Parametric decay of plasma waves near the upper-hybrid resonance

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Abstract

An intense X wave propagating perpendicularly to a dc magnetic field is unstable with respect to the parametric decay into an electron Bernstein wave (EBW) and a lower-hybrid wave (LHW). A modified theory of this effect is proposed that extends to the high-intensity regime, where the instability rate γ ceases to be a linear function of the incident-wave amplitude [1]. An explicit formula for γ is derived and expressed in terms of cold-plasma parameters. Theory predictions are in reasonable agreement with the results of the PIC simulations reported in Ref. [2].

[1] I. Y. Dodin and A. V. Arefiev, *Parametric decay of plasma waves near the upper-hybrid resonance*, Phys. Plasmas **24**, 032119 (2017).

[2] A. V. Arefiev, I. Y. Dodin, A. Köhn, E. J. Du Toit, E. Holzhauer, V. F. Shevchenko, R. G. L. Vann, *Kinetic simulations of X-B and O-X-B mode conversion and its deterioration at high input power*, Nucl. Fusion **57**, 116024 (2017).

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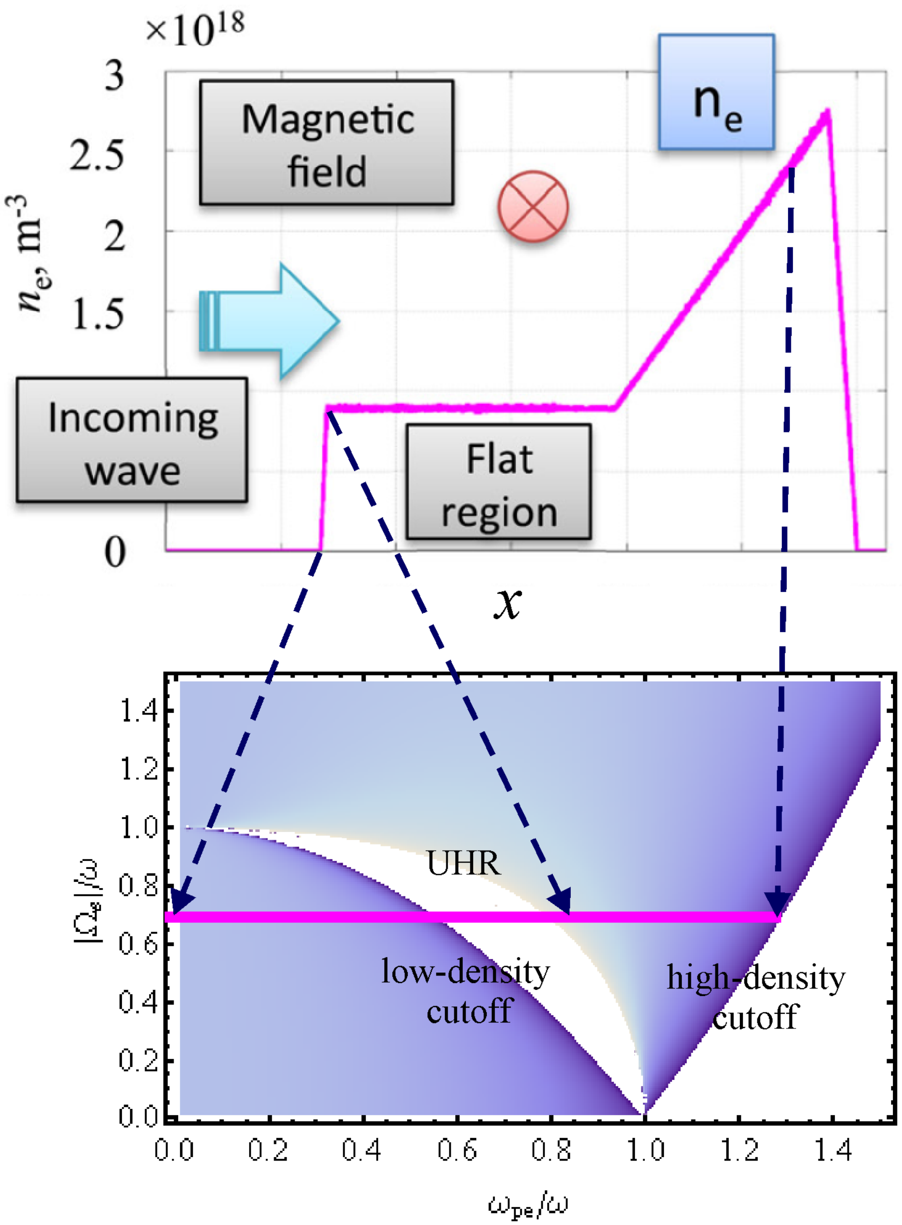
Motivation

Spherical-tokamak plasmas are typically overdense and thus inaccessible to externally-injected RF waves in the electron cyclotron range. This can be overcome by converting RF waves to EBWs, which can access the dense plasma core. But nonlinear EBW physics at high input power has not been sufficiently quantified.

To address this problem, PIC simulations of X-B and O-X-B mode conversion were done by Arefiev *et al* using EPOCH (Nucl. Fusion, 2017). It was observed numerically that, at high enough input power, the ion dynamics becomes important and short-scale oscillations are excited in high-density plasma *prior to the arrival of the EBW*.

The main purpose of this poster is to analytically explain those numerical results (in 1D) and to extend the existing PDI theory.

