Drift-wave turbulence as quantumlike plasma

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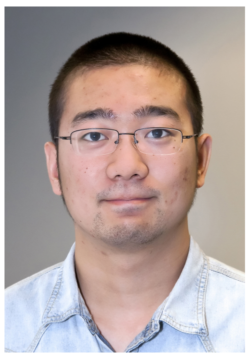


in collaboration with

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Seminar
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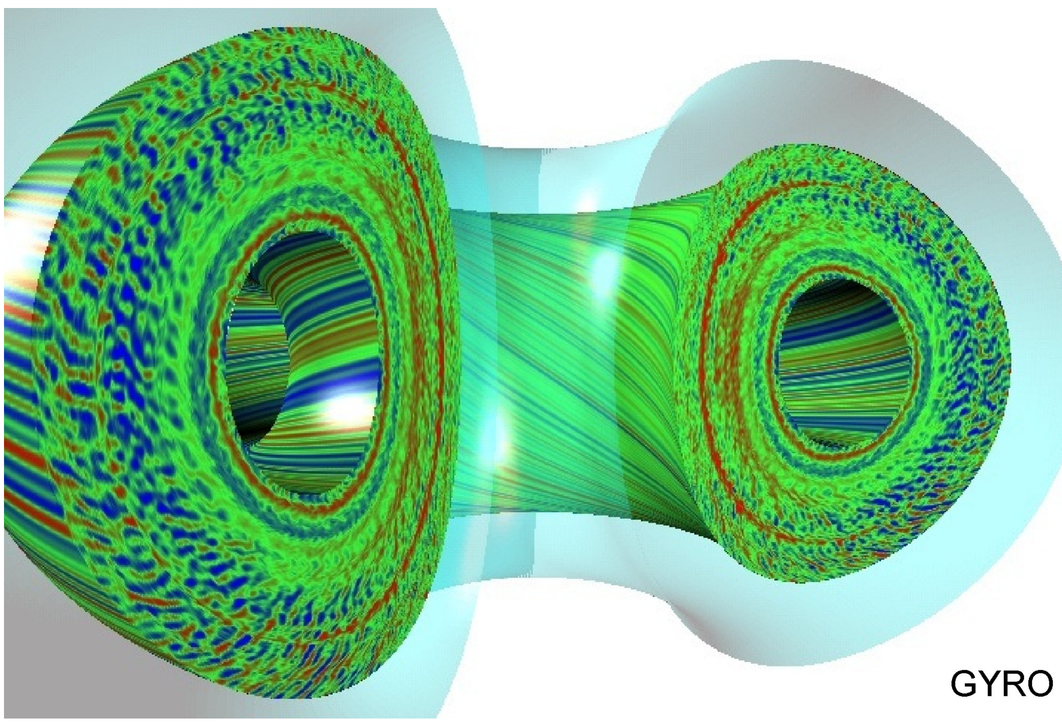
Related papers

- **Overview:** H. Zhu and I. Y. Dodin, *Wave-kinetic approach to zonal-flow dynamics: recent advances*, Phys. Plasmas 28, 032303 (2021).
- H. Zhu, Y. Zhou, and I. Y. Dodin, *Theory of the tertiary instability and the Dimits shift within a scalar model*, J. Plasma Phys. 86, 905860405 (2020).
- H. Zhu, Y. Zhou, and I. Y. Dodin, *Theory of the tertiary instability and the Dimits shift from reduced drift-wave models*, Phys. Rev. Lett. 124, 055002 (2020).
- H. Zhu, Y. Zhou, and I. Y. Dodin, *Nonlinear saturation and oscillations of collisionless zonal flows*, New J. Phys. 21, 063009 (2019).
- Y. Zhou, H. Zhu, and I. Y Dodin, *Formation of solitary zonal structures via the modulational instability of drift waves*, Plasma Phys. Control. Fusion 61, 075003 (2019).
- D. E. Ruiz, M. E. Glinsky, and I. Y. Dodin, *Wave kinetic equation for inhomogeneous drift-wave turbulence beyond the quasilinear approximation*, J. Plasma Phys. 85, 905850101 (2019).
- H. Zhu, Y. Zhou, and I. Y. Dodin, *On the Rayleigh–Kuo criterion for the tertiary instability of zonal flows*, Phys. Plasmas 25, 082121 (2018).
- H. Zhu, Y. Zhou, and I. Y. Dodin, *On the structure of the drifton phase space and its relation to the Rayleigh–Kuo criterion of the zonal-flow stability*, Phys. Plasmas 25, 072121 (2018).
- H. Zhu, Y. Zhou, D. E. Ruiz, and I. Y. Dodin, *Wave kinetics of drift-wave turbulence and zonal flows beyond the ray approximation*, Phys. Rev. E 97, 053210 (2018).
- D. E. Ruiz, J. B. Parker, E. L. Shi, and I. Y. Dodin, *Zonal-flow dynamics from a phase-space perspective*, Phys. Plasmas 23, 122304 (2016).

Introduction

- Drift-wave (DW) turbulence is ubiquitous in magnetized plasmas. In fusion science, DW turbulence is actively studied because it affects plasma confinement.
- DW turbulence can spontaneously generate zonal flows (ZF), which are sheared $E \times B$ flows with $k_{\|} = 0$. ZFs reduce turbulent transport but can be unstable.

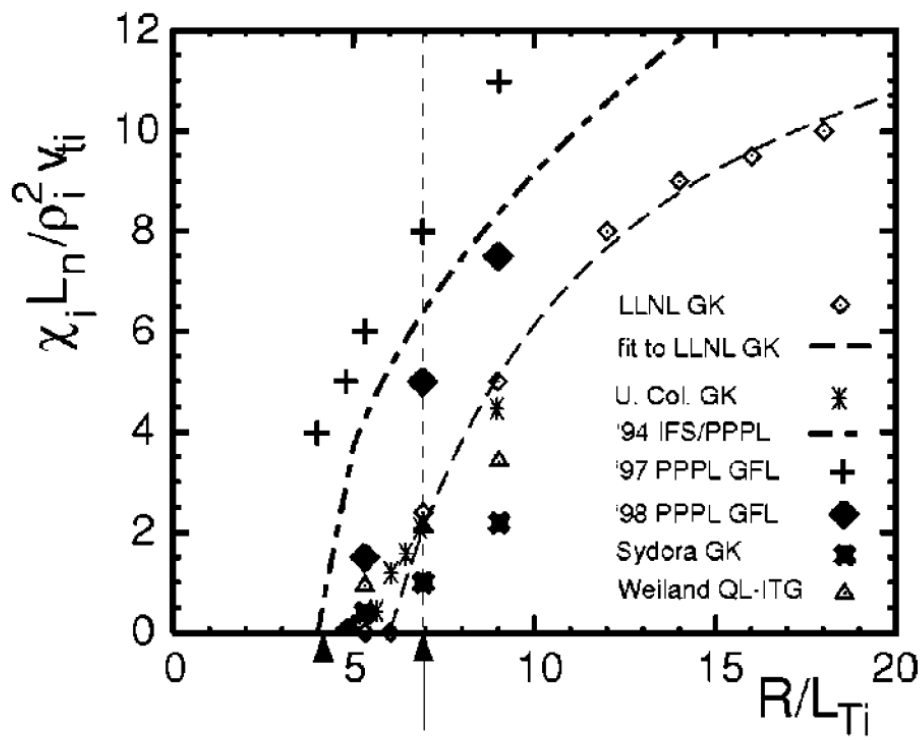


primary instabilities (PI)
pump up turbulence
↓
secondary instability (SI)
creates zonal flows
↓
zonal flows saturate,
oscillate, or exhibit a
tertiary instability (TI)

Image taken from https://w3.pppl.gov/~hammett/viz/viz.html

Dimits shift is a problem that encompasses all DZ–ZF physics

- Predictions of nonlinear simulations differ from predictions of nonlinear simulations. Dimits shift = difference in the critical temperature gradients ($\sim 1/L_{Ti}$).
- Apparently, zonal flows stabilize turbulence to some extent. How do they do it?
- Answering this requires understanding of many aspects of DW–ZF interactions. Here, we do it within a simple model that allows for a complete analytical theory.



Dimits *et al.* (2000)

Outline

- Turbulence model
- Quantumlike formulation
- Parameter space of zonal flows
- Explanation of the Dimits shift
- Other applications

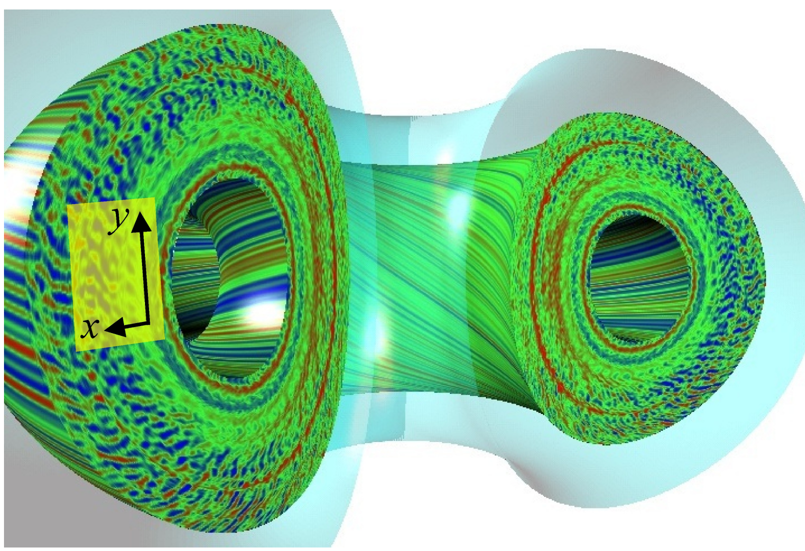
Hasegawa–Mima model (HMM) captures the basic interactions.

