Variational Methods in Modeling Plasma Waves: Basic Physics and Applications

I. Y. Dodin

PPPL, Princeton University

In collaboration with:

N. J. Fisch, D. E. Ruiz, C. Liu, I. Barth,
P. F. Schmit (Sandia), A. I. Zhmoginov (UC Berkeley)

National Nuclear Security Administration

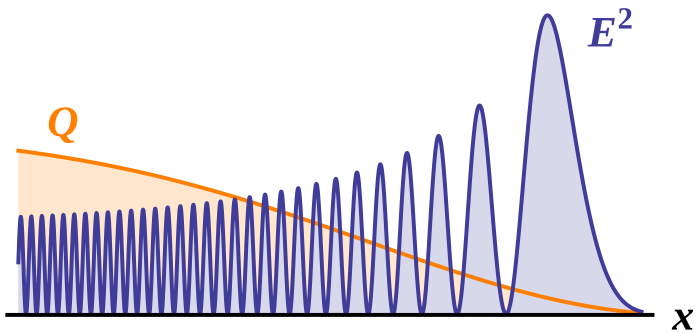
Research & Review Seminar

PPPL, 2014

PRINCETON PLASMA PHYSICS LABORATORY

Wave theory has been neglected but is still full of challenges

- Suppose an RF wave in stationary homogeneous plasma with $k^2 = Q(\omega, \mathcal{N})$. Then suppose slow $\mathcal{N}(x)$. Replacing $k \to -i\partial_x$ leads to a WKB solution.

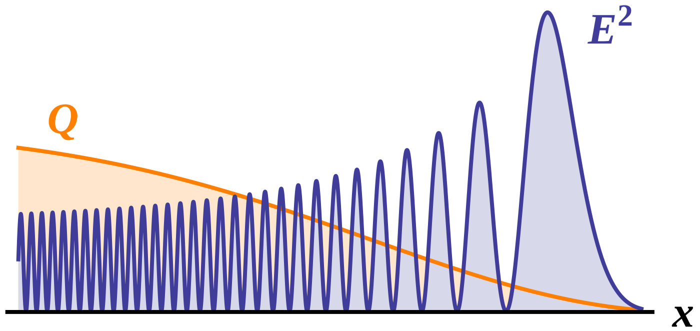


$$\frac{\partial^2 E}{\partial x^2} + Q(x)E = 0$$

$$E \propto \frac{1}{Q(x)^{1/4}} \exp\left(i\int^x Q(\tilde{x})^{1/2}\, d\tilde{x}\right)$$

Wave theory has been neglected but is still full of challenges

1. The substitution $k \to -i\partial_x$ can lead to wrong results. More severe and complex manifestations: at mode conversion in tokamaks, waves in nonstationary plasmas.

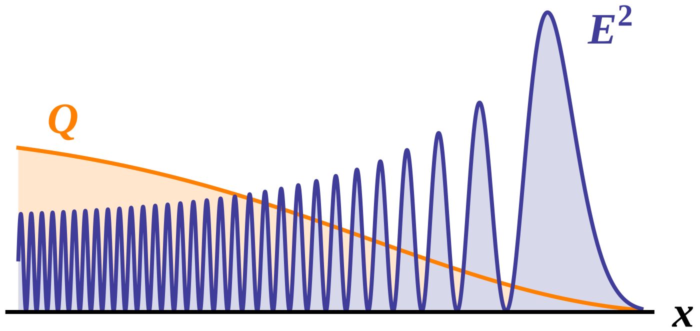


$$\frac{\partial^2 E}{\partial x^2} + 2\alpha \frac{Q'(x)}{Q(x)} \frac{\partial E}{\partial x} + Q(x)E = 0$$

$$E \propto \frac{1}{Q(x)^{\alpha+1/4}} \exp\left(i \int^x Q(\tilde{x})^{1/2}\, d\tilde{x}\right)$$

Wave theory has been neglected but is still full of challenges

1. The substitution $k \to -i\partial_x$ can lead to wrong results. More severe and complex manifestations: at mode conversion in tokamaks, waves in nonstationary plasmas.



$$\frac{\partial^2 E}{\partial x^2} + 2\alpha \frac{Q'(x)}{Q(x)} \frac{\partial E}{\partial x} + Q(x)E = 0$$

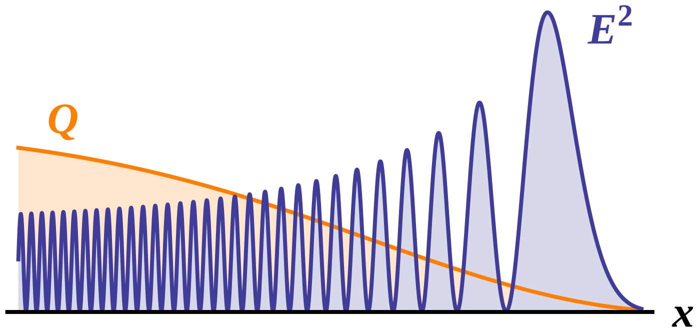
$$E \propto \frac{1}{Q(x)^{\alpha+1/4}} \exp\left(i \int^x Q(\tilde{x})^{1/2}\, d\tilde{x}\right)$$

2. Now consider nonlinear waves. Typically, the dispersion is calculated by assuming a stationary wave, $E(t,x) = \mathcal{E}(\omega t - kx)$, and some $f_0(v)$ with $V = 0$; that gives

$$\omega(k,\mathcal{E}) = \omega_0(k) + \delta\omega_{\mathrm{NL}}(k,\mathcal{E})$$

Wave theory has been neglected but is still full of challenges

1. The substitution $k \to -i\partial_x$ can lead to wrong results. More severe and complex manifestations: at mode conversion in tokamaks, waves in nonstationary plasmas.



$$\frac{\partial^2 E}{\partial x^2} + 2\alpha\,\frac{Q'(x)}{Q(x)}\,\frac{\partial E}{\partial x} + Q(x)E = 0$$

$$E \propto \frac{1}{Q(x)^{\alpha+1/4}}\,\exp\left(i\int^x Q(\tilde{x})^{1/2}\,d\tilde{x}\right)$$

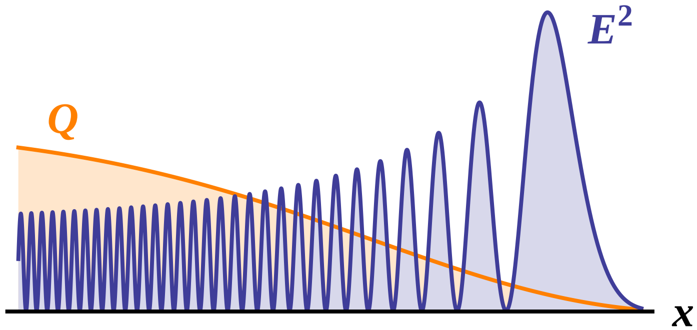
2. Now consider nonlinear waves. Typically, the dispersion is calculated by assuming a stationary wave, $E(t,x) = \mathcal{E}(\omega t - kx)$, and some $f_0(v)$ with $V = 0$; that gives

$$\omega(k,\mathcal{E}) = \omega_0(k) + \delta\omega_{\mathrm{NL}}(k,\mathcal{E}) + kV(k,\mathcal{E})$$

Increasing amplitude accelerates plasma $\to$ new $f_0(v)$ $\to$ Doppler shift. To find it, a nonstationary wave must be considered $\to$ much more complicated PDEs.