1D PIC simulation setup



$$n_e = 0.89 \times 10^{18}\ \text{m}^{-3}$$
$$T_e = 950\ \text{eV}$$
$$B_0 = 0.25\ \text{T}$$

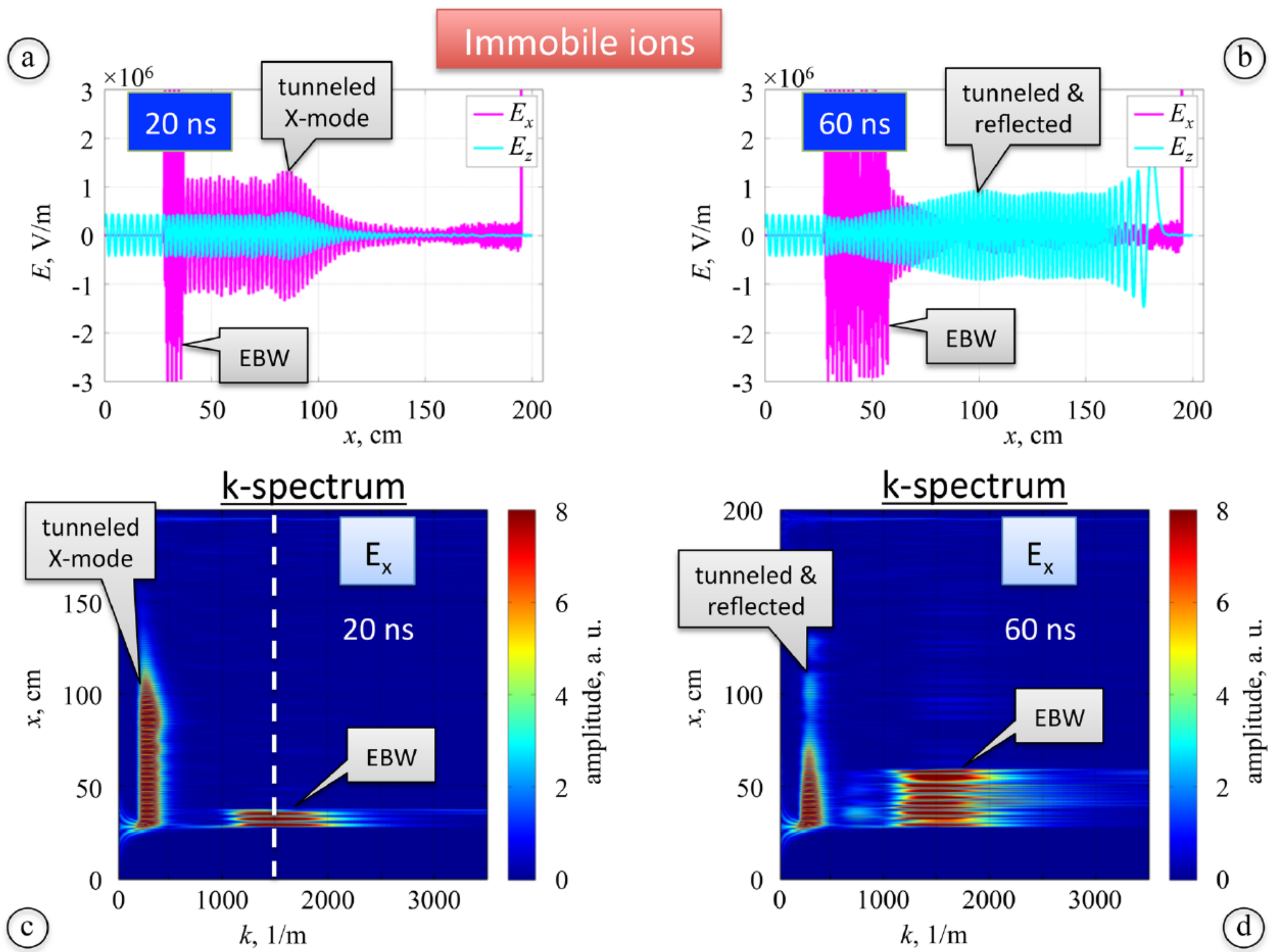
$$f = 10\ \text{GHz} \equiv \omega/2\pi$$
$$E_0 = 8 \times 10^5\ \text{V/m}$$

$$\Omega_e/\omega \approx 0.7, \quad \Omega_e/\omega_{pe} \approx 0.82$$
$$\omega_{\rm UH} \approx 6.9 \times 10^{10}\ \text{s}^{-1} \approx 1.1\omega$$
$$\omega_{\rm LH} \approx 7.9 \times 10^{8}\ \text{s}^{-1} \approx 33.1\Omega_i$$

