasmas: kinetic theory

❖ **Introduction to kinetic theory of plasma waves**

- Probability distribution, variable transformations
- Canonical distribution, Liouville theorem
- Vlasov equation, linearized Vlasov equation
- Longitudinal conductivity
- Phase mixing
- Case–van Kampen modes
- Initial-value problem, quasimodes

❖ **Waves in nonmagnetized plasma**

- Dielectric tensor
- Landau's rule
- Transverse waves in isotropic plasma
- Longitudinal waves in isotropic plasma
- Approximate formulas for ω_r and ω_i

❖ **Electrostatic waves in Maxwellian plasma**

- Plasma dispersion function Z
- Z as an integral over the Landau contour
- Asymptotics of Z
- Langmuir waves in Maxwellian plasma
- Ion acoustic waves in Maxwellian plasma

❖ **Landau damping and kinetic instabilities**

- Passing and trapped trajectories
- Bounce frequency as a function of energy
- Width of the trapping island
- Mechanism of Landau damping
- Nonlinear saturation of Landau damping
- Inverse Landau damping
- BGK waves
- Nyquist theorem
- Stability of distributions with one and two peaks

❖ **Dispersion and dissipation in magnetized plasma**

- Solution of the linearized Vlasov equation on characteristics
- General structure of the dielectric tensor, cold limit
- Conditions of resonant absorption in not-too-hot plasma
- Resonance condition in the single-particle picture
- Landau damping: basic properties
- Transit-time magnetic pumping
    - Physical mechanism, analogy with Landau damping
    - Basic properties, effect of cross terms
- Cyclotron damping
    - Basic properties
    - Diffusion paths
    - Nonlinear saturation

❖ **Waves in magnetized plasma**

- Kinetic dielectric tensor of magnetized isotropic Maxwellian plasma
- O and X waves
- Electrostatic electron Bernstein waves (EBW)
- EBW coupling with X wave and DK waves
- Plasma heating with EBW:
    - CMA diagram for X wave
    - X–B conversion
    - O–X–B conversion
- Electrostatic ion Bernstein waves (EBW)
- Accessibility of the lower-hybrid resonance

❖ **Quasilinear theory**

- Introduction
    - Interaction with one wave: fractional nonlinearities
    - Interaction with two waves: Chirikov criterion
    - Interaction with many waves: statistical approach
- Simplified quasilinear theory for homogeneous plasma
    - Assumptions, quasilinear (QL) approximation
    - Diffusion coefficient: basic properties
    - Equation for the spectral energy density
    - Dispersion relation
    - Conservation laws of QL theory
- Broadband bump-on-tail instability

  - Formation of a quasilinear plateau
  - Momentum conservation
- Dressed particles, ponderomotive effects, and amended QL theory
  - Oscillation centers (OCs), ponderomotive energy
  - OC Hamiltonian
  - Variational principle for GO waves
  - Quasilinear diffusion for the OC distribution
  - Wave kinetic equation for on-shell fields
  - Conservation laws: OCs + waves vs. particles + fields

Homeworks

❖ **Homework 1**

- Electrostatic approximation
- Photon wave function in cold magnetized plasma
- Beam–plasma instability (cold electrostatic limit)
- Surface waves

❖ **Homework 2**

- Single-mode dynamics within geometrical optics
- Coupling of resonant waves, mode conversion

❖ **Homework 3**

- Interferometry, reflectometry, Faraday rotation
- Wave transformation in the ionosphere
- MHD waves in warm fluid plasma

❖ **Homework 4**

- Wave propagation: initial-value problem
- Longitudinal waves in Lorentzian plasma
- Two-stream instability in Lorentzian plasma

❖ **Homework 5**

- Ion acoustic waves in Maxwellian plasma
- Weibel instability in non-magnetized plasma
- Plasma susceptibility in magnetic field

❖ **Homework 6**

- Kinetic waves propagating parallel to magnetic field
- Kinetic whistler waves
- Cyclotron damping