Wave theory has been neglected but is still full of challenges

1. The substitution $k \to -i\partial_x$ can lead to wrong results. More severe and complex manifestations: at mode conversion in tokamaks, waves in nonstationary plasmas.



$$\frac{\partial^2 E}{\partial x^2} + 2\alpha \frac{Q'(x)}{Q(x)} \frac{\partial E}{\partial x} + Q(x) E = 0$$

$$E \propto \frac{1}{Q(x)^{\alpha + 1/4}} \exp\left(i \int^x Q(\tilde{x})^{1/2}\, d\tilde{x} \right)$$

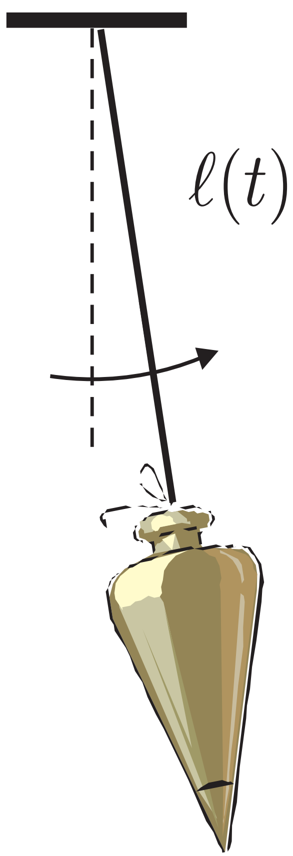
2. Now consider nonlinear waves. Typically, the dispersion is calculated by assuming a stationary wave, $E(t,x) = \mathcal{E}(\omega t - kx)$, and some $f_0(v)$ with $V = 0$; that gives

$$\omega(k,\mathcal{E}) = \omega_0(k) + \delta\omega_{\mathrm{NL}}(k,\mathcal{E}) + kV(k,\mathcal{E})$$

   Increasing amplitude accelerates plasma $\to$ new $f_0(v)$ $\to$ Doppler shift. To find it, a nonstationary wave must be considered $\to$ much more complicated PDEs.

3. What is the wave energy-momentum? No (unambiguous) answer in Maxwell's eqs. Do electrostatic waves have momentum?.. nonlinear forces on the medium?..

Plasma wave theory needs qualitatively new approaches



$$\boxed{\mathcal{E}/\omega = \mathrm{inv}}$$

- Conventional theories of plasma waves ignore special symmetries (adiabatic invariants) that nondissipative linear and nonlinear waves have as dynamic objects.

  ↳ Dissipation can be added as a perturbation, but the local symmetries can, nevertheless, be critical.

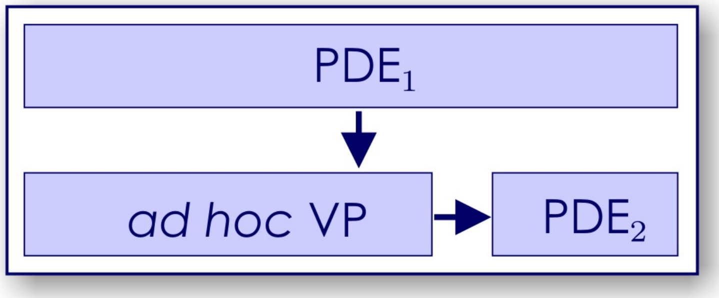
  ↳ Building on these symmetries can help modernize wave theory and find new physics.

- Investments in modernizing foundations today is an opportunity to lead wave theory in the future.

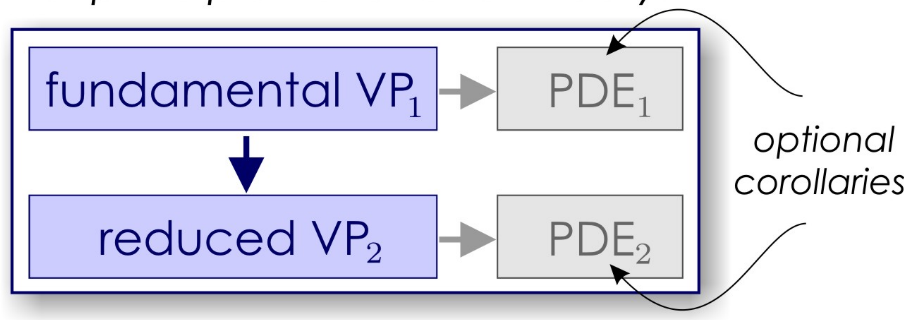
- It also helps continue the tradition of innovation that PPPL has had in wave physics and Hamiltonian dynamics:
  - current drive for waves in all frequency regimes;
  - alpha channeling for tokamaks and mirror machines;
  - classic research by Stix, Dewar, Kruskal, Bernstein, Greene...

The way to deal with symmetries is to develop a field theory

common variational theories:



first-principle variational theory:



- Dealing with *given* variational principles (VP) is understood, especially for linear nondissipative waves. Recent book: *Tracy, Brizard, Richardson, & Kaufman (2014)*.

$$\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x_i}\right)^2\tilde{n}+\omega_p^2\tilde{n}-C_{j\ell}\frac{\partial^2\tilde{n}}{\partial x_j\,\partial x_\ell}+\frac{2k_\ell}{k^2}\left(\frac{\partial\tilde{n}}{\partial t}+u_i\frac{\partial\tilde{n}}{\partial x_i}\right)\left(\frac{\Omega}{\omega_p}\frac{\partial\omega_p}{\partial x_\ell}+k_j\frac{\partial u_j}{\partial x_\ell}\right)-\left(\delta_{js}+\frac{k_jk_s}{k^2}\right)\frac{\partial C_{s\ell}}{\partial x_j}\frac{\partial\tilde{n}}{\partial x_\ell}=0$$

- But, except in simplest cases, VPs are hard to guess *ad hoc*[†].
  - general linear waves in inhomogeneous nonstationary plasma; e.g., Langmuir ↑
  - nonlinear, dissipative, and non-eikonal waves are even worse. . .

Goal: replace ad hoc VPs with more general and unified first-principle VPs

† *Dodin et al., PoP (2009); Dodin and Fisch, PRD (2010). . .*

Waves as oscillations on (unbounded) manifolds: Lagrangians

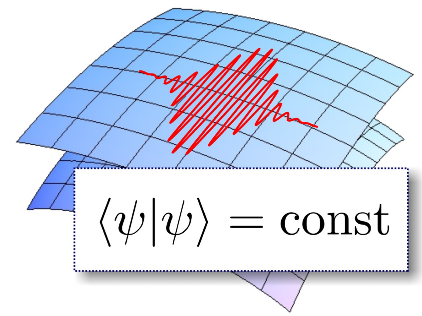
- Any wave has a "photon wave function" ψ and a Hermitian H such that

$$\boxed{\mathfrak{L} = (i/2)[\psi^\dagger(\partial_t\psi) - (\partial_t\psi^\dagger)\psi] - \psi^\dagger H(t, \mathbf{x}, -i\nabla)\psi + \text{parametric/nonlinear}}$$

$$i\partial_t\psi = H(t, \mathbf{x}, -i\nabla)\psi, \quad \psi^\dagger\psi = \text{action operator}$$

- Quantum mechanics emerges as a special case $\to$ can treat particles and waves on *precisely* the same footing.

Everything has a unified interface, $\mathfrak{L}$. What remains is to describe the coupling through this unified interface.



- Sufficient semiclassical ($\equiv$ cold-fluid) approximation:

$$\boxed{\psi = e^{i\theta}\sqrt{\mathcal{I}} \quad \xrightarrow{\text{GO}} \quad \mathfrak{L}[\theta, \mathcal{I}] = -[\partial_t\theta + H(t, \mathbf{x}, \nabla\theta)]\mathcal{I}}$$

Classical limit *is redundant* yet subsumed too: $\mathcal{I} = \delta(\mathbf{x} - \mathbf{X}(t))$,

$$\int \mathfrak{L}\, d^3x = \mathbf{P} \cdot \dot{\mathbf{X}} - H(t, \mathbf{X}, \mathbf{P}), \quad \mathbf{P} \doteq \nabla\theta(t, \mathbf{X})$$

Dodin, PLA (2014); cf. Hayes (1973); Whitham (1974)...

traditional wave theory:

wave equations
wave

Maxwell's eqs.	Newton's eqs.
EM field	particles

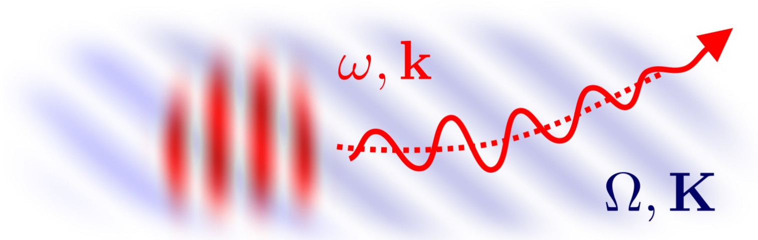
first-principle variational theory:

$\mathfrak{L}_3 = \mathfrak{L}_1 + \mathfrak{L}_2$
wave

$\mathfrak{L}_1$	$\mathfrak{L}_2$
wave	wave

Test wave experiencing cross-phase modulation (XPM)