$$2\pi/\omega_{\rm LH} \approx 7.9\ \text{ns}$$

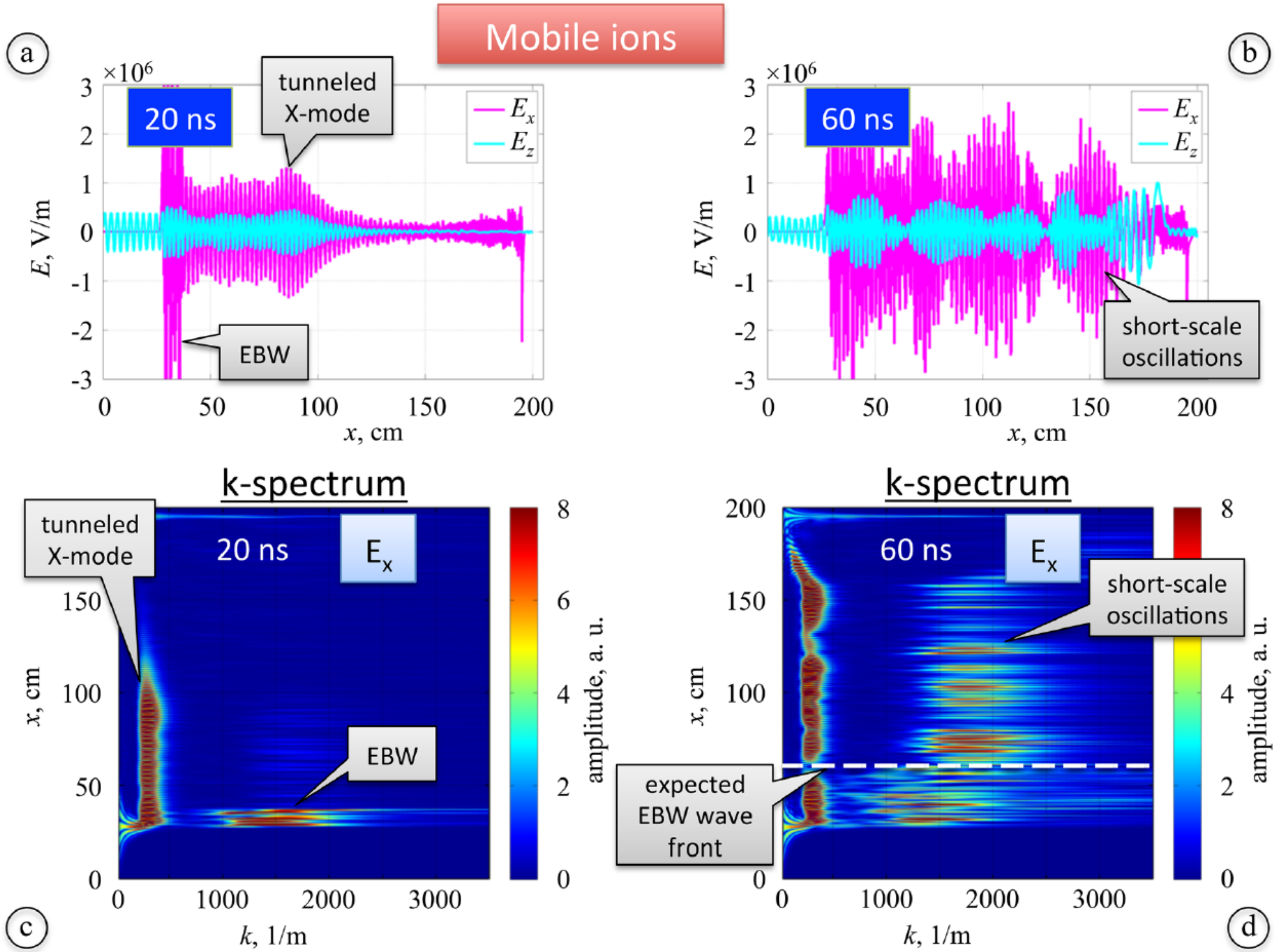
$$\lambda_e \approx 0.34, \quad \lambda_i \approx 317$$

Simulations with immobile ions [from Arefiev *et al*, NF (2017)]



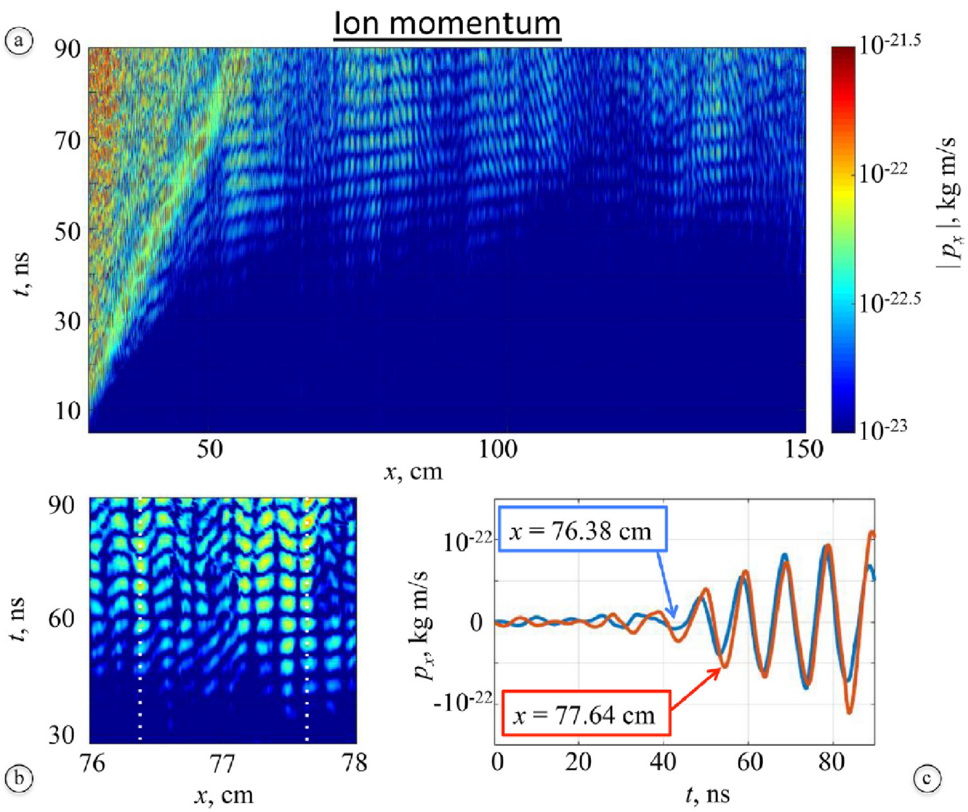
PIC simulations

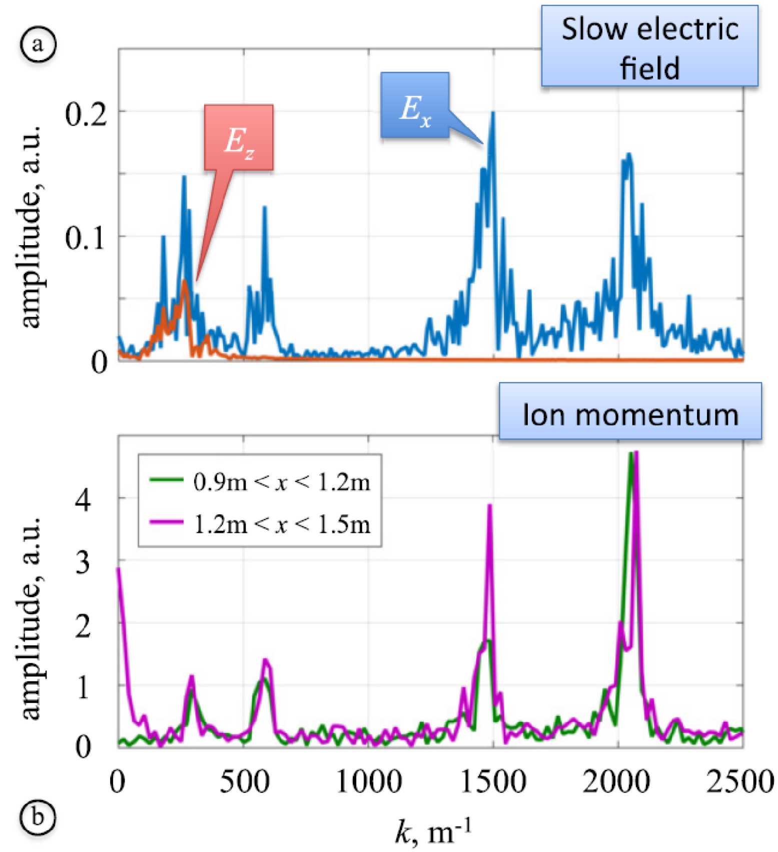
Simulations with mobile ions [from Arefiev *et al*, NF (2017)]