- Slab approximation, $(x, y) \perp \mathbf{B}$, incompressible $\mathbf{E}\times\mathbf{B}$ flow, cold ions, hot electrons:

$$\partial_t n_i + \mathbf{v}_{E\times B} \cdot \nabla n_i = 0, \quad n_i = -\beta x + w(t, x, y) \quad -\nabla \cdot (1 + \widehat{\chi}_e)\nabla\varphi = 4\pi w$$

$$\boxed{\partial_t w + \{\varphi, w\} - \beta\partial_y\varphi = 0, \qquad (\nabla_\perp^2 - \widehat{a})\varphi = w}$$

- Electrons respond adiabatically to DW ($k_\parallel \neq 0$) and do not respond to ZF ($k_\parallel = 0$):



$$\widehat{a}_{\mathrm{dw}} = 1, \qquad \widehat{a}_{\mathrm{zf}} = 0$$

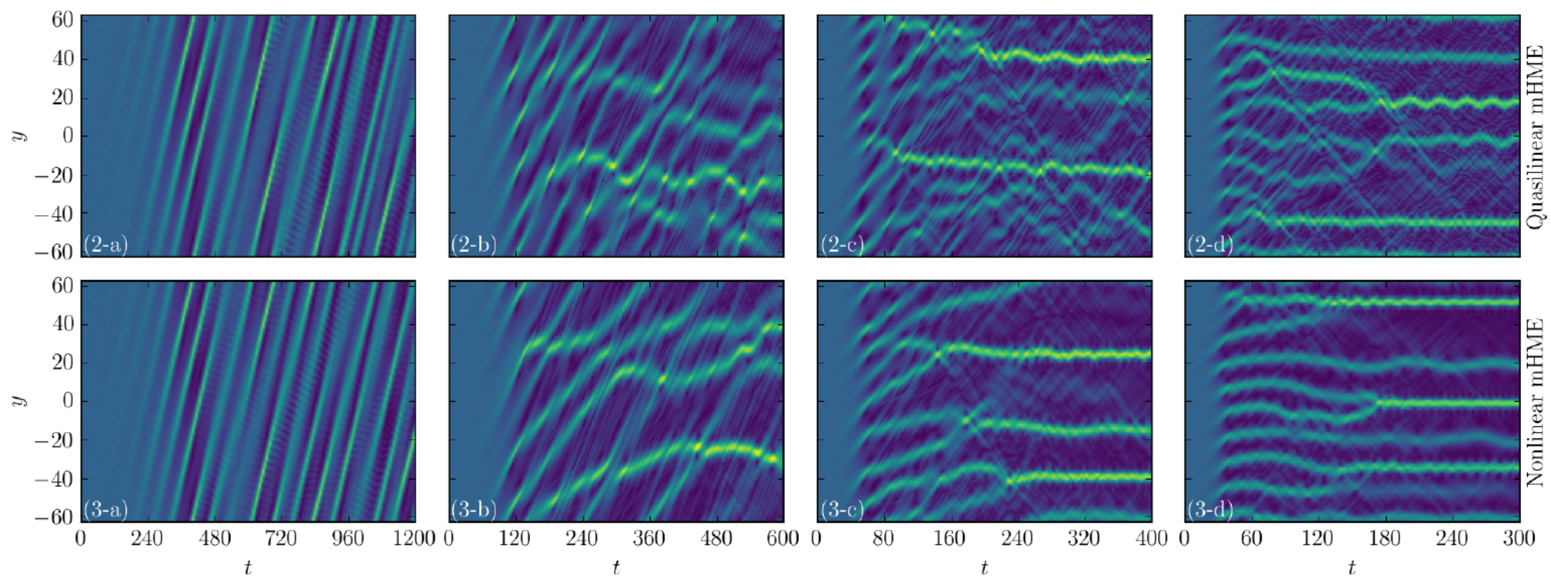
- HMM has no primary instabilities and thus no Dimits shift either.
- But one can use HMM to study other physics that contributes to the Dimits shift. Let's!

The quasilinear approximation captures DW–ZF interactions.

- Let us split the equation for w into the zonal average and fluctuations:

ZF velocity: $$\partial_t U + \partial_x \overline{\tilde{v}_x \tilde{v}_y} = 0, \qquad \overline{(\dots)} = \int_0^{L_y} (\dots)\, \mathrm{d}y / L_y$$

DW: $$\partial_t \tilde{w} + U \partial_y \tilde{w} - [\beta + (\partial_x^2 U)] \partial_y \tilde{\varphi} = \underbrace{\overline{\tilde{\mathbf{v}} \cdot \nabla \tilde{\boldsymbol{w}}} - \tilde{\mathbf{v}} \cdot \nabla \tilde{\boldsymbol{w}}}_{\text{neglected (QL model)}}$$



- Using $\tilde{\varphi} = (\nabla_\perp^2 - 1)^{-1} \tilde{w}$, one can express eqn for w as 'drifton' Schrödinger eqn:

$$i\partial_t \tilde{w} = \widehat{\mathcal{H}} \tilde{w}, \qquad \widehat{\mathcal{H}} = \widehat{k}_y \widehat{U} + (\beta + \widehat{U}'') \widehat{k}_y (1 + \widehat{k}_\perp^2)^{-1}, \qquad \widehat{\mathbf{k}} = -i\nabla$$

Ruiz *et al.* (2016); Zhou *et al.* (2019)

Kelvin–Helmholtz instability as drifton-vacuum breakdown

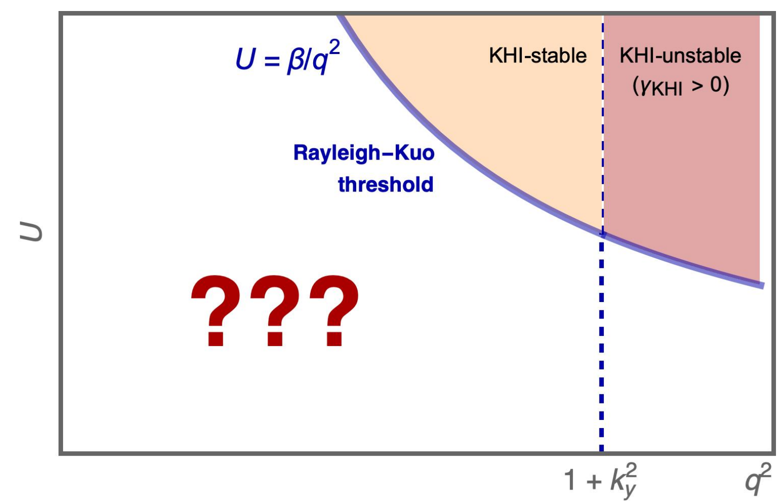
- The Hamiltonian $\widehat{\mathcal{H}}$ is pseudo-Hermitian: using $\widehat{Q} \doteq \beta + \widehat{U}''$, one has a transformation $\tilde{w} = \widehat{Q}^{1/2}\eta$ that makes the Hamiltonian Hermitian, as $\widehat{\mathcal{H}}\widehat{Q} = \widehat{Q}\widehat{\mathcal{H}}^\dagger$:

$$i\partial_t \tilde{w} = \widehat{\mathcal{H}}\tilde{w} \quad \Rightarrow \quad i\partial_t \eta = \underbrace{[\widehat{Q}^{-1/2}(\widehat{\mathcal{H}}\widehat{Q})\widehat{Q}^{-1/2}]}_{\text{Hermitian if } \widehat{Q}^{-1} \text{ exists}}\eta \qquad (\text{for } \partial_t U = 0)$$

- When $|U''| > \beta$, i.e. $\boldsymbol{U > \beta/q^2}$, then $\widehat{Q}^{-1}$ does not exist $\Rightarrow$ pseudo-Hermiticity breaks $\Rightarrow$ "drifton-vacuum breakdown", a.k.a. Kelvin–Helmholtz instability (KHI).

$$\gamma_{\text{KHI}} \approx |k_y U_0| \left(1 - \frac{1 + k_y^2}{q^2}\right)\sqrt{1 - \frac{\beta^2}{U_0^2 q^4}}$$

- KHI $\neq$ tertiary instability! Actually, the regime $U \lesssim \beta/q^2$ will be more relevant.



Let's introduce some machinery...