- Scalar waves, geometrical-optics limit: $\mathfrak{L}[\theta, \mathcal{I}] = -[\partial_t\theta + H(t, \mathbf{x}, \nabla\theta)]\mathcal{I}$
  - $\delta_\theta \mathfrak{L} = 0 \longrightarrow$ action conservation theorem: $\partial_t \mathcal{I} + \nabla \cdot (\mathcal{I}\mathbf{v}_g) = 0$
  - $\delta_{\mathcal{I}} \mathfrak{L} = 0 \longrightarrow$ Hamilton-Jacobi/dispersion: $\underbrace{\partial_t\theta}_{-\omega} + H(t, \mathbf{x}, \underbrace{\nabla\theta}_{\mathbf{k}}) = 0$



- Consider a test wave modulated weakly at some $\Omega \doteq -\partial_t\Theta$ and $\mathbf{K} \doteq \nabla\Theta$:

$$\omega = \bar{\omega} + \mathrm{Re}\left[\tilde{\omega}_c(t, \mathbf{x}, \mathbf{k})\, e^{-i\Theta(t,\mathbf{x})}\right]$$

- Averaging over Θ gives a "dressed" Lagrangian, $\bar{\mathfrak{L}} = -[\partial_t\bar{\theta} + \mathcal{H}(t, \mathbf{x}, \nabla\bar{\theta})]\,\bar{\mathcal{I}}$. Instantaneous Kerr effect: now conservative & through the *linear* $\omega(\mathbf{k})$.

$$\boxed{\mathcal{H} = \bar{\omega} + \frac{\mathbf{K}}{4} \cdot \frac{\partial}{\partial \bar{\mathbf{k}}} \left(\frac{|\tilde{\omega}_c|^2}{\Omega - \mathbf{K} \cdot \bar{\mathbf{v}}_g} \right)}$$

$$d_t\bar{\mathbf{x}} = \partial_{\bar{\mathbf{k}}}\mathcal{H}, \quad d_t\bar{\mathbf{k}} = -\partial_{\bar{\mathbf{x}}}\mathcal{H}$$

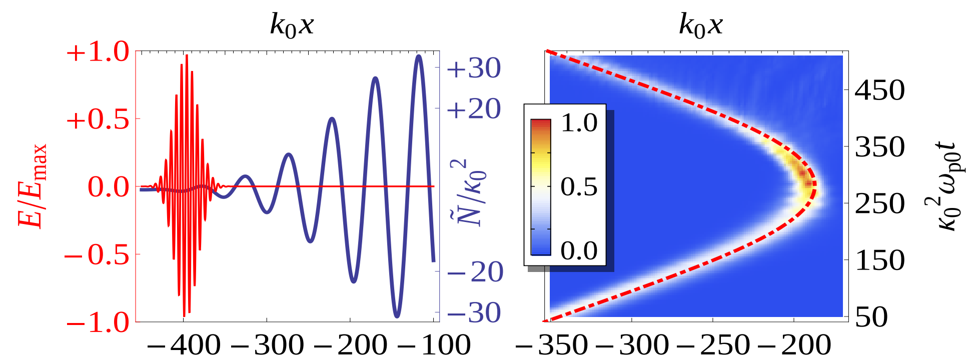
Dodin and Fisch, PRL (2014)

Ponderomotive forces on rays and particles. Asymmetric barriers

- Photons/plasmons, $\omega_0 = (\omega_p^2 + k^2c^2)^{1/2}$

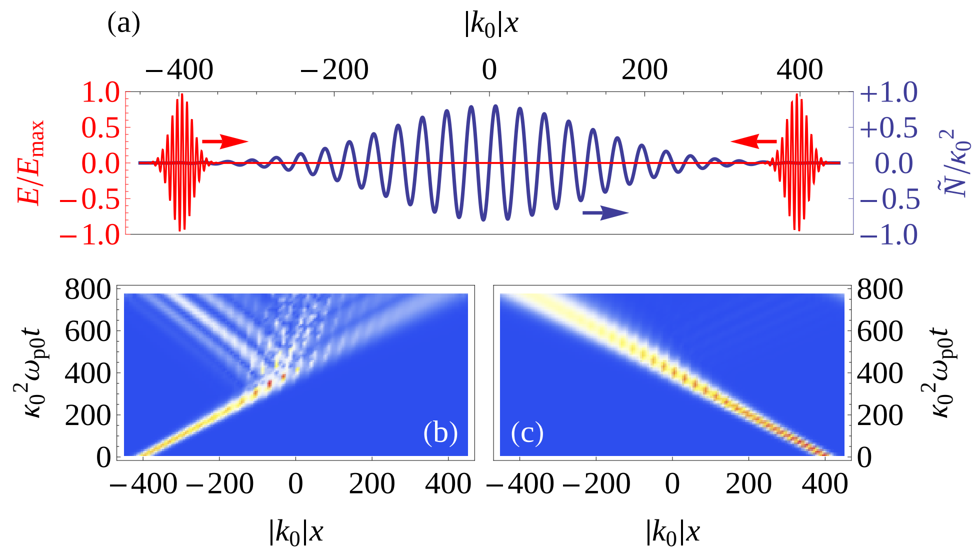
$$\mathcal{H} = \omega_0\left[1 - \underbrace{\left(\frac{\omega_p^2}{4\omega_0^2}\,\frac{\tilde{n}}{n_0}\right)^2 \frac{(\Omega^2 - K^2c^2)}{(\Omega - \mathbf{K}\cdot\bar{\mathbf{v}}_g)^2}}_{\text{Kerr (ponderomotive) term, } \lessgtr 0}\right]$$

Nonreciprocal propagation due to the "group resonance" at $\Omega = \mathbf{K}\cdot\bar{\mathbf{v}}_g$.



- Electrons, $\hbar\omega_0 = \hbar^2k^2/2m + e\tilde{\varphi}$

$$\mathcal{H} = \frac{P^2}{2m} + \frac{e^2|\tilde{E}|^2/4m}{(\Omega - \mathbf{K}\cdot\mathbf{V})^2 \underbrace{-(\hbar K^2/2m)^2}_{\text{beyond GO}}}$$



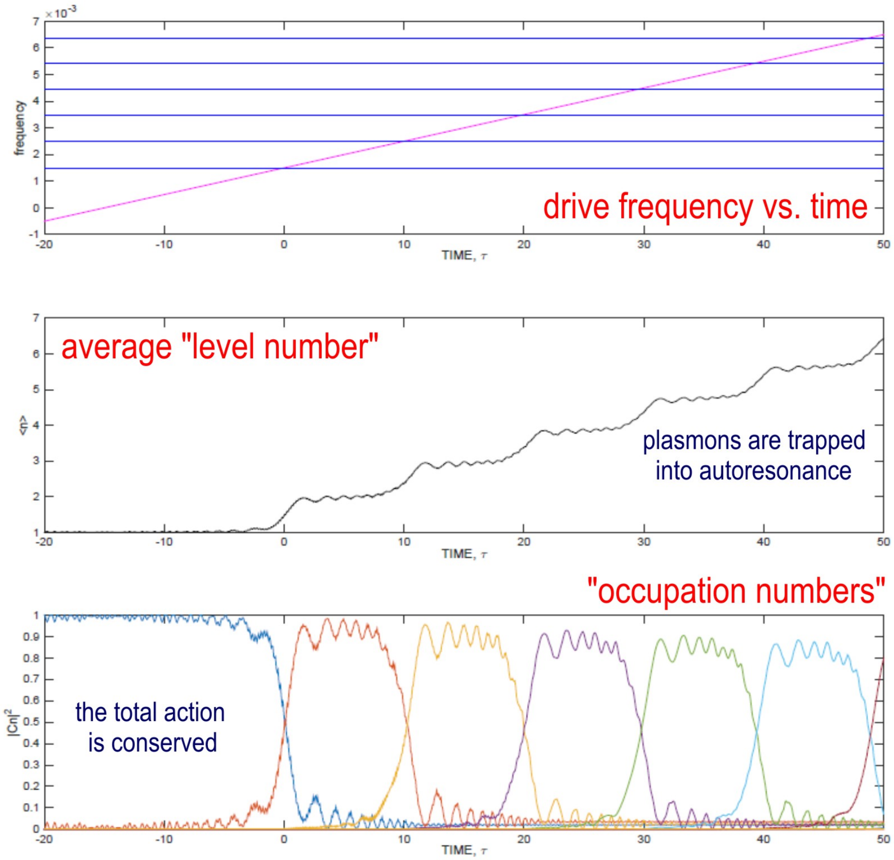
- Quantumlike derivations are often simpler, since equations are kept linear.
- EM waves may be better modeled with *vector rays*. "Spin" effects at ray tracing?

