Supplementary Material to
$t-$statistic based correlation and heterogeneity robust inference
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We report here two additional tables on the Monte Carlo performance of the t-statistic approach

- for inference about the location parameter of time dependent symmetric stable random variables;
- for inference about a regression coefficient in a panel with both time series and cross sectional dependence.

Specifically, Table A-1 reports the small sample performance of the subsampling and $t-$statistic approach in the Monte Carlo design considered in McElroy and Politis (2002). As McElroy and Politis (2002), we find that the size control of the subsampling approach strongly depends on the choice of the subsample length b, on which there is little theoretical guidance. In contrast, the $t-$statistic based tests are mostly moderately undersized, with pronounced distortions only in the case of the MA process and a sample size of $T = 100$. We report non-size adjusted power because the $t-$statistic approach has asymptotic rejection probability under the null hypothesis below the nominal level in this data generating process, with potentially adverse effects on power. For the considered sample sizes, however, the $t-$statistic approach is typically more powerful than subsampling when both control size, at least for $q \geq 8$.

Table A-2 compares $t-$statistic approach to inference with clustering and Fama-MacBeth standard errors for two designs considered by Thompson (2006) in a panel with $N = 50$ and $T = 25$. Both data generating processes generate some dependence in both dimensions. As noted by Thompson (2006), an approach that clusters in both dimensions (also see Cameron, Gelbach, and Miller (2006)) has poor small sample properties for these values of N and T, even in absence of any time series correlation ($\rho = 0$). In contrast, the $t-$statistic approach has reasonable size control as long as the time series dependence is not extreme ($\rho = 0.9$), and has favorable size control properties compared to parametric or non-parametric corrections to the Fama-MacBeth approach. Unreported results show that this remains true also under the inclusion of additional fixed effects in either or both dimensions. As can be seen from Table A-2, these advantages in size control of the $t-$statistic approach come at a certain cost in size adjusted power, though, especially for q small. The higher power of the Fama-MacBeth approaches when ρ is small stems from the inherent time fixed effect in that estimator; the other approaches have similar size adjusted power when time fixed effects are included (see Petersen (2009) for similar results on efficiency of alternative estimators).

Table A-1: Small Sample Results in a Time Series Location Model with Symmetric α-Stable Disturbances

				$t-$statistic (q)				subsampled $t-$statistic (b)			
				2	4	8	16	2	4	8	16
DGP	α	T	β	Size							
MA	1.2	100	0	3.8	3.4	4.7	9.8	0.0	0.5	3.4	18.7
MA	1.2	1000	0	3.6	3.1	2.9	3.1	2.0	3.3	7.1	16.5
MA	1.8	100	0	4.9	5.0	6.7	12.1	0.0	0.1	1.6	9.4
MA	1.8	1000	0	4.9	4.7	5.0	5.0	0.1	0.1	0.2	0.9
AR	1.2	100	0	3.9	3.2	3.7	5.4	0.2	7.0	18.6	32.6
AR	1.2	1000	0	4.1	3.0	2.5	3.0	2.9	10.5	20.5	26.3
AR	1.8	100	0	4.7	4.4	5.3	6.9	0.0	1.1	6.5	14.9
AR	1.8	1000	0	4.7	4.6	4.8	4.8	0.1	0.3	1.9	4.8
DGP	α	T	β	Non-Size Adjusted Power							
MA	1.2	100	1.0	10.4	30.1	45.3	56.4	0.6	10.5	28.6	49.4
MA	1.2	1000	1.0	14.2	45.7	58.9	63.8	2.5	19.0	44.1	59.1
MA	1.8	100	0.4	14.2	38.4	58.9	73.5	0.0	2.9	20.6	49.2
MA	1.8	1000	0.2	19.0	56.8	76.0	82.0	0.1	0.1	15.8	51.1
AR	1.2	100	2.0	10.8	29.2	42.5	50.5	3.5	36.2	51.3	60.2
AR	1.2	1000	2.0	14.9	45.0	58.3	63.1	5.2	52.3	65.7	69.5
AR	1.8	100	0.8	14.2	37.7	55.4	65.6	0.1	24.1	51.2	66.4
AR	1.8	1000	0.4	18.7	57.2	75.6	81.3	0.1	34.6	70.0	78.6

Notes: Rejection probabilities of nominal 5% level two-sided tests about β in the model $y_t = \beta + u_t$, $t = 1, \cdots, T$, where $u_t = 0.5u_{t-1} + S_t$ and $u_0 = 0$ (AR) and $u_t = \sum_{j=0}^{10} \psi_j S_{t-j}$ with $\{\psi_j\}_{j=0}^{10} = \{.03, .05, .07, .1, .15, .2, .15, .1, .07, .05, .03\}$ (MA), and S_t are i.i.d. mean zero α-symmetric stable distributed. The subsampled $t-$statistic rejects if the full sample OLS $t-$statistic falls outside the 2.5% and 97.5% quantiles of the empirical distribution function of OLS $t-$statistics computed on all $T - b + 1$ consecutive subsamples of length b, as described in detail in McElroy and Politis (2002). Based on 10,000 replications.