PIC simulations

Ion dynamics [from Arefiev *et al*, NF (2017)]

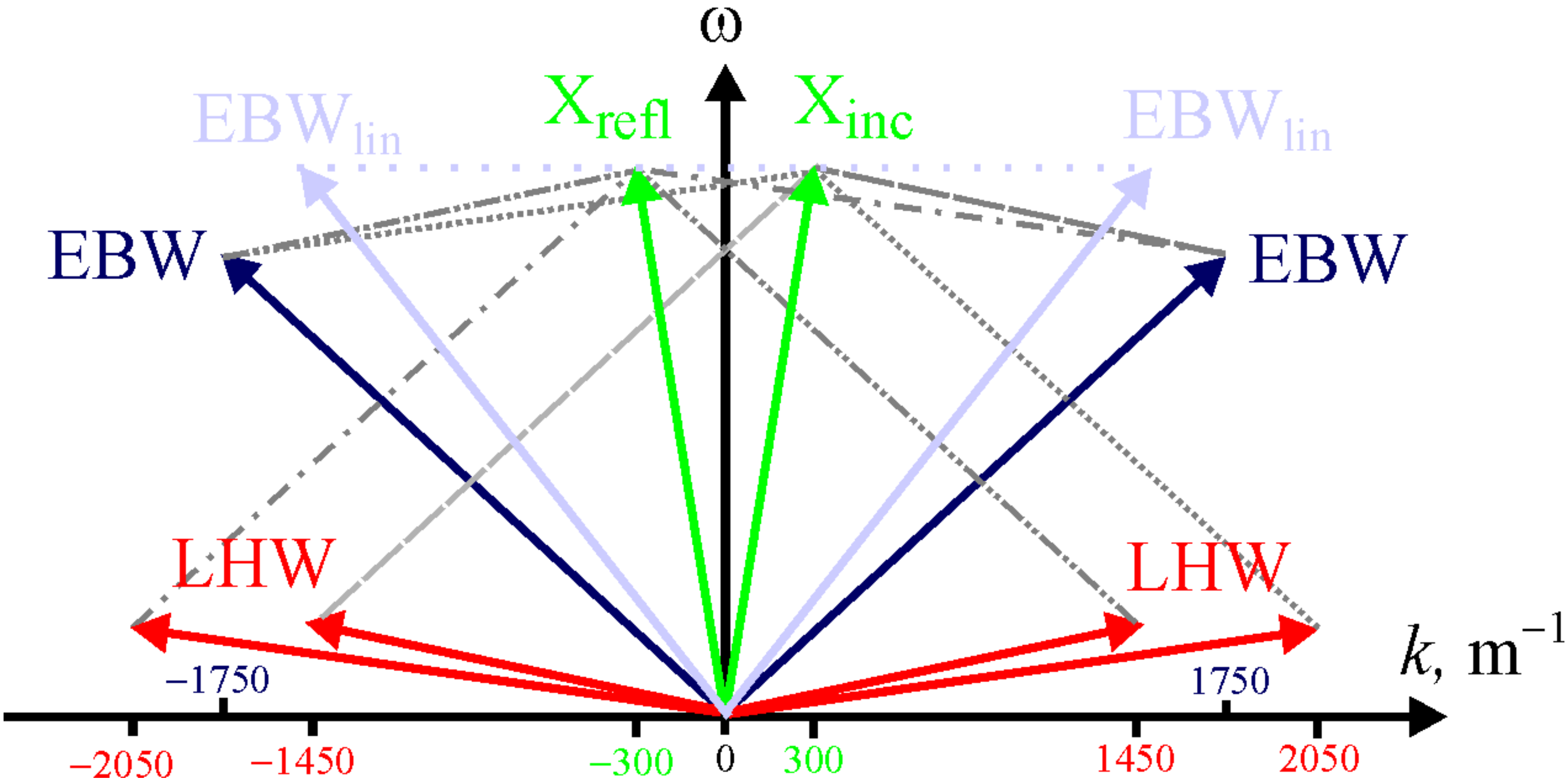




After the HF dynamics is filtered out, well-defined peaks in the spatial spectrum of the electric field and ion-momentum field are identified in the LH range.

We identified this as the X → EBW + LHW parametric decay instability.
It is the first time this instability is observed in first-principle simulations.