Dodin and Fisch, PRL (2014); Ruiz and Dodin, in preparation

Hamiltonian dynamics of multiple coherent waves. Mode conversion

- Example: Langmuir waves coupled via low-frequency drive $\sim \eta(t)\cos(k_N x)$

$$\boxed{i\dot{\psi}_n = \sum_m H_{nm}(t)\psi_m = \omega_n\psi_n + \eta(t)\left(\psi_{n-N} + \psi_{n+N}\right)}$$



- Plasma mode conversion $\equiv$ quantum Landau-Zener problem. The total action is conserved manifestly:

$$\sum_n |\psi|^2 = \text{const}$$

- Autoresonant trapping of plasmons at

$$\Omega \approx K v_g$$

- Quantum-ladder-climbing emulation in a classical system. BGK waves made of plasmons/quantum particles

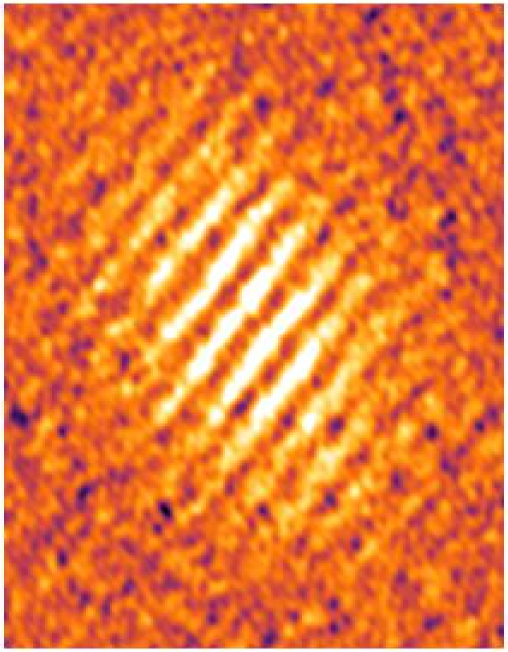
Barth, Dodin, and Fisch, in preparation

Self-consistent dynamics and dispersion. Photons are polarizable

- Low-frequency wave $(\Theta, \mathscr{J})$ interacts with many high-frequency waves $(\theta_n, \mathcal{I}_n)$:

$$\mathfrak{L} = -[\partial_t\Theta + \Omega_0]\mathscr{J} - \textstyle\sum_n[\partial_t\bar{\theta}_n + \underbrace{(\omega_n + \beta_n \mathscr{J})}_{\text{ponderomotive effect}}]\bar{\mathcal{I}}_n$$

$$= -[\partial_t\Theta + \underbrace{(\Omega_0 + \textstyle\sum_n \beta_n \bar{\mathcal{I}}_n)}_{\text{self-consistent frequency, }\Omega}]\mathscr{J} - \underbrace{\textstyle\sum_n \cancel{(\partial_t\bar{\theta}_n)\bar{\mathcal{I}}_n}}_{\text{independent of }(\Theta,\, \mathscr{J})}$$



- EM wave interacting with particles and HF waves:

$$\boxed{\mathfrak{L} = \frac{1}{16\pi}\tilde{\mathbf{E}}^* \cdot (1 + \underbrace{\hat{\boldsymbol{\chi}}_{\text{particles}}}_{\text{standard }\hat{\boldsymbol{\epsilon}}} + \underbrace{\hat{\boldsymbol{\chi}}_{\text{waves}}}_{\text{“anomalous”}}) \cdot \tilde{\mathbf{E}} - \frac{|\tilde{\mathbf{B}}|^2}{16\pi}}$$

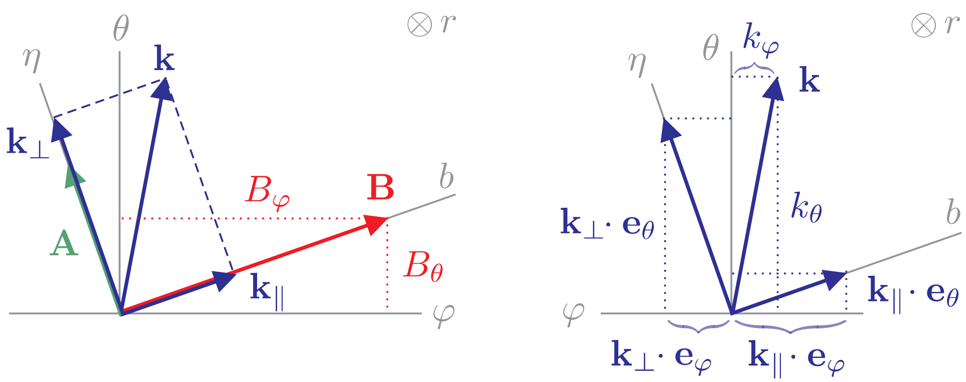
- Wave quanta contribute to linear $\hat{\boldsymbol{\epsilon}}$ just like electrons and ions. An improved formula for a *Langmuir wave in a photon bath*: extra term + evolution allowed.

$$\boxed{\Omega(\mathbf{K}) \approx \Omega_0(\mathbf{K}) - \frac{K^2\omega_{p0}^4}{8m_e n_0 \Omega}\int \frac{\nabla_{\bar{\mathbf{k}}} \cdot [\mathbf{K} f_0(\bar{\mathbf{k}})]}{(\Omega - \mathbf{K}\cdot\bar{\mathbf{v}}_g)\omega_0^2}\, d^3\bar{k} - \frac{K^2\omega_{p0}^4}{8m_e n_0 \Omega}\int \frac{f_0(\bar{\mathbf{k}})}{\omega_0^3}\, d^3\bar{k}.}$$

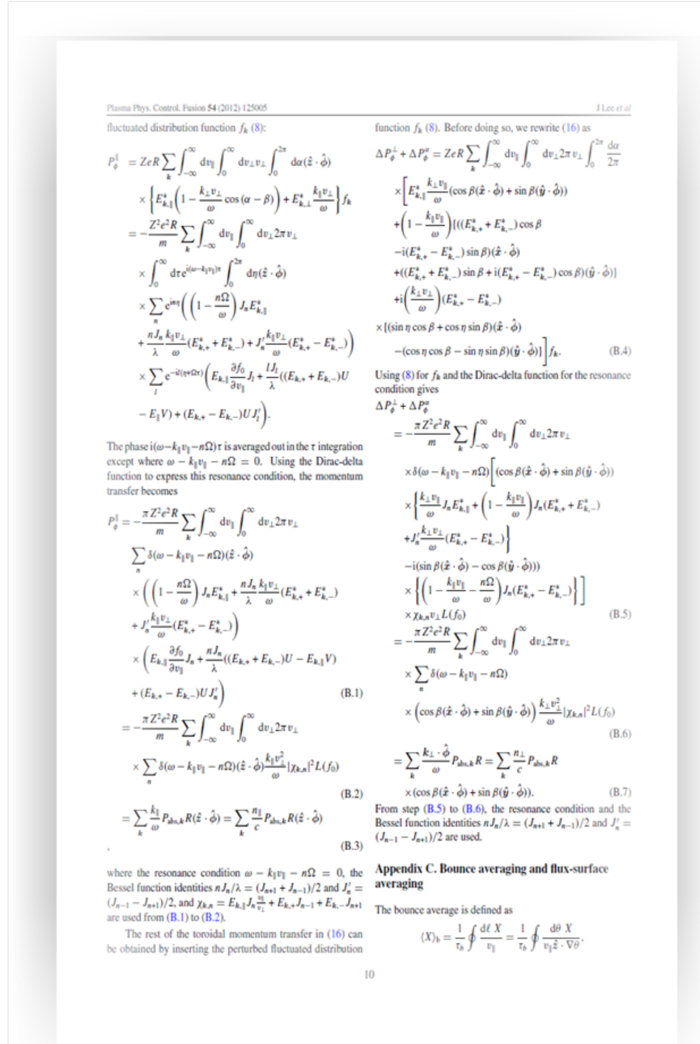
cf. Tsytovich (1970); Bingham et al. (1997); Mendonça (2000); Dylov and Fleischer (2008)...