- Any operator $\widehat{A}\psi(\mathsf{x}) = \int \mathsf{A}(\mathsf{x},\mathsf{x}')\,\psi(\mathsf{x}')\,\mathrm{d}\mathsf{x}'$ can be expressed through its *Weyl symbol* using $\widehat{\mathsf{x}} = \mathsf{x}$ and $\widehat{\mathsf{k}} = -i\nabla$:

$$A(\mathsf{x},\mathsf{k}) = \int \mathsf{A}(\mathsf{x}+\mathsf{s}/2,\mathsf{x}-\mathsf{s}/2)\,e^{-i\mathsf{k}\cdot\mathsf{s}}\,\mathrm{d}\mathsf{s}$$

$$\widehat{A} = \frac{1}{(2\pi)^{2\mathsf{n}}}\int A(\mathsf{x}',\mathsf{k}')\,e^{i\mathsf{k}''\cdot(\mathsf{x}'-\widehat{\mathsf{x}})-i\mathsf{x}''\cdot(\mathsf{k}'-\widehat{\mathsf{k}})}\,\mathrm{d}\mathsf{x}'\,\mathrm{d}\mathsf{k}'\,\mathrm{d}\mathsf{x}''\,\mathrm{d}\mathsf{k}''$$

$$\widehat{1} \Leftrightarrow 1$$
$$\widehat{\mathsf{x}} \Leftrightarrow \mathsf{x}$$
$$\widehat{\mathsf{k}} \Leftrightarrow \mathsf{k}$$
$$\widehat{\mathsf{A}}^\dagger \Leftrightarrow \mathsf{A}^\dagger$$
$$\widehat{\mathsf{A}}\widehat{\mathsf{B}} \Leftrightarrow \mathsf{A} \star \mathsf{B}$$

- **Example 1:** The dielectric tensor $\boldsymbol{\epsilon}(t,\mathbf{x},\omega,\mathbf{k})$ is actually the Weyl symbol of $\widehat{\boldsymbol{\epsilon}}$, at least up to $\mathcal{O}(1/\omega\tau, 1/kL)$.

- **Example 2:** Spectrum of the 2-point correlation function of any ψ is the symbol of $\widehat{W} = |\psi\rangle\langle\psi|$, a.k.a. *Wigner function:*

$$W(t,\mathbf{x},\omega,\mathbf{k}) = (2\pi)^{-4}\int \mathrm{d}\tau\,\mathrm{d}\mathbf{s}\,e^{i\omega\tau - i\mathbf{k}\cdot\mathbf{s}} \times \psi(t+\tau/2,\mathbf{x}+\mathbf{s}/2)\,\psi^*(t-\tau/2,\mathbf{x}-\mathbf{s}/2)$$

* $(\widehat{\mathsf{x}},\widehat{\mathsf{k}}) \equiv (t,\mathbf{x},-i\partial_t,-i\nabla)$

Quantumlike kinetic theory for the Wigner function W

- The Schrödinger equation for $\tilde{w}$ → the von Neumann equation for $\widehat{W} = |\tilde{w}\rangle\langle\tilde{w}|$:

$$i\partial_t\,|\tilde{w}\rangle = \widehat{\mathcal{H}}\,|\tilde{w}\rangle \quad \Rightarrow \quad \partial_t\widehat{W} = [\widehat{\mathcal{H}}, \widehat{W}] \quad \Rightarrow \quad W = \langle \text{symb}\,\widehat{W}\rangle$$

- The Wigner function $W(t, x, \mathbf{k}) \doteq \int \mathrm{d}\mathbf{s}\, e^{-i\mathbf{k}\cdot\mathbf{s}}\langle\tilde{w}(t, \mathbf{x}+\mathbf{s}/2)\tilde{w}(t, \mathbf{x}-\mathbf{s}/2)\rangle$ satisfies

$$\frac{\partial W}{\partial t} = \{\{\mathcal{H}_\mathrm{H}, W\}\} + [\![\mathcal{H}_\mathrm{A}, W]\!], \qquad \frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^2}\,\frac{1}{1+k_\perp^2}\star k_x k_y W \star \frac{1}{1+k_\perp^2}$$

$$\mathcal{H}_\mathrm{H} = k_y U + \frac{\beta k_y}{1+k_\perp^2} + \frac{1}{2}\left[\!\!\left[U'', \frac{k_y}{1+k_\perp^2}\right]\!\!\right], \qquad \mathcal{H}_\mathrm{A} = \frac{1}{2}\left\{\!\!\left\{U'', \frac{k_y}{1+k_\perp^2}\right\}\!\!\right\}$$

- Geometrical-optics limit: improved wave kinetic equation (iWKE) **with new terms**:

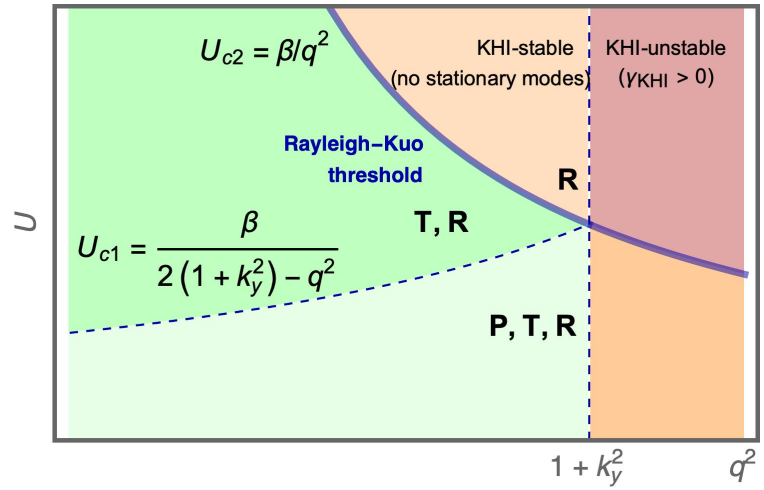
$$\frac{\partial W}{\partial t} = \{\mathcal{H}_\mathrm{H}, W\} + \boldsymbol{2\mathcal{H}_\mathrm{A} W}, \qquad \frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^2}\,\frac{k_x k_y W}{(1+k_\perp^2)^2}$$

$$\mathcal{H}_\mathrm{H} \approx k_y U + k_y(\beta + \boldsymbol{U''})/(1+k_\perp^2), \qquad \mathcal{H}_\mathrm{A} \approx -\boldsymbol{U'''} k_x k_y/(1+k_\perp^2)^2$$

Ruiz *et al.* (2016); Parker (2016); cf. Smolyakov and Diamond (1999); Krommes and Kim (2000)...

Topology of the drifton phase space in the GO regime $q^2 \lesssim 1 + k_y^2$

- We are mostly interested in $q^2 \lesssim 1 + k_y^2$, where geometrical optics (GO) works. From the ray eqs, one finds that the drifton phase space (x, k_x) changes topology at

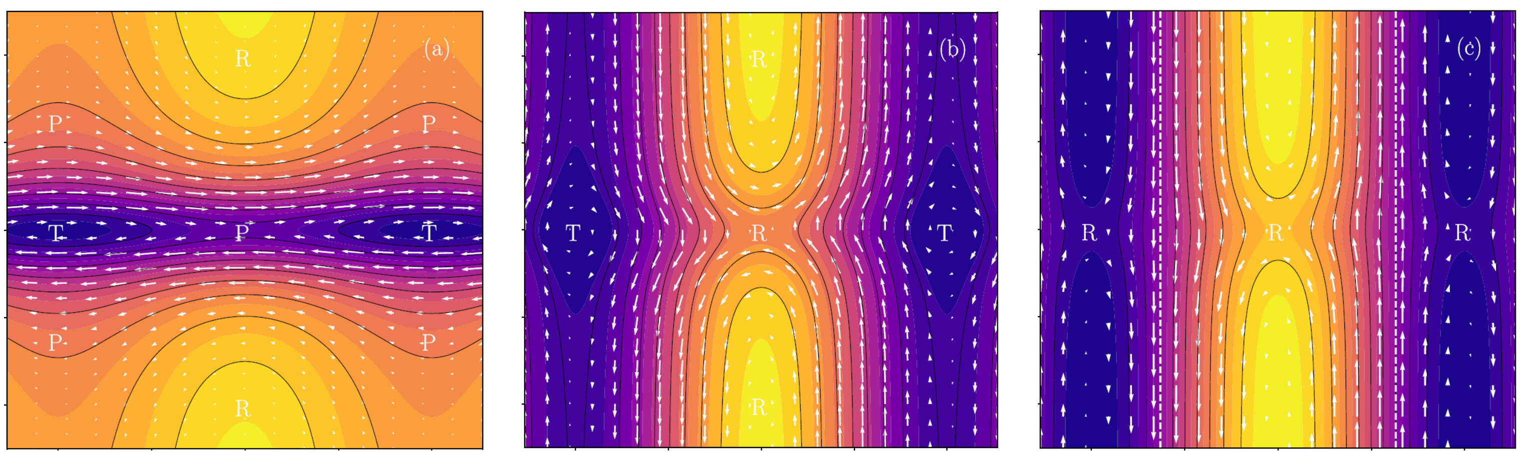


$$U_{c1} = \frac{\beta}{2(1+k_y^2) - q^2}, \quad U_{c2} = \frac{\beta}{q^2} < U_{c1}$$

No passing (P) trajectories at $U > U_{c1}$.

No trapped (T) trajectories at $U > U_{c2}$.