Multiple resonant triplets {X, EBW, LHW} are involved



$$\omega^{(\mathrm{x})} \approx \omega^{(\mathrm{ebw})} + \omega^{(\mathrm{lh})}, \quad \mathbf{k}^{(\mathrm{x})} \approx \mathbf{k}^{(\mathrm{ebw})} + \mathbf{k}^{(\mathrm{lh})}$$

[yet we allow $\omega^{(\mathrm{x})} - \omega^{(\mathrm{ebw})} - \omega^{(\mathrm{lh})}$ to be of order $\omega^{(\mathrm{lh})}$]

Variational principle

- The wave Lagrangian density can be written in terms of the unperturbed dielectric tensor $\hat{\boldsymbol{\epsilon}}_0$ and the LHW-driven complexified perturbation to the plasma susceptibility $\hat{\boldsymbol{\chi}}_{\mathrm{c}}^{(\mathrm{int})}$ as follows (see Appendix A):

$$\mathfrak{L} = \mathfrak{L}^{(\mathrm{x})} + \mathfrak{L}^{(\mathrm{ebw})} + \mathfrak{L}^{(\mathrm{lh})} + \mathfrak{L}^{(\mathrm{int})},$$

$$\mathfrak{L}^{(q)} = \frac{1}{16\pi}\mathbf{E}_{\mathrm{c}}^{(q)*} \cdot \hat{\boldsymbol{\mathfrak{D}}} \cdot \mathbf{E}_{\mathrm{c}}^{(q)}, \quad \hat{\boldsymbol{\mathfrak{D}}} \doteq \frac{c^2}{\hat{\omega}^2}\big[\hat{\mathbf{k}}\hat{\mathbf{k}} - \mathbf{1}(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}})\big] + \hat{\boldsymbol{\epsilon}}_0,$$

$$\hat{\omega} \doteq i\partial_t, \quad \hat{\mathbf{k}} \doteq -i\nabla,$$

$$\mathfrak{L}^{(\mathrm{int})} = \frac{1}{16\pi}\,\mathrm{Re}\left\{\mathbf{E}_{\mathrm{c}}^{(\mathrm{x})*} \cdot \hat{\boldsymbol{\chi}}_{\mathrm{c}}^{(\mathrm{int})} \cdot \mathbf{E}_{\mathrm{c}}^{(\mathrm{ebw})}\right\}.$$

- Further simplifications: the X-wave amplitude remains constant in space and time, $\mathbf{B}_0 = B_0\mathbf{e}_z =$ const, $\mathbf{E}^{(\mathrm{ebw})} \| \mathbf{k}^{(\mathrm{ebw})}$, and $\mathbf{E}^{(\mathrm{lh})} \| \mathbf{k}^{(\mathrm{lh})}$. Then,

$$\mathfrak{L} = \mathfrak{L}^{(\mathrm{ebw})} + \mathfrak{L}^{(\mathrm{lh})} + \mathfrak{L}^{(\mathrm{int})}, \quad \mathfrak{L}^{(q)} = \frac{1}{16\pi} E_{\mathrm{c}}^{(q)*}\,\hat{\mathfrak{D}}\,E_{\mathrm{c}}^{(q)}, \quad \hat{\mathfrak{D}} = \hat{\epsilon}_{0,xx},$$

$$\mathfrak{L}^{(\mathrm{int})} = \frac{1}{16\pi}\,\mathrm{Re}\left\{\big[\mathbf{E}_{\mathrm{c}}^{(\mathrm{x})*} \cdot \hat{\boldsymbol{\chi}}_{\mathrm{c}}^{(\mathrm{int})} \cdot \mathbf{e}_x\big] E_{\mathrm{c}}^{(\mathrm{ebw})}\right\}.$$

Cold-plasma approximation for $\mathfrak{L}^{(\mathrm{int})}$

- To the leading order, one can use the cold-plasma approximation for $\hat{\chi}_{\mathrm{c}}^{(\mathrm{int})}$. Then,

$$\hat{\boldsymbol{\chi}}_{\mathrm{c}}^{(\mathrm{int})} \approx \sum_s \frac{\partial \hat{\boldsymbol{\chi}}_s^{(\mathrm{uh})}}{\partial n_s} n_{s,c}^{(\mathrm{lh})} = \sum_s \frac{n_{s,c}^{(\mathrm{lh})}}{n_s} \hat{\boldsymbol{\chi}}_s^{(\mathrm{uh})} \approx \frac{\hat{k}^{(\mathrm{lh})} \hat{\chi}_{e,xx}^{(\mathrm{lh})}}{4\pi i q_e n_e} E_{\mathrm{c}}^{(\mathrm{lh})} \boldsymbol{\chi}_s(\omega_{\mathrm{UH}}).$$

- This leads to the following density of the interaction Lagrangian:

$$\mathfrak{L}^{(\mathrm{int})} \approx \frac{1}{8\pi} \operatorname{Re}\left\{ i E^{(\mathrm{ebw})} \hat{\beta} E_{\mathrm{c}}^{(\mathrm{lh})} \right\},$$

$$\hat{\beta} \doteq -\left[\mathbf{E}_c^{(\mathrm{x})*} \cdot \boldsymbol{\chi}_s(\omega_{\mathrm{UH}}) \cdot \mathbf{e}_x\right] \frac{\hat{k}^{(\mathrm{lh})} \hat{\chi}_{e,xx}^{(\mathrm{lh})}}{8\pi q_e n_e} = \frac{k^{(\mathrm{lh})} E_c^{(\mathrm{x})*}}{8\pi q_e n_e} \frac{\omega_{pe}^2}{\Omega_e^2}.$$

- Hence, $\mathfrak{L}^{(\mathrm{int})}$ can be expressed in terms of a coupling constant β and phase θ_{x}:

$$\mathfrak{L}^{(\mathrm{int})} \approx \frac{1}{8\pi} \operatorname{Re}\left[i\beta E^{(\mathrm{ebw})} E^{(\mathrm{lh})} e^{-i\theta_{\mathrm{x}}} \right], \quad \beta = \frac{k^{(\mathrm{lh})} \mathcal{E}_c^{(\mathrm{x})*}}{8\pi q_e n_e}, \quad E_{\mathrm{c}}^{(q)} = \mathcal{E}^{(q)} e^{i\theta_q}.$$

Weak pump

- If the pump is weak, both dispersion operators can be Weyl-expanded:

$$e^{-i\theta_q}\hat{\mathfrak{D}}E_{\mathrm{c}}^{(q)} = \mathfrak{D}\left(\omega^{(q)} + \hat{\omega}, k^{(q)}\right)\mathcal{E}^{(q)} \approx \underbrace{\mathfrak{D}(\omega^{(q)}, k^{(q)})}_{=0} + \mathfrak{D}_{\omega}^{(q)}\hat{\omega}\mathcal{E}^{(q)} = \mathfrak{D}_{\omega}^{(q)} i\partial_t \mathcal{E}^{(q)}$$