Wave energy and momentum. Effect of $k_\perp$ on RF-driven rotation

- The variational approach is the *only* unambiguous and reliable way to define the wave energy-momentum, both "canonical" and "kinetic".
  - Example: The effect of $k_\perp$ has been unclear, since $k_\perp$ does not cause resonant acceleration along $\mathbf{B}_0$.



  - Quasilinear calculation: [Lee, Parra, Parker, and Bonoli, PPCF (2012)]. But detailed dynamics is unimportant. The symmetry of $\mathfrak{L}$ already gives



$$\Delta P_\perp = k_\perp \Delta \mathcal{E}/\omega \;\rightarrow\; \Delta r_{\mathrm{gc}} = c k_\perp \Delta \mathcal{E}/(eB_0\omega)$$

$\rightarrow$ charge separation $\rightarrow$ Poynting momentum

Guan, Dodin, Qin, Liu, and Fisch, PoP (2013)

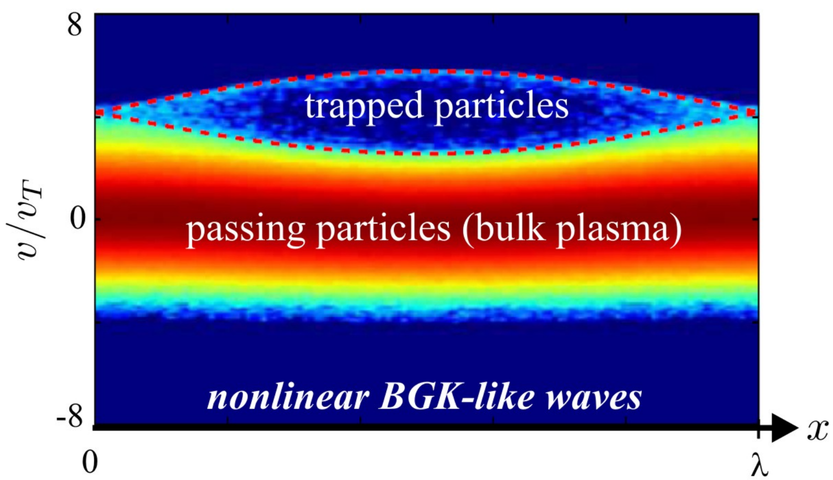
Same ideas apply to nonlinear waves, where $\mathcal{H}$ is more complicated

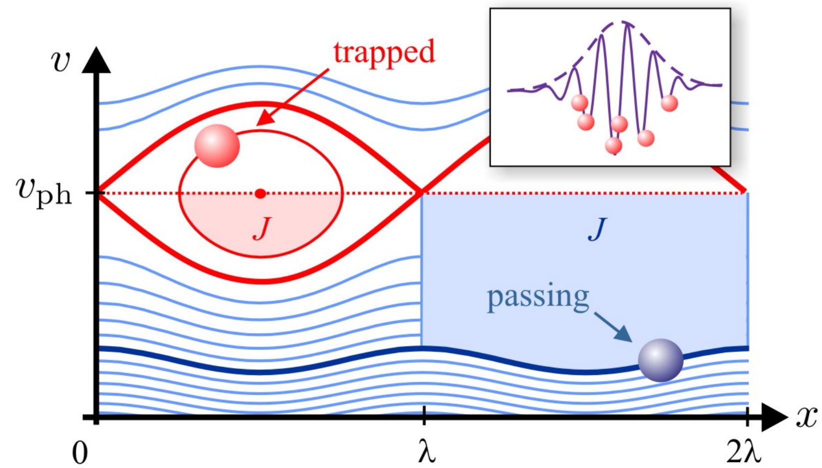
- Klein-Gordon Hamiltonian → Chen-Sudan Lagrangian, Akhiezer-Polovin waves

$$\omega^2 \approx k^2c^2 + \omega_p^2\,[1 + \langle (e\mathbf{A}/mc^2)^2\rangle]^{-1/2} \quad \textit{(same in the Dirac model)}$$

Ruiz and Dodin, in preparation

- Advanced kinetic nonlinearities: waves with autoresonant trapped particles





$$\mathfrak{L} = \frac{\langle E^2\rangle}{8\pi} - \Big[n_{\rm p}\underbrace{\langle \varepsilon_{\rm p}(J) \pm \omega J + m v_{\rm ph}^2/2\rangle}_{\text{generalized ponderomotive potential}} + \underbrace{n_{\rm t}\langle \varepsilon_{\rm t}(J)\rangle}_{\text{bouncing}} \Big] + \underbrace{n_{\rm t} m v_{\rm ph}^2/2}_{\text{t.p. inertia}}$$

Dodin, Fusion Sci. Tech. (2014); Dodin and Fisch, PRL (2011), PoP (2012a,b,c); Schmit et al., PRL (2013)

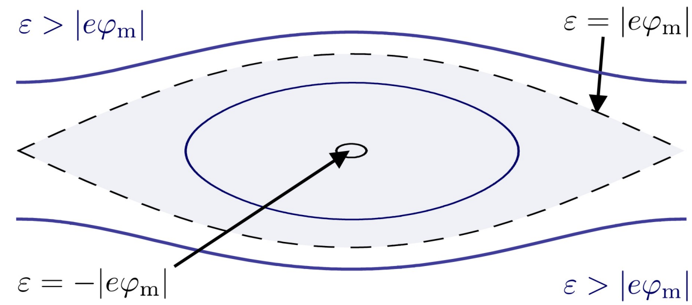
Example: BGK wave dispersion in the limit when $E \sim e^{ikx}$

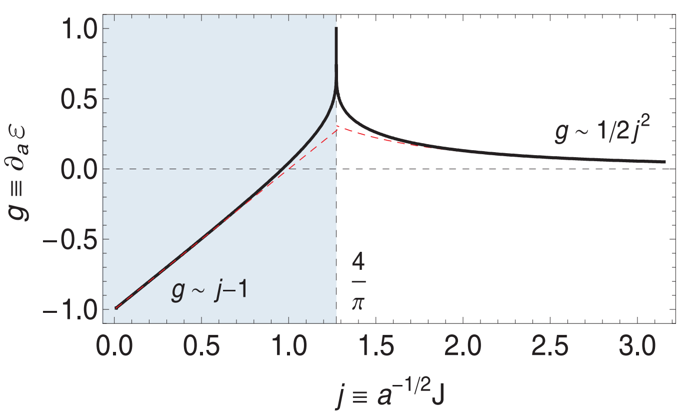
$$\mathfrak{L}(a, \omega, k) = E_{\mathrm{m}}^2/16\pi - \left[n_{\mathrm{p}}\langle\varepsilon_{\mathrm{p}}\rangle + \cancel{n_{\mathrm{p}}\langle m v_{\mathrm{ph}}^2/2 \pm \omega J\rangle} + n_{\mathrm{t}}\langle\varepsilon_{\mathrm{t}}\rangle\right] + \cancel{n_{\mathrm{t}} m v_{\mathrm{ph}}^2/2}$$

$a \equiv keE_{\mathrm{m}}/(m\omega^2)$, ε is the single-particle energy in the wave rest frame

$$0 = \partial_a \mathfrak{L} = \partial_a\left[E_{\mathrm{m}}^2/16\pi - n\langle\varepsilon\rangle\right]$$

$$\boxed{\omega^2 = \frac{2\omega_p^2}{a}\int_0^\infty g(J)\, \underbrace{F(J)\, dJ}_{\text{ensemble averaging}}}$$





↳ Deeply trapped particles: $\omega^2 \approx \omega_{\mathrm{L}}^2 - 2\omega_{\mathrm{t}}^2 a^{-1}$