Only runaway (R) trajectories – KHI stabilized

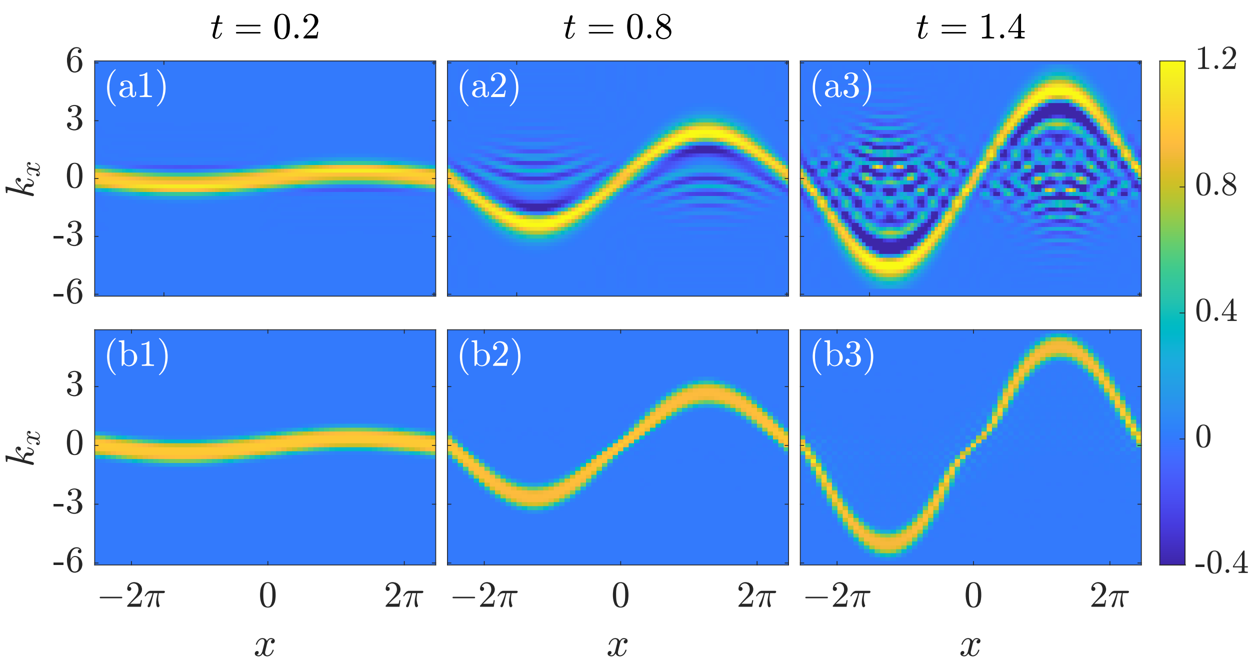


$U < U_{c1}$ $\quad$ $U_{c1} < U < U_{c2}$ $\quad$ $U > U_{c2}$

Onset of ZFs: secondary (modulational, zonostrophic) instability

- ZFs form spontaneously at small amplitudes ($U \ll U_{c1}$) = linear instability of drifton plasma. Its dispersion relation is derived just like for Langmuir waves:*

$$1 - \frac{q^2}{\omega}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^2}\,\frac{1}{\omega - q v_\mathrm{g}}\,\frac{\beta k_x k_y^2}{(1+k_\perp^2)^2}\,\frac{\partial}{\partial k_x}\left[\left(1 - \frac{q^2}{1+k_\perp^2}\right)W_0\right] = 0$$



- Simulations show that ZFs saturate with the same q that corresponds to the maximum growth rate. What is the typical saturation amplitude? Let's derive it!

*For the general expression beyond the GO limit, see Ruiz *et al.* (2016); Zhou *et al.* (2019).

Step 1: equation of state at $U < U_{c2}$, i.e., $|U''| \ll \beta$

- Let us rewrite iWKE in the following form using the group velocity $v_{\mathrm{g}} = \partial \mathcal{H}_{\mathrm{H}}/\partial k_x$:

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial x}(W v_{\mathrm{g}}) = \frac{\partial}{\partial k_x}\left(W \frac{\partial \mathcal{H}_{\mathrm{H}}}{\partial x}\right) + \underbrace{\frac{U'''}{\beta + U''} W v_{\mathrm{g}}}_{\text{negligible}}$$

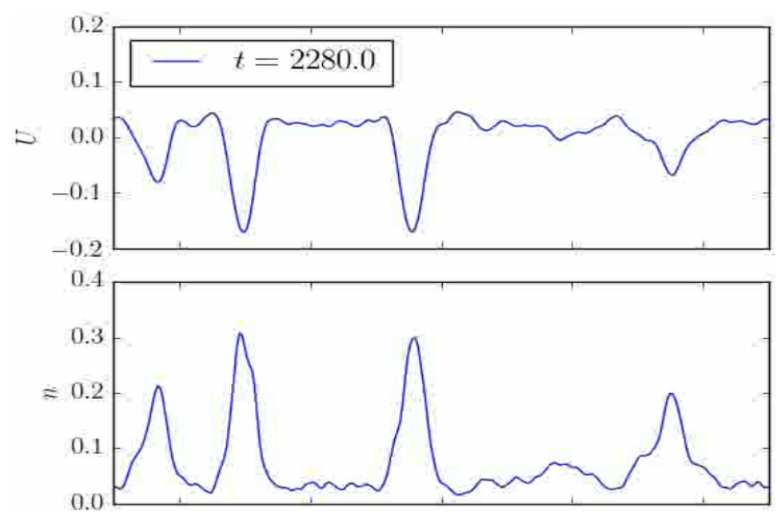
- Integration over $\mathbf{k}$ leads to the continuity equation for the drifton density N:

$$\partial_t N + \partial_x J \approx 0, \qquad N \doteq \int W \, \mathrm{d}\mathbf{k}, \qquad J \doteq \int W v_{\mathrm{g}} \, \mathrm{d}\mathbf{k}$$

- Using $\partial_t N \approx -\partial_x J$, one can express U as a local function of N ("equation of state"):

$$\frac{\partial U}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{J}{2(\beta + U'')}\right] \approx -\frac{\partial_x J}{2\beta} \approx \frac{\partial_t N}{2\beta}$$

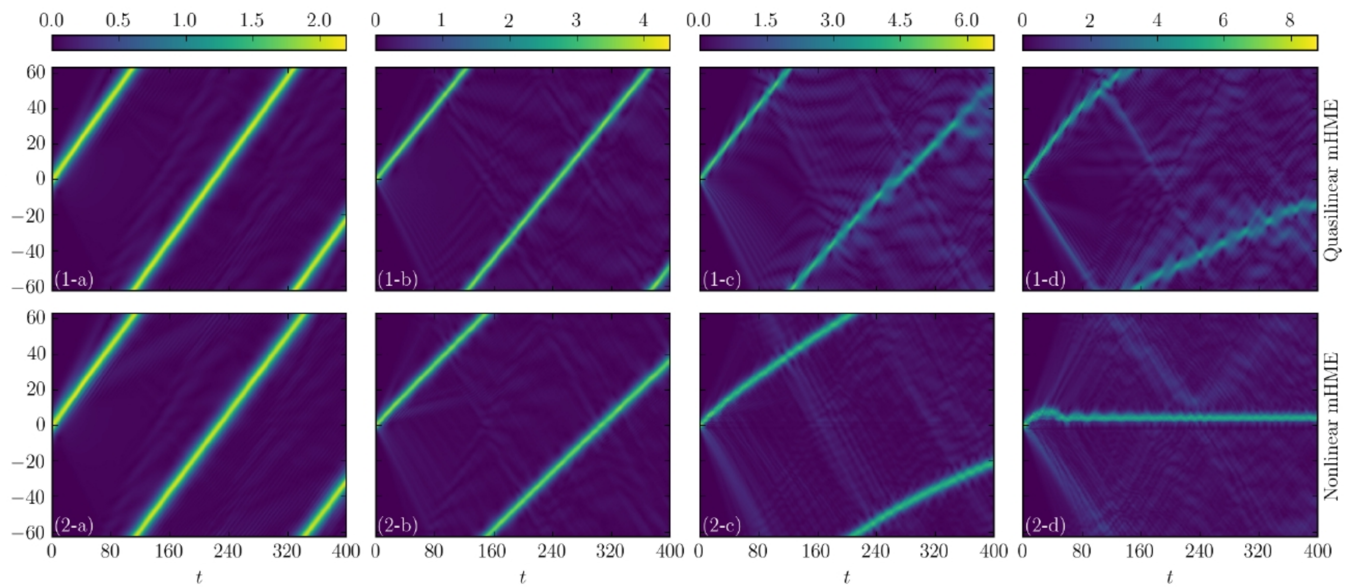
$$U \approx \frac{N}{2\beta} = \frac{\langle \tilde{w}^2 \rangle}{2\beta}$$



Step 2: nonlinear Schrödinger equation (NLSE) and typical q

- Quasimonochromatic DW: $\tilde{w} = e^{i\mathbf{k}\cdot\mathbf{x}}\psi$ and $\boldsymbol{U} \approx \langle|\boldsymbol{\psi}|^2\rangle/4\boldsymbol{\beta}$ → NLSE model:

$$\mathcal{H} \approx \mathcal{H}_0 + \frac{\partial \mathcal{H}}{\partial k_x}\Delta k_x + \frac{\partial^2 \mathcal{H}}{\partial k_x^2}\frac{(\Delta k_x)^2}{2} \quad \Rightarrow \quad i\left(\frac{\partial}{\partial t} + v_{\mathrm{g}}\frac{\partial}{\partial x}\right)\psi \approx -\chi\frac{\partial^2 \psi}{\partial x^2} + k_y \boldsymbol{U}\psi$$



NLSE solitons at small amplitudes, quasistationary ZFs at larger amplitudes

- The linear grow rate is maximized at $q \sim (1 + k_y^2)\sqrt{N}/\beta$. The equation of state says that $N \sim U\beta$. From here, one gets $\boldsymbol{U} \sim \boldsymbol{\beta q^2/(1 + k_y^2)^2}$.