- This leads to the following Lagrangian density and Euler-Lagrange equations:

$$16\pi\mathfrak{L} = \mathcal{E}^{(\mathrm{ebw})*}\mathfrak{D}_{\omega}^{(\mathrm{ebw})} i\partial_t \mathcal{E}^{(\mathrm{ebw})} + \mathcal{E}^{(\mathrm{lh})*}\mathfrak{D}_{\omega}^{(\mathrm{lh})} i\partial_t \mathcal{E}^{(\mathrm{lh})} + i\beta\mathcal{E}^{(\mathrm{ebw})}\mathcal{E}^{(\mathrm{lh})}e^{i\Delta\omega t} - i\beta^*\mathcal{E}^{(\mathrm{ebw})*}\mathcal{E}^{(\mathrm{lh})*}e^{-i\Delta\omega t}.$$

$$\delta\mathcal{E}^{(\mathrm{ebw})}: \quad \mathfrak{D}_{\omega}^{(\mathrm{ebw})}\partial_t\mathcal{E}^{(\mathrm{ebw})*} = \beta\mathcal{E}^{(\mathrm{lh})}\, e^{i\Delta\omega t},$$

$$\delta\mathcal{E}^{(\mathrm{lh})*}: \quad \mathfrak{D}_{\omega}^{(\mathrm{lh})}\partial_t\mathcal{E}^{(\mathrm{lh})} = \beta^*\mathcal{E}^{(\mathrm{ebw})*}\, e^{-i\Delta\omega t}.$$

- Using the cold-plasma approximation for $\mathfrak{D}$ (Appendix B), one obtains the following growth rate of the parametric decay instability:

$$\gamma = \sqrt{\gamma_0^2 - \frac{(\Delta\omega)^2}{4}}, \quad \gamma_0 \approx \omega_{\mathrm{LH}}\frac{\omega_{pe}\omega_{pi}}{|\Omega_e\Omega_i|}\sqrt{\frac{\omega_{\mathrm{LH}}}{\omega_{\mathrm{UH}}}}\frac{\left|k^{(\mathrm{lh})}\mathcal{E}_x^{(\mathrm{x})}\right|}{16\pi e n_e}$$

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General case

- At a stronger pump, $\hat{\mathfrak{D}}\bar{\mathcal{E}}^{(\mathrm{lh})}$ cannot be easily Weyl-expanded. Then,

$$16\pi\mathfrak{L} = \mathcal{E}^{(\mathrm{ebw})*}\mathfrak{D}_\omega^{(\mathrm{ebw})} i\partial_t \mathcal{E}^{(\mathrm{ebw})} + \bar{\mathcal{E}}^{(\mathrm{lh})*}\hat{\mathfrak{D}}\bar{\mathcal{E}}^{(\mathrm{lh})}$$
$$+ i\beta\mathcal{E}^{(\mathrm{ebw})}\bar{\mathcal{E}}^{(\mathrm{lh})}e^{i\vartheta t} - i\beta^*\mathcal{E}^{(\mathrm{ebw})*}\bar{\mathcal{E}}^{(\mathrm{lh})}e^{-i\vartheta t},$$

$$\bar{\mathcal{E}}^{(\mathrm{lh})}(t) \doteq E_c^{(\mathrm{lh})}(t,x)\exp[-i\mathbf{k}^{(\mathrm{lh})}\cdot\mathbf{x}],$$

$$\vartheta \doteq \omega^{(\mathrm{x})} - \omega^{(\mathrm{ebw})} = \omega^{(\mathrm{lh})} + \Delta\omega, \quad \hat{\mathfrak{D}} = \hat{\omega}^{-2}(\hat{\omega}^2 - \omega_{\mathrm{LH}}^2)\,\omega_{pi}^2/\omega_{\mathrm{LH}}^2.$$

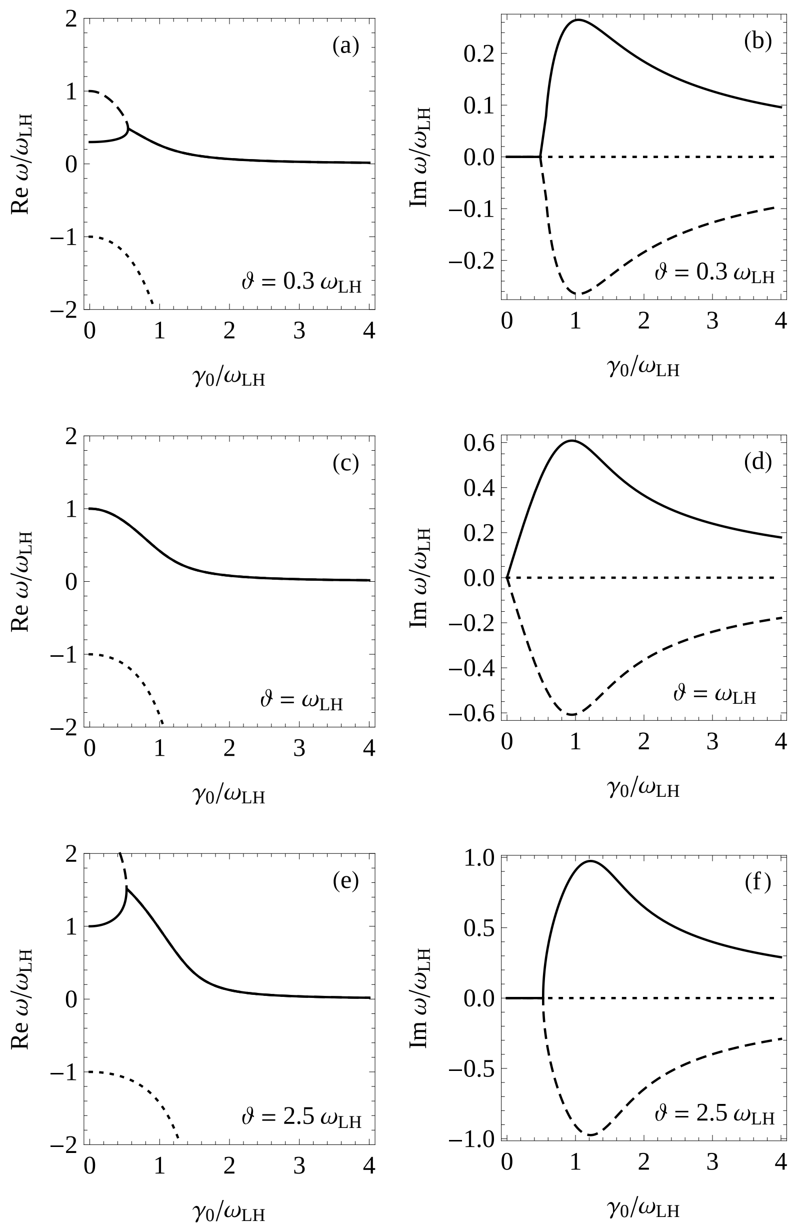
- The corresponding Euler-Lagrange equations are as follows:

$$\delta\mathcal{E}^{(\mathrm{ebw})}: \quad \mathfrak{D}_\omega^{(\mathrm{ebw})}\partial_t\mathcal{E}^{(\mathrm{ebw})*} = \beta\bar{\mathcal{E}}^{(\mathrm{lh})}\,e^{i\vartheta t},$$
$$\delta\bar{\mathcal{E}}^{(\mathrm{lh})*}: \quad \hat{\mathfrak{D}}\bar{\mathcal{E}}^{(\mathrm{lh})} = i\beta^*\mathcal{E}^{(\mathrm{ebw})*}\,e^{-i\vartheta t}.$$

- After excluding $\mathcal{E}^{(\mathrm{ebw})}$, one obtains the following equation for $\bar{\mathcal{E}}^{(\mathrm{lh})}$:

$$\boxed{\left[(\hat{\omega}-\vartheta)(\hat{\omega}^2-\omega_{\mathrm{LH}}^2) + 2\gamma_0^2\hat{\omega}^2\right]\bar{\mathcal{E}}^{(\mathrm{lh})} = 0.}$$

General case (continued)



- The corresponding dispersion relation is

$$(\omega - \vartheta)(\omega^2 - \omega_{\rm LH}^2) + 2\gamma_0^2\omega^2 = 0,$$

$$g \doteq \frac{\gamma_0}{\omega_{\rm LH}} \approx \frac{\omega_{pe}\omega_{pi}}{|\Omega_e\Omega_i|}\sqrt{\frac{\omega_{\rm LH}}{\omega_{\rm UH}}}\,\frac{\left|k^{(\rm lh)}\mathcal{E}_x^{(\rm x)}\right|}{16\pi e n_e}$$

- Asymptotics at weak pump ($g \ll 1$):

$$\frac{\omega_{1,2}}{\omega_{\rm LH}} \approx 1 \pm ig, \qquad \frac{\omega_3}{\omega_{\rm LH}} \approx -1 - \frac{g^2}{2}$$

- Asymptotics at strong pump ($g \gg 1$):

$$\frac{\omega_{1,2}}{\omega_{\rm LH}} \approx \pm\frac{i}{g\sqrt{2}}, \qquad \frac{\omega_3}{\omega_{\rm LH}} \approx -2g^2 + 1$$

- Simulation parameters: $g \approx 0.17$, $\gamma \approx \gamma_0$, and $\gamma^{-1} \approx 7.6\,$ns. This is in reasonable agreement with the PIC simulations.

Conclusions

In summary, we proposed a modified theory of the instability that is caused by the resonant scattering, or parametric decay, of an intense X wave into an EBW and LHW. Our theory extends to the high-intensity regime, where the instability rate γ ceases to be a linear function of the incident-wave amplitude. We derived an explicit formula for γ and expressed it in terms of cold-plasma parameters. Predictions of our theory are in reasonable agreement with the results of the PIC simulations in [Arefiev $et\ al$, Nucl. Fusion (2017)].

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Appendix A: General variational principle

- The Lagrangian density of the field-plasma system can be averaged over the high-frequency (UH) oscillations in time and over the least wavelength in space:

$$\mathfrak{L} = \langle \mathfrak{L}_{\text{em}} \rangle_{t,x} + \langle \mathfrak{L}_p \rangle_{t,x}, \quad \mathfrak{L}_{\text{em}} = (E^2 - B^2)/(8\pi).$$

- $\mathfrak{L}_p$ is the plasma contribution, which consists of the 0th-order term independent of the UH field $\mathbf{E}_{\text{c}}^{(\text{uh})} \doteq \mathbf{E}_{\text{c}}^{(\text{x})} + \mathbf{E}_{\text{c}}^{(\text{ebw})}$ and some bilinear functional $\mathcal{U}$ of $\mathbf{E}_{\text{c}}^{(\text{uh})}$:

$$\langle \mathfrak{L}_p \rangle_{t,x} = \langle \mathfrak{L}_0 \rangle_x + \frac{1}{8\pi} \langle \mathcal{U} \rangle_{t,x}, \quad \langle \mathcal{U} \rangle_{t,x} = \frac{1}{2} \langle \mathbf{E}_{\text{c}}^{(\text{uh})*} \cdot \hat{\boldsymbol{\chi}} \cdot \mathbf{E}_{\text{c}}^{(\text{uh})} \rangle_x.$$

- The Hermitian operator $\hat{\boldsymbol{\chi}}$ can consist of a slow part $\hat{\boldsymbol{\chi}}_0$ and the remaining part $\hat{\boldsymbol{\chi}}^{(\text{int})}$ that is linear in the LH field, so

$$\langle \mathcal{U} \rangle_{t,x} = \frac{1}{2} \,\text{Re}\, \big[\mathbf{E}_{\text{c}}^{(\text{x})*} \cdot \hat{\boldsymbol{\chi}}_{\text{c}}^{(\text{int})} \cdot \mathbf{E}_{\text{c}}^{(\text{ebw})} \big].$$