Table A-2: Small Sample Results in a Panel with $N = 50$, $T = 25$ and Correlation in Both Dimensions

	Individual Persistence				Common Persistence			
ρ	0	0.5	0.7	0.9	0	0.5	0.7	0.9
				Size				
$t-$statistic $q = 2$	4.9	5.3	5.0	6.0	4.9	5.1	5.3	6.3
$t-$statistic $q = 4$	4.9	5.2	5.4	9.8	4.1	5.0	5.3	10.4
$t-$statistic $q = 8$	4.6	5.3	6.4	17.1	3.9	4.9	7.1	16.8
Fama-MacBeth with Newey-West	12.6	13.6	19.8	34.8	11.4	12.3	14.2	23.4
Fama-MacBeth with AR(1) corr.	9.6	9.9	13.5	22.5	8.8	9.1	10.4	18.0
cluster by i and t	9.3	8.9	8.8	7.0	10.2	19.0	29.9	49.5
cluster by i and t + common pers.	16.3	16.2	14.9	12.1	17.0	21.3	26.4	38.3
				Size Adjusted Power				
$(\beta - \beta_0)/\sqrt{nT}$	7	7	7	7	25	30	30	45
$t-$statistic $q = 2$	12.9	13.0	16.2	14.9	20.3	20.9	18.1	20.8
$t-$statistic $q = 4$	30.5	35.2	45.5	45.3	58.4	63.5	58.4	60.6
$t-$statistic $q = 8$	50.9	57.7	67.6	61.3	59.5	73.2	67.3	68.2
Fama-MacBeth with Newey-West	100	99.5	91.6	58.9	57.3	58.4	47.4	47.1
Fama-MacBeth with AR(1) corr.	100	99.2	88.8	51.4	57.6	60.2	46.7	44.8
cluster by i and t	46.8	55.4	67.6	74.8	86.3	83.4	66.2	70.2
cluster by i and t + common pers.	31.7	39.4	52.7	69.8	69.6	70.0	53.6	60.1

Notes: The entries are rejection probabilities of nominal 5% level two-sided $t-$tests about the coefficient β of $x_{i,t}$ in the linear regression $y_{i,t} = \mathbf{X}'_{i,t}\boldsymbol{\theta} + u_{i,t}$, $i = 1, \cdots, N$, $t = 1, \cdots, T$, where $\mathbf{X}_{i,t} = (x_{i,t}, 1)'$. The DGPs correspond to Panels B and C of Thompson (2006), where under "Individual Persistence", $u_{i,t} = \xi_t + \eta_{i,t}$, $\eta_{i,t} = \rho\eta_{i,t-1} + \varepsilon_{i,t}$, $\eta_{i,0} = 0$, ξ_t and $\varepsilon_{i,t}$ are mutually independent and distributed i.i.d. $\mathcal{N}(0, 1)$, and under "Common Persistence" $u_{i,t} = h_i f_t + \varepsilon_{i,t}$, $f_t = \rho f_{t-1} + \xi_t$, $f_0 = 0$, and the disturbances are mutually independent and $\varepsilon_{i,t} \sim i.i.d.\ \mathcal{N}(0, 0.01)$, $h_i \sim i.i.d.\ \mathcal{N}(1, 0.25)$, $\xi_t \sim i.i.d.\ \mathcal{N}(0, 1)$. In both cases, $x_{i,t}$ is an independent draw of the same distribution as $u_{i,t}$ (with the same h_i under common persistence). The considered tests are: the $t-$statistic approach with groups $\mathcal{G}_j = \{(i, t) : (j-1)T/q < t \leq jT/q\}$; Fama-MacBeth standard errors with a Newey West correction with 5 lags; Fama-MacBeth standard errors multiplied by $\sqrt{(1 + \hat{\rho})/(1 - \hat{\rho})}$, where $\hat{\rho}$ is the first order autocorrelation coefficient of $\hat{\beta}_j$, $j = 1, \cdots, T$ (see Fama and French (2002)); and inference based on clustering in both dimensions as suggested in Thompson (2006), where in the "+ common pers."-row, the clustering allows for a persistence common shock with lag length 2. For all approaches other than the $t-$statistic, critical values from a standard normal were employed. Based on 10,000 replications.

References

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