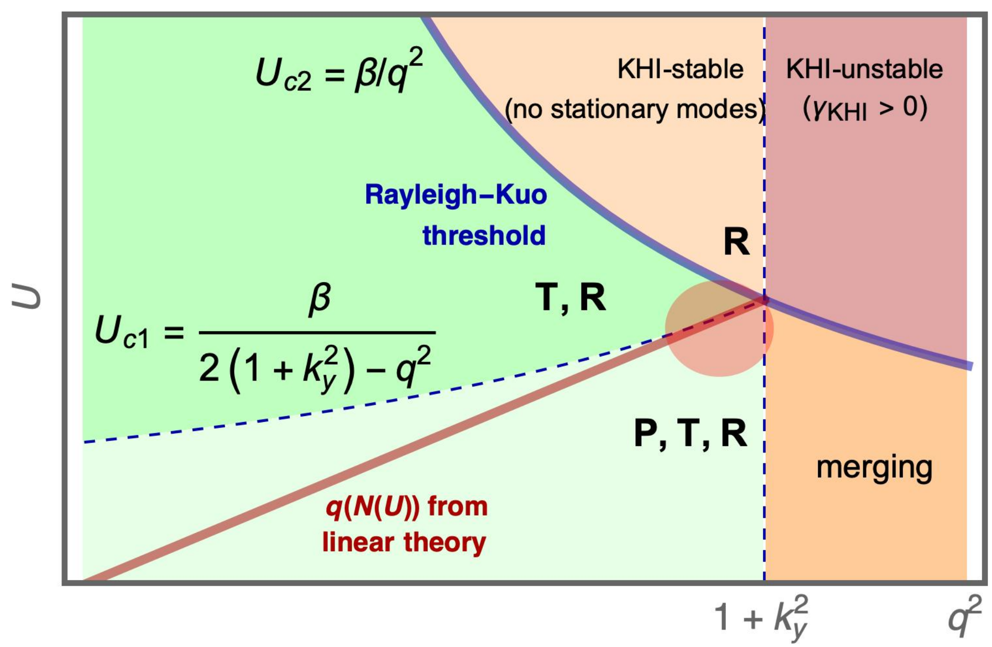
Zhou *et al.* (2019)

Typical parameters of zonal flows at saturation (estimates)

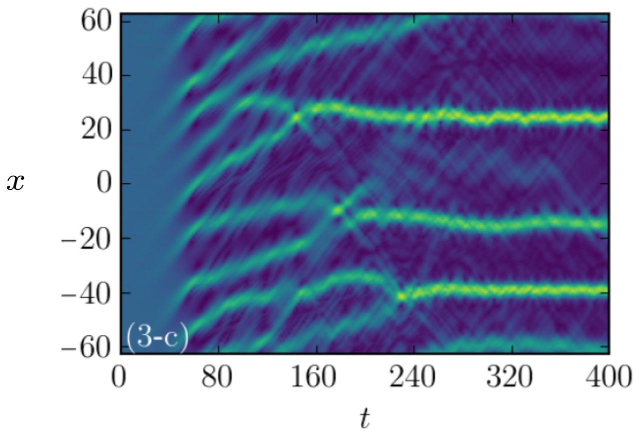
- So, let's summarize: NLSE gives $U \sim \beta q^2/(1+k_y^2)^2$, if $U < U_{c2}$. If more turbulence energy is available, then ZFs approach $U \sim U_{c2}$ and dissipate the rest via the KHI.
- Thus, saturated ZFs typically have $q^2 \sim 1 + k_y^2$ and $U \sim U_{c1}$ at this q:

$$U \sim \beta/(1+k_y^2) \equiv U_*, \qquad q^2 \sim 1 + k_y^2 \equiv q_*^2$$

- In the original units: assuming $k_y \sim \rho_s^{-1}$, one has $U \sim cT/eBL_n$, so $k_y U \sim \omega_*$.



*Last unexplored regime: ZF merging at $U \lesssim U_{c2}$ and $q^2 \gtrsim 1 + k_y^2$

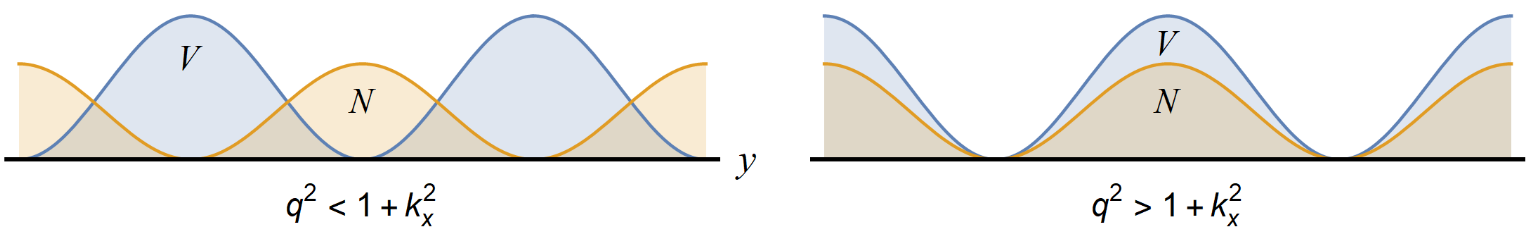


- The iWKE is only marginally applicable to ZF formation but can explain it qualitatively.

$$\mathcal{H} = \frac{k_y(\beta + U'')}{1 + k_x^2 + k_y^2} + k_y U, \quad U \approx \frac{N}{2\beta} + \text{const}$$

$$k_x^2 \ll 1 + k_y^2, \quad q^2 \doteq -U''/U, \quad k_y = \text{const}$$

$$\mathcal{H} \approx C_1\left(\frac{k_x^2}{2m} + V\right) + C_2, \quad m \doteq \frac{(1+k_y^2)^2}{2\beta^2}, \quad V \doteq \left(\frac{q^2}{1+k_y^2} - 1\right)\frac{N}{2}$$



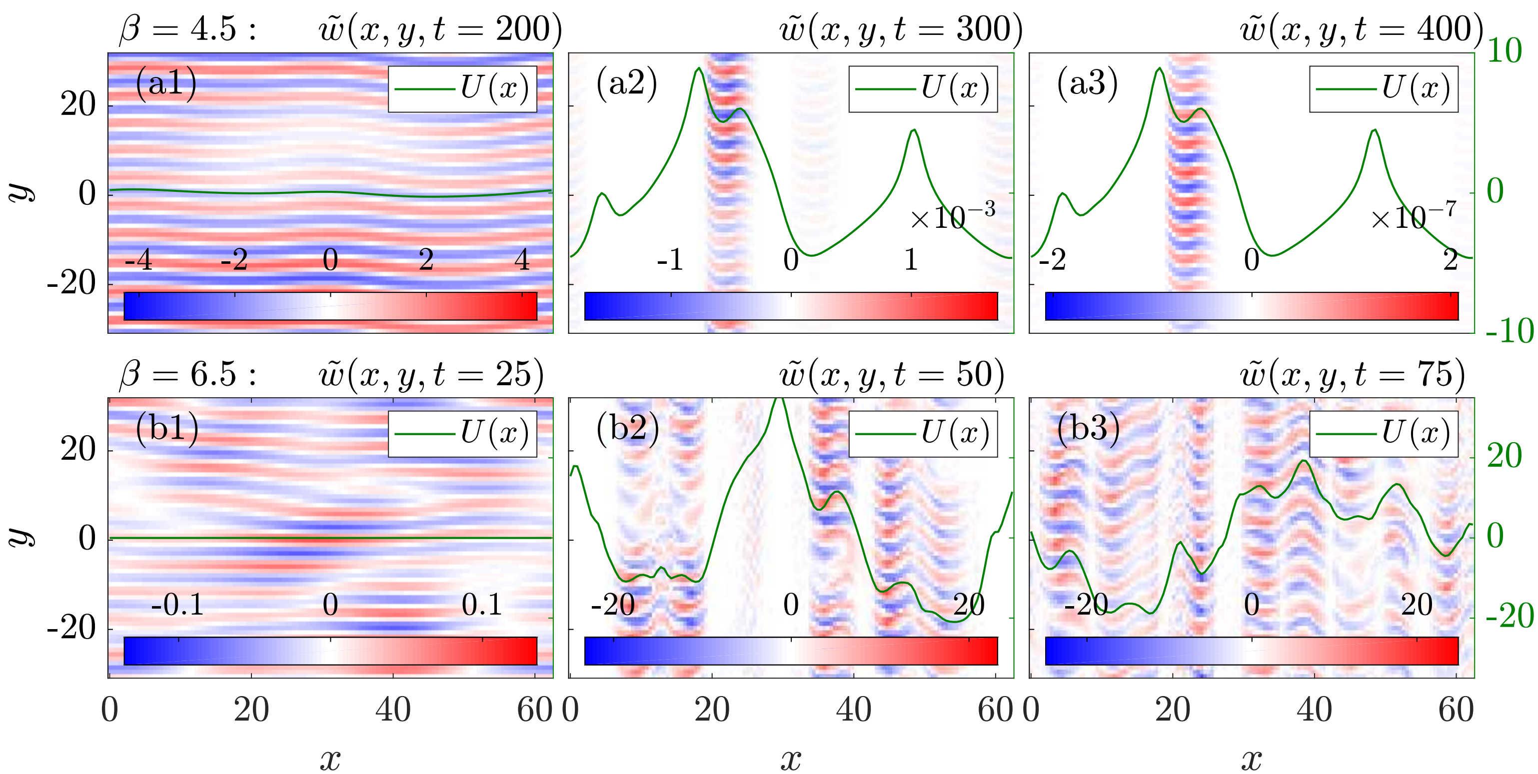
- If $q^2 < 1 + k_y^2$, driftons reside near minima of V, so the system is stable.
- If $q^2 > 1 + k_y^2$, driftons reside near maxima of V. The system can lower the energy by bifurcating to a lower-q state, so it is unstable to ZF merging.