- Likewise, $\langle \mathfrak{L}_0 \rangle_x$ contains terms scaling as the 0th and 2nd powers of the LH field:

$$\langle \mathfrak{L}_0 \rangle_x = \mathfrak{L}_{p0} + \frac{1}{8\pi} \langle \mathbf{E}^{(\text{lh})} \cdot \hat{\boldsymbol{\chi}}_0 \cdot \mathbf{E}^{(\text{lh})} \rangle_{t,x}.$$

Appendix A: Variational principle (continued)

- After combining all these terms together, we obtain:

$$\mathfrak{L} = \mathfrak{L}^{(\mathrm{x})} + \mathfrak{L}^{(\mathrm{ebw})} + \mathfrak{L}^{(\mathrm{lh})} + \mathfrak{L}^{(\mathrm{int})},$$

$$\mathfrak{L}^{(q)} = \frac{1}{16\pi}\left\{|\mathbf{E}_{\mathrm{c}}^{(q)}|^2 - |\mathbf{B}_{\mathrm{c}}^{(q)}|^2 + \mathbf{E}_{\mathrm{c}}^{(q)*} \cdot \hat{\boldsymbol{\chi}}^{(q)} \cdot \mathbf{E}_{\mathrm{c}}^{(q)}\right\},$$

$$\mathfrak{L}^{(\mathrm{int})} = \frac{1}{16\pi}\,\mathrm{Re}\left\{\mathbf{E}_{\mathrm{c}}^{(\mathrm{x})*} \cdot \hat{\boldsymbol{\chi}}_{\mathrm{c}}^{(\mathrm{int})} \cdot \mathbf{E}_{\mathrm{c}}^{(\mathrm{ebw})}\right\}.$$

- Next, use $\mathbf{B}_{\mathrm{c}}^{(q)} = \hat{\omega}^{-1}\, c\hat{\mathbf{k}} \times \mathbf{E}_{\mathrm{c}}^{(q)}$ and $\hat{\boldsymbol{\epsilon}}_0 \doteq \mathbf{1} + \hat{\boldsymbol{\chi}}_0$. Then,

$$\mathfrak{L}^{(q)} = \frac{1}{16\pi}\,\mathbf{E}_{\mathrm{c}}^{(q)*} \cdot \hat{\boldsymbol{\mathfrak{D}}} \cdot \mathbf{E}_{\mathrm{c}}^{(q)}, \quad \hat{\boldsymbol{\mathfrak{D}}} \doteq \frac{c^2}{\hat{\omega}^2}\big[\hat{\mathbf{k}}\hat{\mathbf{k}} - \mathbf{1}(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}})\big] + \hat{\boldsymbol{\epsilon}}_0.$$

- Also, in cold plasma, $\hat{\boldsymbol{\chi}}_{\mathrm{c}}^{(\mathrm{int})}$ can be recognized as the LHW-driven perturbation to the plasma susceptibility, *which is well known*.

Appendix B: Justification of the cold-plasma approximation

- Electrons have $\lambda_e \doteq (kv_{Te}/\Omega_e)^2 \approx 0.34$, which is small enough. Then,

$$\mathfrak{D}^{(\mathrm{ebw})}(\omega, k) \approx 1 - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2}\,\Theta(\lambda_e), \quad \Theta(\lambda_e) \doteq \frac{2I_n(\lambda_e)}{\lambda_e}\,e^{-\lambda_e} \approx 1 - \lambda_e.$$

- Ions have $\lambda_i \doteq (kv_{Ti}/\Omega_i)^2 \approx 317$. This number is far too large to allow for an asymptotic small-argument expansion of the modified Bessel functions. Likewise, the large-argument expansion is inapplicable because of the large value of $a \doteq \omega/\Omega_i \approx 33.1$, which determines the number of relevant harmonics ($n \sim a$). Thus, a different approach is needed to justify the cold-plasma approximation.

- We use an alternative expression for χ_{xx} that does not contain an infinite series[†]:

$$\chi_{xx} = \frac{\omega_p^2}{\omega\Omega}\int v_\perp \frac{\partial f_0(v)}{\partial v_\perp}\,\mathfrak{T}_{xx}\,2\pi v_\perp\,dv_\perp\,dv_\parallel, \quad \mathfrak{T}_{xx} = \frac{a}{z^2}\left[\frac{\pi a}{\sin(\pi a)}\,J_{-a}(z)J_a(z) - 1\right].$$

- If a is large and $z \doteq kv_\perp/\Omega_i$ is small (in the sense yet to be defined), then

$$\mathfrak{T}_{xx} = \frac{a}{2(a^2-1)} + \frac{3az^2}{8(a^2-1)(a^2-2^2)} + \frac{5az^4}{16(a^2-1)(a^2-2^2)(a^2-3^2)} + \dots$$

[†] H. Qin, C. K. Phillips, and R. C. Davidson, Phys. Plasmas **14**, 092103 (2007).

Appendix B (continued)

- Then, after integrating over a Maxwellian f_0, one obtains

$$\chi_{xx} = -\frac{\omega_p^2}{\Omega^2}\left[\frac{1}{a^2-1} + \frac{3\lambda}{(a^2-1)(a^2-2^2)} + \frac{15\lambda^2}{(a^2-1)(a^2-2^2)(a^2-3^2)} + \ldots\right].$$

- At noninteger a, this expansion requires $\lambda \ll a^2$ (roughly). Notably, this is less restrictive than the requirement $\lambda \ll a$ that is needed to expand $J_a(z)$.

- For the simulation parameters, this is satisfied, albeit marginally. Hence,

$$\chi_{xx} \approx -\frac{\omega_p^2}{\Omega^2}\frac{1}{a^2-1} = -\frac{\omega_p^2}{\omega^2-\Omega^2},$$

which coincides with the cold-plasma limit.

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