↳ Flat-top beam: $\omega^2 \approx \omega_{\mathrm{L}}^2 - \beta\, a^{-1/2}$

↳ Smooth distribution: $\omega_{\mathrm{NL}} \approx C_1\, a^{1/2} + C_2 \ln a$

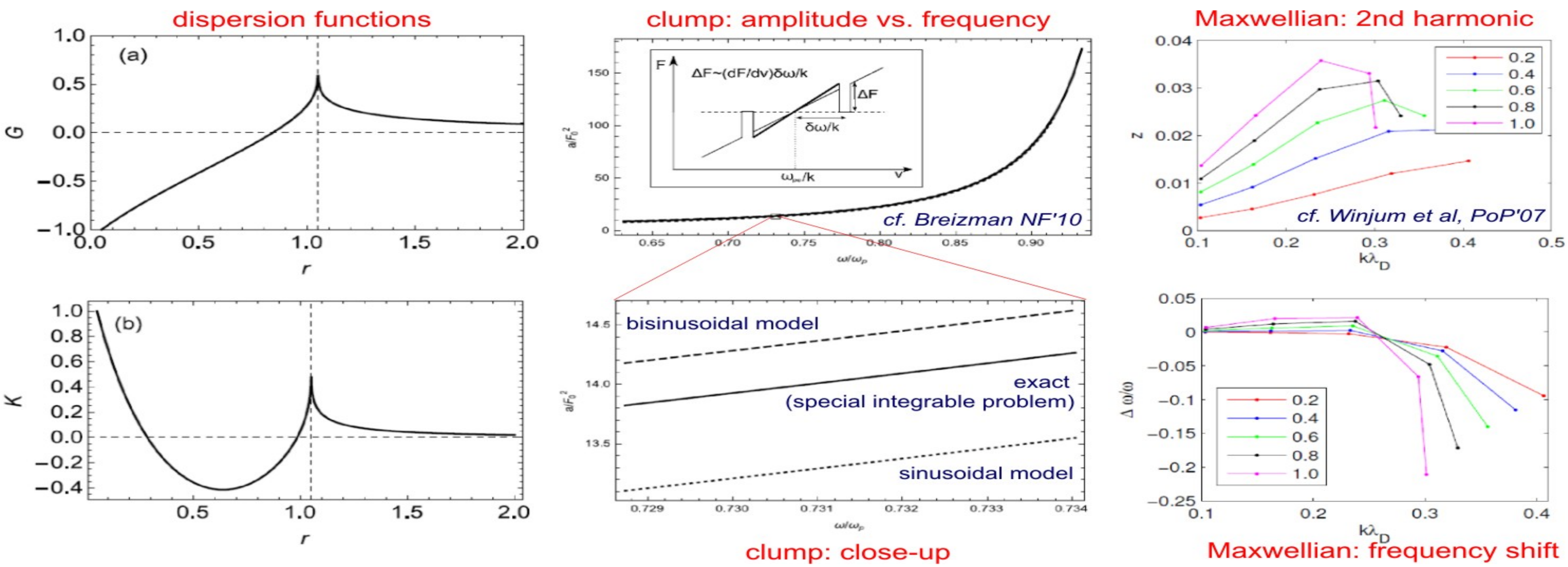
cf. Goldman and Berk (1971); Manheimer and Flynn (1971); Dewar (1972); Rose and Russell (2001); Khain and Friedland (2007); Krasovsky (2007). . .

- There are 3-4 communities working on this and largely ignoring each other.

Anharmonic waves: clumps & holes, Maxwellian distribution

- Bisinusoidal-wave model: $\phi = a_1 \cos\theta + a_2 \cos 2\theta$, two independent amplitudes

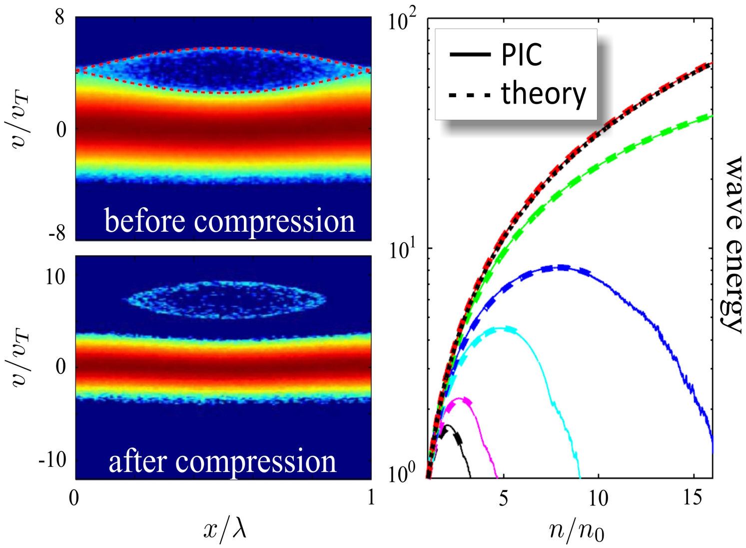
$$\boxed{\frac{\omega^2}{\omega_p^2} = \frac{2}{a_1}\int_0^\infty G(J, a_1, a_2)\, F(J)\, dJ = \frac{1}{2a_2}\int_0^\infty K(J, a_1, a_2)\, F(J)\, dJ}$$



- Indistinguishable from Breizman's exact solution for a clump. Automatically accounts for both fluid and kinetic nonlinearities, unlike in *ad hoc* models.

Liu and Dodin (in preparation); cf. Winjum et al. (2007); Krasovsky (2007); Breizman (2010)

Adiabatic dynamics of BGK-like waves



- Wave action ($\sim$ momentum) conservation in compressing plasma: e.g., k = const

$$v_{\text{ph}} = \omega/k \approx \omega_p(t)/k \sim \sqrt{n(t)}$$

$$\boxed{\text{inv} = \int \partial_\omega \mathfrak{L}\, dx \;\sim\; E_{\text{m}}^2\, \partial_\omega \epsilon/(16\pi) + n_{\text{t}} m v_{\text{ph}}/k}$$

Schmit et al., PRL (2013); Dodin and Fisch, PoP (2012a,b,c)

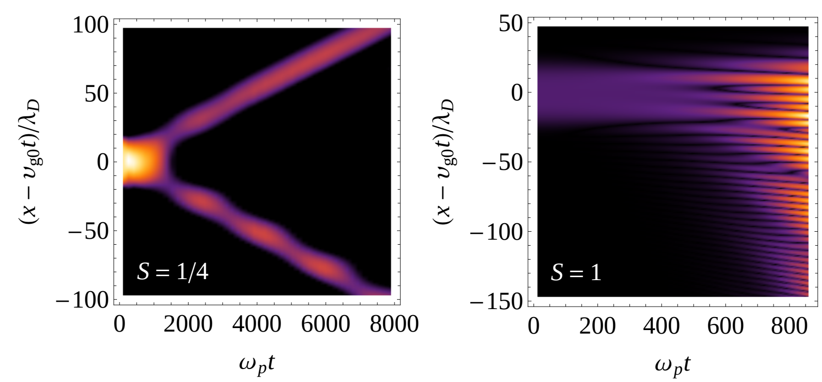
- Self-action is *not* described by the NLSE

$$i(\partial_t \psi + v_{g0}\, \partial_x \psi) + \tfrac{1}{2}\, v'_{g0}\, \partial^2_{xx} \psi \neq \omega_{\text{NL}} \psi$$

- The modulational instability is revised:

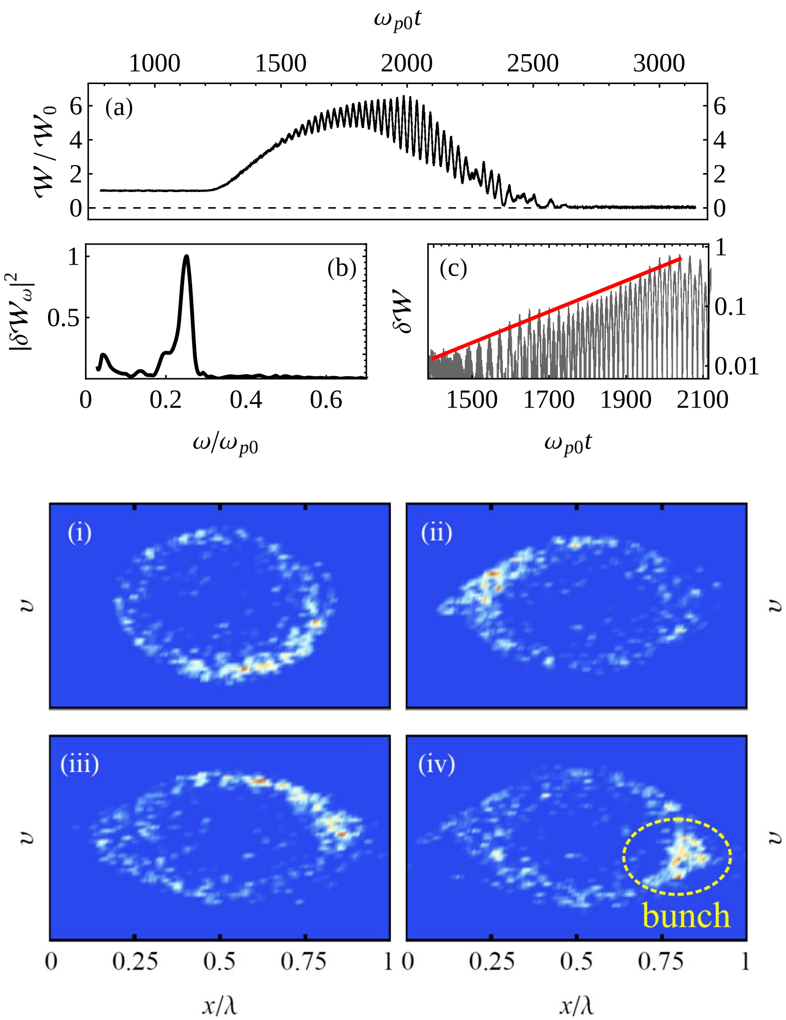
$$\gamma \approx v_{g0}\, \Delta k \sqrt{\left(\frac{eE_{\text{m}}k}{m\omega_p^2}\right) \frac{S}{2}\, (2S - 1)}\;\Big]$$

cf. Dewar et al. (1972); Rose (2005)...



$$S = \frac{\text{trapped-}e\text{ energy flux}}{\text{passing-}e\text{ energy flux}}$$

Nonadiabatic dynamics. Negative-mass instability (NMI)



cf. Smith and Pereira (1978); Adam et al. (1981); Tennyson et al. (1994); Herrera et al. (2013)...

- New instability is identified for BGK waves: trapped particle *bunching* due to $\Omega'(J) < 0$

  cf. Kruer et al. (1969); Goldman and Berk (1971); Liu (1972). . .

$$\boxed{1 = \frac{\omega_t^2}{\epsilon_{\rm eff}} \int_0^{J_*} \frac{J F_t'(J)}{\Omega^2(J) - (\tilde{\omega} - \tilde{k} v_{\rm ph})^2}\, dJ}$$

$$1/\epsilon_{\rm eff} \equiv 1/\epsilon(\tilde{\omega} + \omega, \tilde{k} + k) + 1/\epsilon(\tilde{\omega} - \omega, \tilde{k} - k)$$

- Applies also to aperiodic waves. Akin to the NMI in accelerators, FELs, ion traps. . .

  Kolomenskii and Lebedev (1959); Strasser et al. (2002). . .

- Follow-up studies by other groups

  Brunner et al. (2014); Hara et al, submitted (2014)

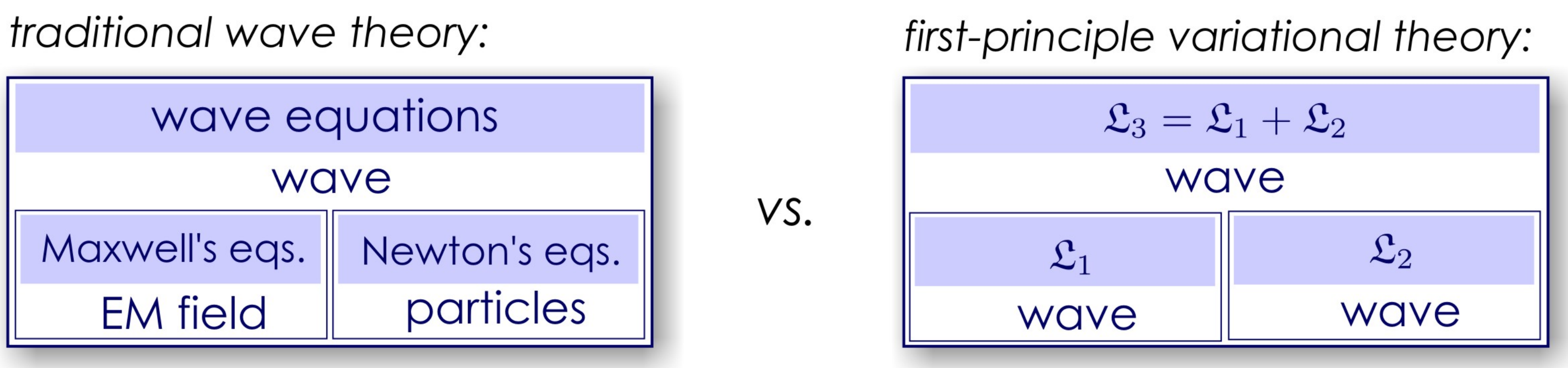
- Connection with KEEN waves

  Dodin and Fisch, PoP (2014)

Dodin, Schmit, Rocks, and Fisch, PRL (2013)

Summary

- New "object-oriented" approach to linear and nonlinear plasma waves is being developed that blends variational methods of fluid and quantum theories.
  - Every object is a wave, every interaction is ponderomotive, every equation roots in a VP.
  - Lagrangians are unified and serve as building blocks to model multiple-wave coupling.



- Immediate advantages:
  - Unified and manifestly conservative equations, even beyond geometrical optics.
  - Lowest-order wave-wave coupling gives *linear* dispersion, "photons are polarizable".
  - Energy-momentum is well defined, even dissipation enters naturally.
- Applications to nonlinear waves (e.g., BGK-like) and quantum plasmas

PRINCETON PLASMA PHYSICS LABORATORY

Research status and plans

- Present: developing robust understanding of variational principles (VPs) for waves
  - VPs for dissipative waves; photon polarization & Landau damping *(with A. I. Zhmoginov, UCB)*
  - Origin and limitations of reduced VPs explored through quantum parallels *(with D. E. Ruiz)*
  - Resonant ponderomotive effect on photons: BGK modes comprised of plasmons *(with I. Barth)*
  - Nonlinear dispersion of anharmonic nonlinear waves, including BGK modes *(with C. Liu)*
  - KEEN waves as the NMI-saturated modes *(with A. H. Hakkim and U. Verma)*
  - Reduced analytic model for nonlinear dissipation of BGK-like waves through separatrix crossing...
- Future: improving robustness of modeling linear RF waves in fusion and other plasmas; synergistic nonlinear physics of energetic-particle modes and LPI
  - Manifestly conservative analytical framework for modeling linear RF waves in realistic settings
  - Symplectic algorithms for modeling linear RF waves (quantum methods?), test simulations
  - Nonlinear waves: manifestly conservative reduced nonlinear models + trapped particles
  - Working on synergistic applications in RF physics (as in the case of RF-driven plasma rotation)
- Teaching: updating the "waves" course [cf. Tracy, Brizard, Richardson, & Kaufman, *Ray tracing and beyond: phase space methods in plasma wave theory* (2014)]

The work follows the recently approved 5-year plan for the BPPG effort in modernizing RF basic theory. Synergies with campus grants and teaching provide 50% of funding.

References

[*Adam et al. (1981)*]
J. C. Adam, G. Laval, and I. Mendonca, *Time-dependent nonlinear Langmuir waves*, Phys. Fluids **24**, 260 (1981).

[*Akhiezer and Polovin (1956)*]
A. I. Akhiezer and R. V. Polovin, *Theory of wave motion of an electron plasma*, Zh. Eksp. Teor. Fiz. **30**, 915 (1956) [Sov. Phys. JETP **3**, 696 (1956)].

[*Bernstein et al. (1957)*]
I. B. Bernstein, J. M. Greene, and M. D. Kruskal, *Exact nonlinear plasma oscillations*, Phys. Rev. **108**, 546 (1957).

[*Bingham et al. (1997)*]
R. Bingham, J. T. Mendonça, and J. M. Dawson, *Photon Landau damping*, Phys. Rev. Lett. **78**, 247 (1997).

[*Breizman (2010)*]
B. N. Breizman, *Nonlinear travelling waves in energetic particle phase space*, Nucl. Fusion **50**, 084014 (2010).