Let's add primary instability & dissipation: the Terry–Horton model

- In the Terry–Horton model, two additional operators are introduced: $\widehat{\delta}$ is responsible for the primary instability, and $\widehat{D}$ models friction and viscosity.

$$\partial_t w + \{\varphi, w\} = \beta \partial_y \varphi - \widehat{\boldsymbol{D}} w$$

$$w = (\nabla_\perp^2 - \widehat{a} + \boldsymbol{i}\widehat{\boldsymbol{\delta}})\varphi, \qquad \widehat{\delta} = \delta(\widehat{k}_y), \qquad \widehat{D} = 1 - \beta \nabla_\perp^2$$

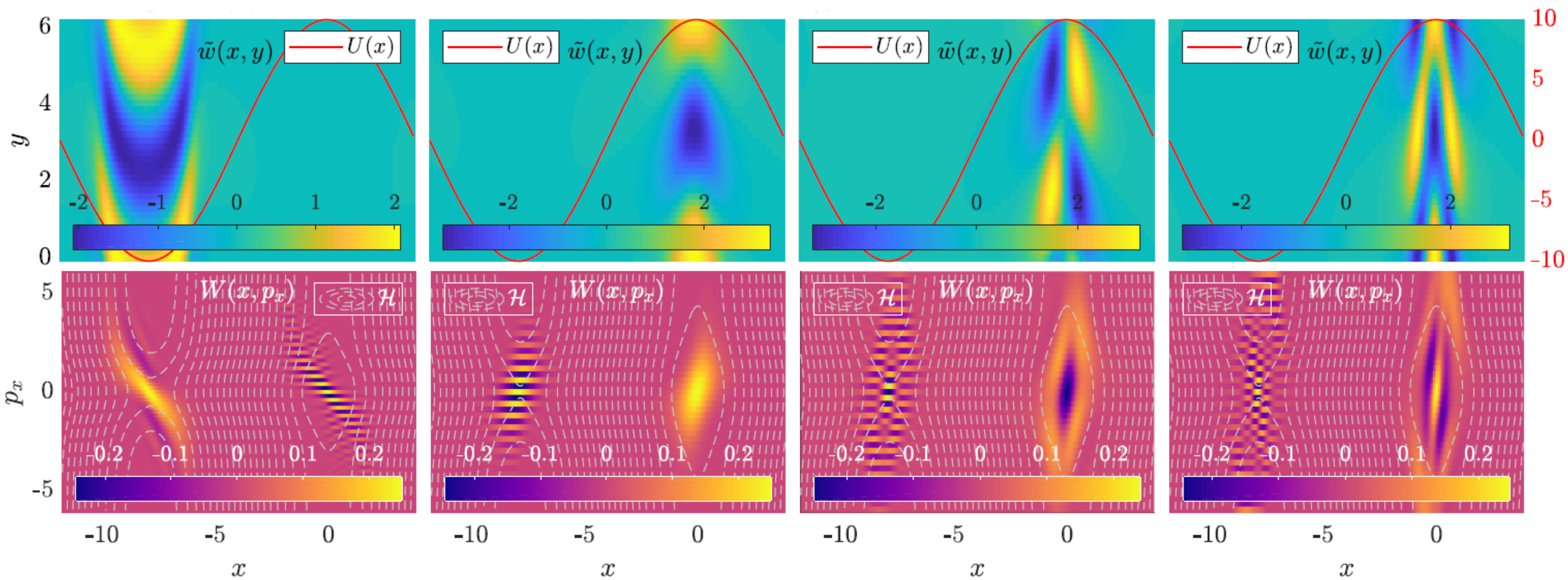


For this modified Terry–Horton model, see St-Onge (2017). Various $\delta(k_y)$ can be used, depending on the physics of interest.

Primary waves in inhomogeneous zonal flows

- The linear primary waves are governed by drifton Schrödinger equation:

$$i\partial_t \tilde{w} = \widehat{\mathcal{H}} \tilde{w}, \qquad \widehat{\mathcal{H}} = k_y \widehat{U} + k_y(\beta + \widehat{U}'')[1 + \widehat{k}_x^2 + k_y^2 - \boldsymbol{i\delta(k_y)}]^{-1} - \boldsymbol{i\widehat{D}}$$



- The lowest-order modes have the largest growth rates. They are localized* in (x, k_x), so the drifton Hamiltonian can be approximated with its Taylor expansion:

$$\partial_t W = \{\{\mathcal{H}_\mathrm{H}, W\}\} + [\![\mathcal{H}_\mathrm{A}, W]\!], \qquad \mathcal{H} \approx c_0 + c_1 x^2 + c_2 k_x^2$$

*DWs tend to be sheared away in (or propagate out from) regions of large velocity shear $|U'|$.

DW modes satisfy the equation of a quantum harmonic oscillator.

- Since $\widehat{\mathcal{H}} \approx c_0 + c_1 \widehat{x}^2 + c_2 \widehat{k}_x^2$, a DW is just a *quantum harmonic oscillator* with complex coefficients and the spectrum that satisfies $\boldsymbol{\varepsilon_n = (2n+1)\vartheta}$:

$$\left(-\vartheta^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} + x^2\right)\tilde{w} = \varepsilon \tilde{w}, \qquad \tilde{w}_n \sim H_n\left(\frac{x}{\sqrt{\vartheta}}\right) e^{-x^2/2\vartheta}$$

$$\vartheta \doteq -\frac{i\sqrt{2(1+\beta/U_0'')}}{1+k_y^2 - i\delta}, \qquad \varepsilon \doteq \frac{2}{k_y U_0''}\left[\omega_{\mathrm{TI}} - k_y U_0 + iD_0 - \frac{k_y(\beta + U_0'')}{1+k_y^2 - i\delta}\right]$$

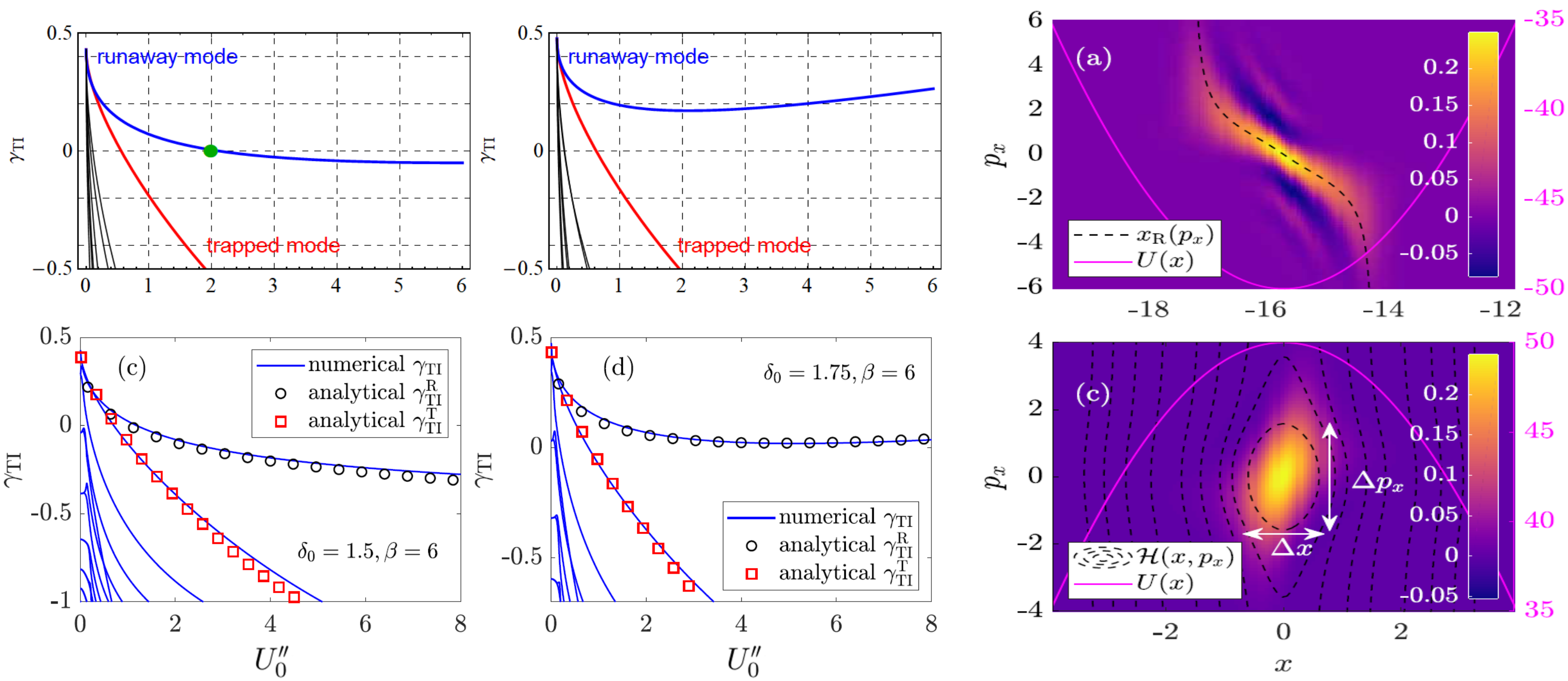
- Using U and $q^2 \doteq -U''/U$ from our results for the Hasegawa–Mima model, one can calculate the growth rate explicitly. The predicted rate agrees with simulations.

$$\gamma_{\mathrm{TI}} = -D_0 + \mathrm{Im}\left[\frac{k_y(\beta + \boldsymbol{U_0''}) - ik_y \boldsymbol{U_0''}\sqrt{(1+\beta/\boldsymbol{U_0''})/2}}{1+k_y^2 - i\delta}\right] \equiv \gamma_{\mathrm{primary}}^{(\mathrm{linear})} + \Delta\gamma(\boldsymbol{U_0''})$$

- In summary, DW are localized near extrema of the zonal velocity U. Trapped modes have $\gamma = \gamma_0 + \Delta\gamma(U'')$, so U'' can affect primary instabilities.

The predicted rates agree with linear simulations with prescribed U''.

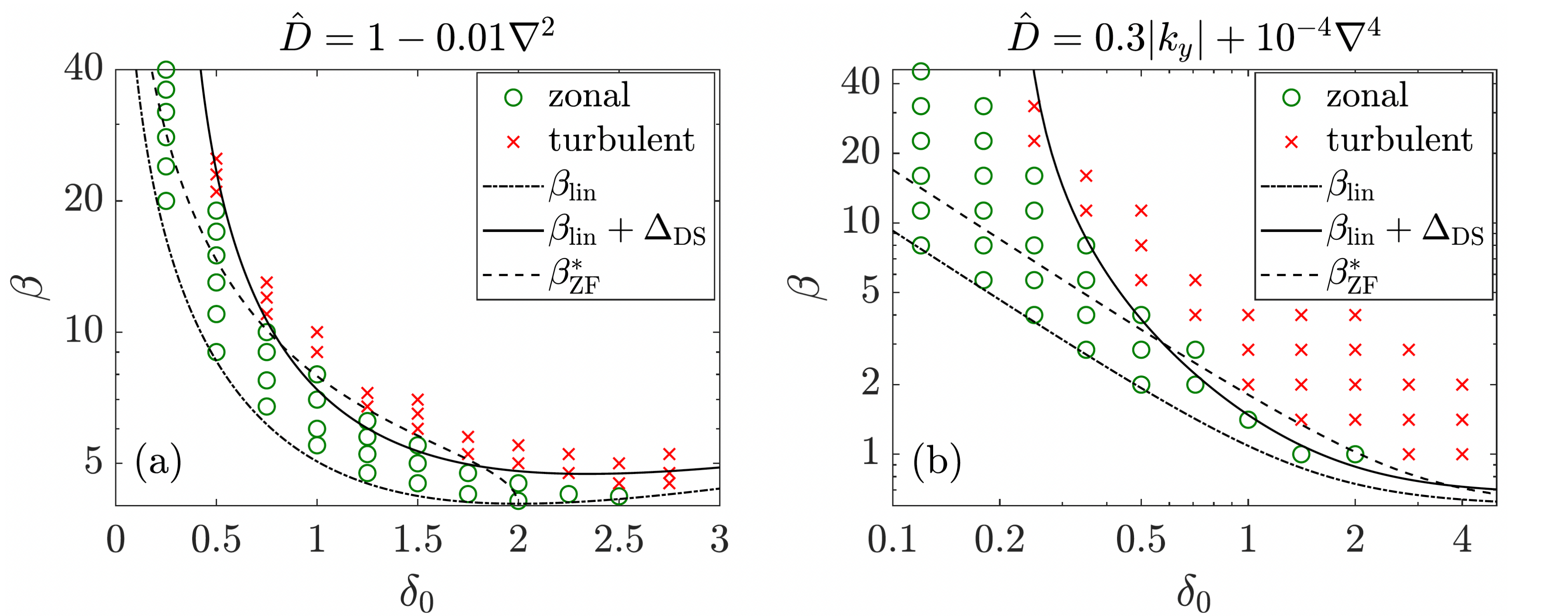
$$\gamma_{\mathrm{TI}} = -D_0 + \mathrm{Im}\left[\frac{k_y(\beta + U_0'') - ik_y U_0''\sqrt{(1+\beta/U_0'')/2}}{1 + k_y^2 - i\delta}\right] \equiv \gamma_{\mathrm{primary}}^{(\mathrm{linear})} + \Delta\gamma(U_0'')$$



- The tertiary instability can be viewed as the primary instability modified by ZFs.
  - If $\gamma_{\mathrm{TI}} < 0$, turbulence is suppressed; ZFs survive, assuming $\hat{D}$ acts only on DWs.
  - If $\gamma_{\mathrm{TI}} > 0$, the system ends up in a turbulent state. **$\Delta\gamma$ is the Dimits shift!**

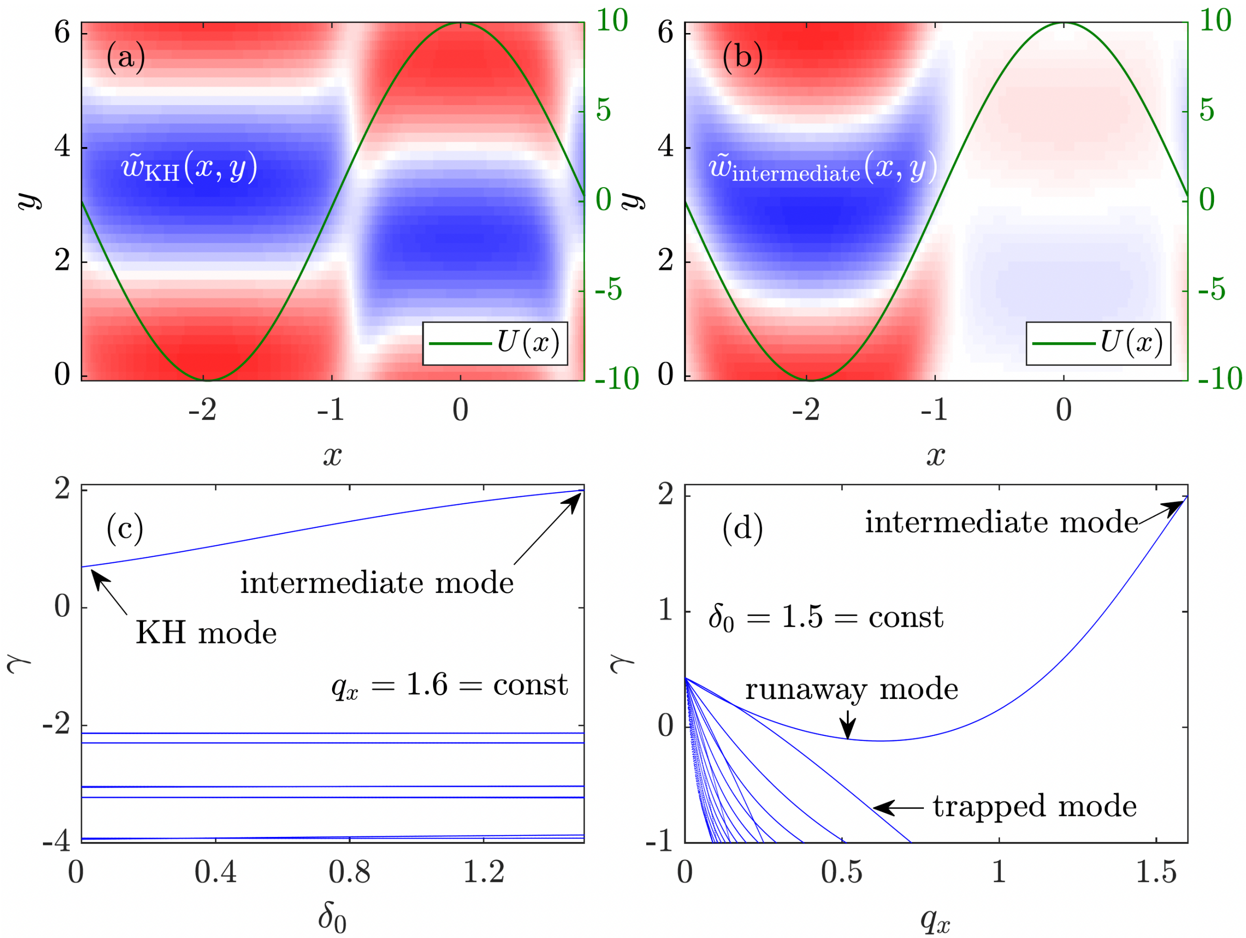
Using our estimate for U'', we are also match nonlinear simulations.

- We calculate the values of β that correspond to $\gamma^{(\text{linear})}_{\text{primary}} = 0$ and $\gamma_{\text{TI}} = 0$ using $U_0'' \sim q_*^2 U_*$. The difference between these values is the **Dimits shift (green)**.
- Compared with related results from St-Onge (2017), denoted β^*_{ZF}, our model is a better fit at both large and small δ. (We assume $\widehat{\delta} = \delta_0 \widehat{k}_y$.)