[*Brunner et al. (2014)*]
S. Brunner, R. L. Berger, B. I. Cohen, L. Hausammann, and E. J. Valeo, *Kinetic simulations and reduced modeling of longitudinal sideband instabilities in non-linear electron plasma waves*, Phys. Plasmas **21**, 102104 (2014).

[*Chen and Sudan (1993)*]
X. L. Chen and R. N. Sudan, *Two-dimensional self-focusing of short intense laser pulse in underdense plasma*, Phys. Fluids B **5**, 1336 (1993).

[*Dewar (1972)*]
R. L. Dewar, *Frequency shift due to trapped particles*, Phys. Fluids **15**, 712 (1972).

[*Dewar et al. (1972)*]
R. L. Dewar, W. L. Kruer, and W. M. Manheimer, *Modulational instabilities due to trapped electrons*, Phys. Rev. Lett. **28**, 215 (1972).

[*Dodin (2014)a*]
I. Y. Dodin, *Geometric view on noneikonal waves*, Phys. Lett. A **378**, 1598 (2014).

[*Dodin (2014)b*]
I. Y. Dodin, *On variational methods in the physics of plasma waves*, Fusion Sci. Tech. **65**, 54 (2014).

[*Dodin and Fisch (2010)*]
I. Y. Dodin and N. J. Fisch, *On the evolution of linear waves in cosmological plasmas*, Phys. Rev. D **82**, 044044 (2010).

[*Dodin and Fisch (2011)*]
I. Y. Dodin and N. J. Fisch, *Nonlinear dispersion of stationary waves in collisionless plasmas*, Phys. Rev. Lett. **107**, 035005 (2011).

[*Dodin and Fisch (2012)a*]
I. Y. Dodin and N. J. Fisch, *Adiabatic nonlinear waves with trapped particles: I. General formalism*, Phys. Plasmas **19**, 012102 (2012).

[*Dodin and Fisch (2012)b*]
I. Y. Dodin and N. J. Fisch, *Adiabatic nonlinear waves with trapped particles: II. Wave dispersion*, Phys. Plasmas **19**, 012103 (2012).

[*Dodin and Fisch (2012)c*]
I. Y. Dodin and N. J. Fisch, *Adiabatic nonlinear waves with trapped particles: III. Wave dynamics*, Phys. Plasmas **19**, 012104 (2012).

[*Dodin and Fisch (2012)d*]
I. Y. Dodin and N. J. Fisch, *Axiomatic geometrical optics,*

Back

References

Abraham-Minkowski controversy, and photon properties derived classically, Phys. Rev. A **86**, 053834 (2012).

[*Dodin and Fisch (2014)a*]
I. Y. Dodin and N. J. Fisch, *On the nature of kinetic electrostatic electron nonlinear (KEEN) waves*, Phys. Plasmas **21**, 034501 (2014).

[*Dodin and Fisch (2014)b*]
I. Y. Dodin and N. J. Fisch, *Ponderomotive forces on waves in modulated media*, Phys. Rev. Lett. **112**, 205002 (2014).

[*Dodin et al. (2009)*]
I. Y. Dodin, V. I. Geyko, and N. J. Fisch, *Langmuir wave linear evolution in inhomogeneous nonstationary anisotropic plasma*, Phys. Plasmas **16**, 112101 (2009).

[*Dodin et al. (2013)*]
I. Y. Dodin, P. F. Schmit, J. Rocks, and N. J. Fisch, *Negative-mass instability in nonlinear plasma waves*, Phys. Rev. Lett. **110**, 215006 (2013).

[*Dylov and Fleischer (2008)*]
D. V. Dylov and J. W. Fleischer, *Observation of all-optical bump-on-tail instability*, Phys. Rev. Lett. **100**, 103903 (2008).

[*Goldman and Berk (1971)*]
M. V. Goldman and H. L. Berk, *Stability of a trapped particle equilibrium*, Phys. Fluids **14**, 801 (1971).

[*Guan et al. (2013)*]
X. Guan, I. Y. Dodin, H. Qin, J. Liu, and N. J. Fisch, *On plasma rotation induced by waves in tokamaks*, Phys. Plasmas **20**, 102105 (2013).

[*Hayes (1973)*]
W. D. Hayes, *Group velocity and nonlinear dispersive wave propagation*, Proc. R. Soc. Lond. A **332**, 199 (1973).

[*Herrera et al. (2013)*]
M. Herrera, T. M. Antonsen, E. Ott, and S. Fishman, *Dynamic localization of a weakly interacting Bose-Einstein condensate in an anharmonic potential*, Phys. Rev. A **87**, 041603(R) (2013).

[*Khain and Friedland (2007)*]
P. Khain and L. Friedland, *A water bag theory of autoresonant Bernstein-Greene-Kruskal modes*, Phys. Plasmas **14**, 082110 (2007).

[*Kolomenskii and Lebedev (1959)*]
A. A. Kolomenskii and A. N. Lebedev, *Stability of a charged beam in storage systems*, Atomnaya Energiya **7**, 549 (1959) [Sov. J. Atomic Energy **7**, 1013 (1961)].

[*Krasovsky (2007)*]
V. L. Krasovsky, *Steady nonlinear electrostatic plasma wave in a weak transverse magnetic field*, J. Plasma Phys. **73**, 179 (2007).

[*Kruer et al. (1969)*]
W. L. Kruer, J. M. Dawson, and R. N. Sudan, *Trapped-particle instability*, Phys. Rev. Lett. **23**, 838 (1969).

[*Liu (1972)*]
C. S. Liu, *Instability of a large-amplitude plasma wave due to inverted trapped particle population*, J. Plasma Phys. **8**, 169 (1972).

[*Manheimer and Flynn (1971)*]
W. M. Manheimer and R. W. Flynn, *Formation of stationary large amplitude waves in plasmas*, Phys. Fluids **14**, 2393 (1971).

[*Mendonça (2000)*]
J. T. Mendonça, *Theory of photon acceleration* (IOP, Philadelphia, 2000).

Back

References

[*Rose (2005)*]
H. A. Rose, *Langmuir wave self-focusing versus decay instability*, Phys. Plasmas **12**, 012318 (2005).

[*Rose and Russell (2001)*]
H. A. Rose and D. A. Russell, *A self-consistent trapping model of driven electron plasma waves and limits on stimulated Raman scatter*, Phys. Plasmas **8**, 4784 (2001).

[*Schmit et al. (2010)*]
P. F. Schmit, I. Y. Dodin, and N. J. Fisch, *Controlling hot electrons by wave amplification and decay in compressing plasma*, Phys. Rev. Lett. **105**, 175003 (2010).

[*Schmit et al. (2013)*]
P. F. Schmit, I. Y. Dodin, J. Rocks, and N. J. Fisch, *Nonlinear amplification and decay of phase-mixed waves in compressing plasma*, Phys. Rev. Lett. **110**, 055001 (2013).

[*Smith and Pereira (1978)*]
G. R. Smith and N. R. Pereira, *Phase-locked particle motion in a large-amplitude plasma wave*, Phys. Fluids **21**, 2253 (1978).

[*Strasser et al. (2002)*]
D. Strasser, T. Geyer, H. B. Pedersen, O. Heber, S. Goldberg, B. Amarant, A. Diner, Y. Rudich, I. Sagi, M. Rappaport, D. J. Tannor, and D. Zajfman, *Negative mass instability for interacting particles in a 1D box: theory and application*, Phys. Rev. Lett. **89**, 283204 (2002).

[*Tennyson et al. (1994)*]
J. L. Tennyson, J. D. Meiss, and P. J. Morrison, *Self-consistent chaos in the beam-plasma instability*, Physica D **71**, 1 (1994).

[*Tracy et al. (2014)*]
E. R. Tracy and A. J. Brizard and A. S. Richardson and A. N. Kaufman, *Ray tracing and beyond: phase space methods in plasma wave theory* (Cambridge University Press, New York, 2014).

[*Tsytovich (1970)*]
V. N. Tsytovich, *Nonlinear effects in plasma* (Plenum Press, New York, 1970).

[*Whitham (1974)*]
G. B. Whitham, *Linear and nonlinear waves* (Wiley, New York, 1974).

[*Winjum et al. (2007)*]
B. J. Winjum, J. Fahlen, and W. B. Mori, *The relative importance of fluid and kinetic frequency shifts of an electron plasma wave*, Phys. Plasmas **14**, 102104 (2007).