Relation to the Kelvin–Helmholtz instability

- The KHI is subsumed under the main equations, but it is a different instability:
  - KHI: delocalized modes, destabilized by U'', does not rely on dissipation
  - dissipative TI: localized modes, stabilized by U'', relies on dissipation



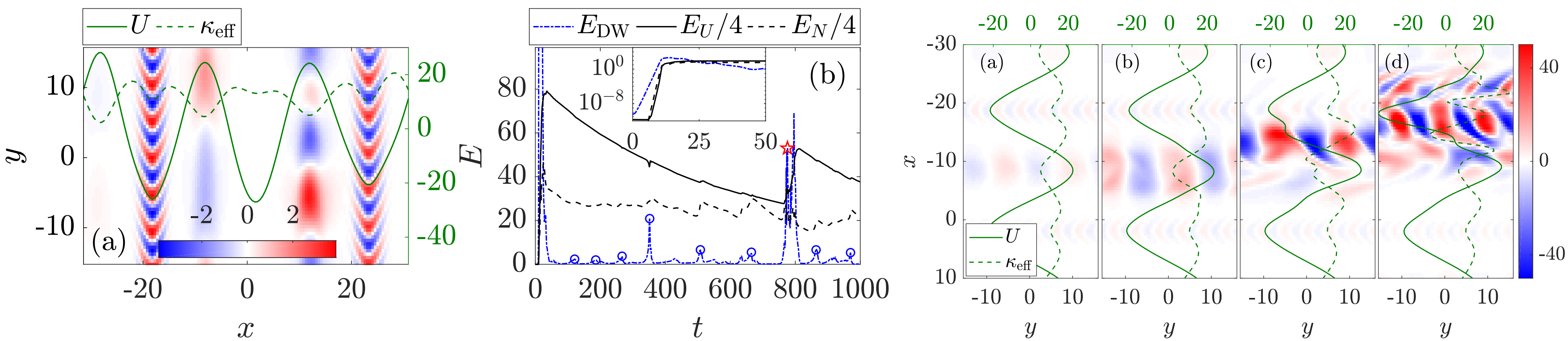
Beyond the adiabatic approximation: Hasegawa–Wakatani model

- Next level of complexity: give up the assumption of adiabatic $n(\varphi)$, treat n as an independent field. For example, Hasegawa–Wakatani model, with $w = \nabla^2\varphi - n$:

$$\partial_t w + \{\varphi, w\} = \beta\partial_y\varphi - \hat{D}w, \qquad \partial_t n + \{\varphi, n\} = \alpha(\tilde{\varphi} - \tilde{n}) - \beta\partial_y\varphi - \hat{D}n$$

- The physics mostly remains the same, but ZF can dissipate. This leads to predator–prey oscillations (PPO).

- Also, since DWs can exchange energy with U and with N, there are two types of PPO, and analytic predictions are more difficult.*



Zhu *et al.* (2020)a

* The secondary instability changes too, cf. Ivanov *et al.* (2020).

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Summary

- Important things missed in earlier studies:
  - Heuristic arguments are not enough, WKE must be derived from first principles.
  - $\boldsymbol{\lambda/L}$ and $\boldsymbol{U''/\beta}$ are not negligible, one must look beyond geometrical optics.
- Scalings for processes that are not directly determined by primary instabilities (PI) and dissipation can be understood from the Hasegawa–Mima model.
- Adding dissipation introduces a new tertiary instability (basically, a modified PI) that is more relevant than the commonly known Kelvin–Helmholtz instability.
- Dimits shift:
  - Dissipation localizes the tertiary modes near the ZF-velocity extrema.
  - Their growth rate can be made negative by U'', leading to the Dimits shift.
  - An analytic theory is developed within the Terry–Horton model.
  - Two-fluid models exhibit additional effects, but the qualitative physics is similar.

*Weyl symbols for dynamo theory without scale separation

- A similar formalism applied to MHD leads to a revised theory of plasma dynamo:

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = (\mathbf{b}\cdot\nabla)\mathbf{b} - \nabla P + \nu\nabla^2\mathbf{v}$$

$$\partial_t \mathbf{b} = \nabla\times(\mathbf{v}\times\mathbf{b}) + \eta\nabla^2\mathbf{b}$$

$$\nabla\cdot\mathbf{v} = 0, \quad \nabla\cdot\mathbf{b} = 0$$

$$i\partial_t \mathbf{W} = \mathbf{H}\star\mathbf{W} - \mathbf{W}\star\mathbf{H}^\dagger - i\tau_c^{-1}\mathbf{W} + \mathbf{T}$$

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}^{++} & \mathbf{H}^{+-} \\ \mathbf{H}^{-+} & \mathbf{H}^{--} \end{pmatrix}$$

$$H_{ij}^{\pm\pm} = \delta_{ij}\left(\bar{z}_l^{\mp}\star k_l - i\nu_+ k^2\right) + i\frac{k_i}{k^2}\star\bar{z}_{l,j}^{\mp}\star k_l$$

$$H_{ij}^{\pm\mp} = -\delta_{ij} i\nu_- k^2 - i\bar{z}_{i,j}^{\pm} + i\frac{k_i}{k^2}\star\bar{z}_{l,j}^{\pm}\star k_l$$

- EMF is a Hodge star of the integrated Elsässer Wigner matrix, $\boldsymbol{\mathcal{E}} = \star\int \mathbf{W}^{-+}\,\mathrm{d}\mathbf{k}$.
- Mean-field equations for $\bar{\mathbf{w}}^{\pm} = \nabla\times(\bar{\mathbf{v}}\pm\bar{\mathbf{b}})$ yield the nonlocal EMF from first principles, subsume known dynamo mechanisms a new one caused by $\langle\tilde{\mathbf{v}}\cdot\tilde{\mathbf{j}}\rangle$.

$$\partial_t\bar{\mathbf{w}}^{\pm} = -\widehat{\mathbf{k}}\times\left\{\left[\left(\widehat{\mathbf{k}}\widehat{k}^{-2}\times\bar{\mathbf{w}}^{\mp}\right)\cdot\widehat{\mathbf{k}}\right]\left(\widehat{\mathbf{k}}\widehat{k}^{-2}\times\bar{\mathbf{w}}^{\pm}\right)\right\} - \widehat{k}^2(\nu_+\bar{\mathbf{w}}^{\pm} + \nu_-\bar{\mathbf{w}}^{\mp}) + \mathbf{S}^{\pm}$$

$$S_i^{\pm} = \epsilon_{ijk}\int\frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\left(k_l k_j\star W_{kl}^{\pm\mp} - k_l\star W_{kl}^{\pm\mp}\star k_j\right)$$

*Weyl symbols for quasilinear theory of wave–particle interactions

- Quasilinear theory of wave–particle interactions: write the QL term in the operator form through the Green's operator $\widehat{G}$ of LVE, then Weyl-expand this operator.
- This leads to a fully conservative equation for the "oscillation-center" distribution $F \doteq \bar{f} + \partial_{\mathbf{p}} \cdot (\Theta\, \partial_{\mathbf{p}} \bar{f})$ captures both QL diffusion *and* ponderomotive forces:

$$\partial_t f = \{\bar{H} + \tilde{H}, f\}$$

$$\tilde{f} = \widehat{G}\{\tilde{H}, \bar{f}\}$$

$$\boxed{\partial_t \bar{f} - \{\bar{H}, \bar{f}\} = \partial_\alpha(\widehat{D}^{\alpha\beta}\partial_\beta \bar{f})}$$

$$\widehat{D}^{\alpha\beta} = \langle \widehat{u}^\alpha \widehat{G} \widehat{u}^\beta \rangle, \quad u^\alpha = J^{\alpha\beta}\partial_\beta \tilde{H}$$

$$\frac{\partial F}{\partial t} = \{\bar{H} + \Phi, F\} + \frac{\partial}{\partial \mathbf{p}} \cdot \left(\mathsf{D}\, \frac{\partial F}{\partial \mathbf{p}}\right)$$

$$\boldsymbol{\Theta} = \frac{\partial}{\partial \vartheta} \fint \mathrm{d}\omega\, \mathrm{d}\mathbf{k}\, \frac{\mathbf{k}\mathbf{k}^\dagger\, \bar{W}_{\tilde{H}}}{2(\omega - \mathbf{k}\cdot\mathbf{v} + \vartheta)}\bigg|_{\vartheta=0}$$

$$\Phi = \frac{\partial}{\partial \mathbf{p}} \cdot \fint \mathrm{d}\omega\, \mathrm{d}\mathbf{k}\, \frac{\mathbf{k}\bar{W}_{\tilde{H}}}{2(\omega - \mathbf{k}\cdot\mathbf{v})}$$

$$\mathsf{D} = \pi \int \mathrm{d}\mathbf{k}\; \mathbf{k}\mathbf{k}^\dagger\, \bar{W}_{\tilde{H}}(t, \mathbf{x}, \mathbf{k}\cdot\mathbf{v}, \mathbf{k}; \mathbf{p})$$

- Conserves nonresonant-wave action. Subsumes many results previously derived ad hoc, including fluctuation theory and Balescu–Lenard collisions (not shown).

Dodin (2022); Dodin (2024)

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